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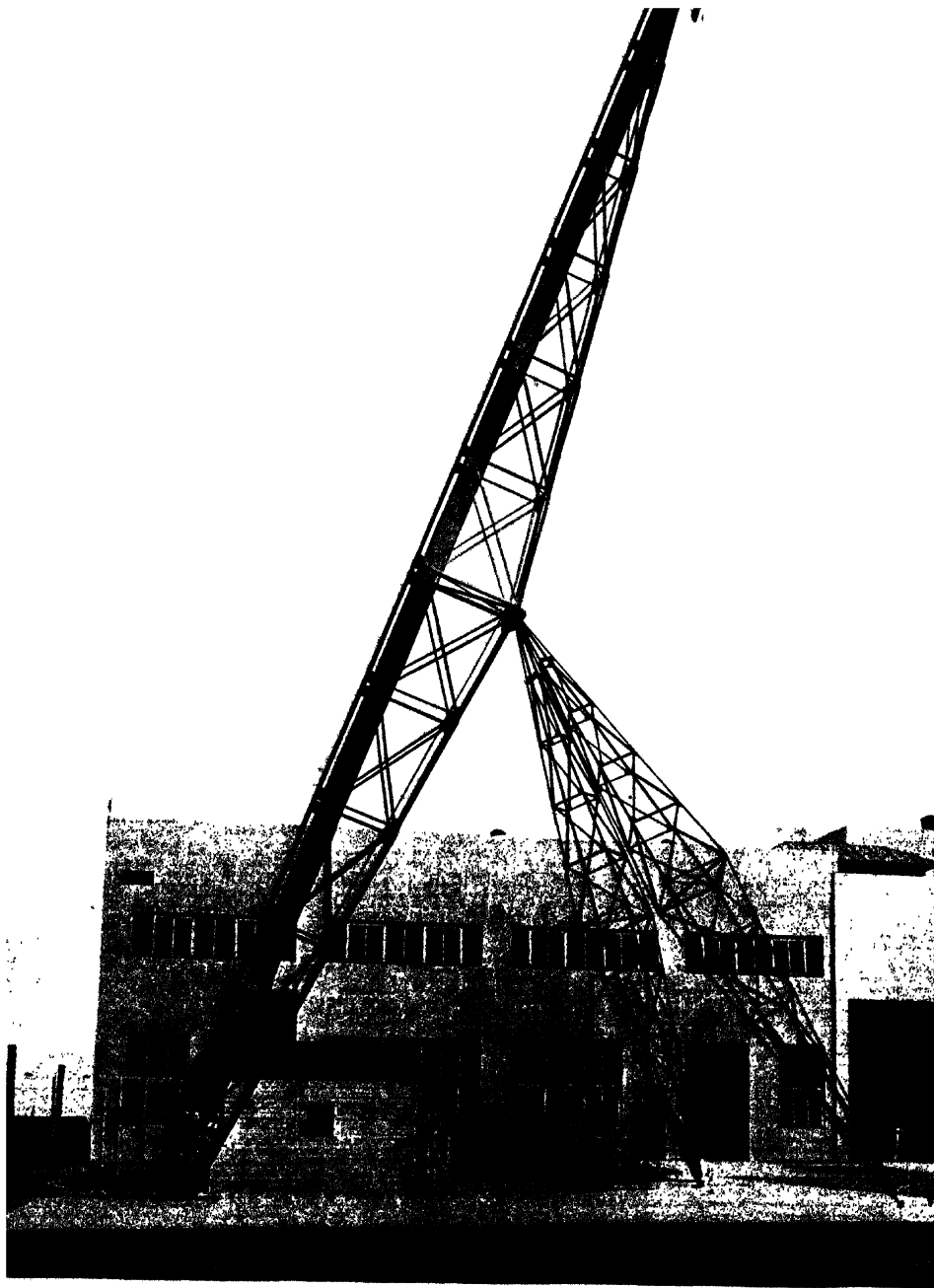
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ENGINEERING SCIENCE SERIES

THEORY OF MODERN STEEL STRUCTURES



Courtesy J. E. Sullivan

STRUCTURAL DESIGN EXTENDS BEYOND BRIDGES AND BUILDINGS
Seat Ejection Tower, Navy Department, Bureau of Aeronautics

THEORY OF MODERN STEEL STRUCTURES

VOLUME I

Statically Determinate Structures

REVISED EDITION

BY

LINTON E. GRINTER, Ph.D., C.E.

DEAN, THE GRADUATE SCHOOL
THE UNIVERSITY OF FLORIDA

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Dedicated to those great teachers of engineering who have so often seemed to reflect the following tenet of James Watt

*Of all things, but proverbially so in mechanics,
the supreme excellence is simplicity.*

PREFACE

The first volume of the Theory of Modern Steel Structures has served widely as a textbook for undergraduate courses in the analysis of statically determinate structures. This revision carries forward the concept of the first edition that the student can be interested in the study of the basic problems of stress analysis by relating them to specific types of structures such as industrial and commercial buildings, highway bridges and railway bridges. The analysis of trusses divorced from the structure itself is mechanics rather than structural engineering.

A small amount of design information concerning loadings and the planning of the structure is integrated into each chapter in order to relate the subject of analysis to structural engineering. If such data had been collected in a single chapter, this purpose would have been defeated. Again, as an aid to integration, graphical methods are not segregated into a chapter of graphic statics but are presented in close parallel with the corresponding algebraic methods. This arrangement effectively combats the tendency of the student to file such seemingly diverse methods in unconnected compartments of the mind. Even when teaching graphical analysis in a drafting room period the instructor will find this arrangement helpful.

Two plans of course organization are provided for by the revised subdivision of the text into eleven chapters. If the students have already studied the theory of beams, or if this study will be completed in a parallel course during the first six weeks of the semester, the chapter order needs no revision. However, if the students' study of beams in strength of materials comes somewhat later in the semester, it may be desirable to delay the assignment of Chapter 4, Industrial Building Bents, until the completion of the analysis of bridge trusses in Chapters 5, 6 and 7. Then the study of Industrial Building Bents, Chapter 4, will be followed immediately by Chapter 8 on Lateral and Portal Bracing of Bridges, which again provides an integrated arrangement.

There is considerable variation between institutions in the use of the material represented by Chapter 9, The Plate Girder, Chapter 10, Office and Commercial Buildings, and Chapter 11, Wind Stresses in Tier Buildings. This subdivision into individual chapters permits the inclusion, omission or transfer of such material as desired. Evidently, many institutions will prefer to treat the plate girder in a later course in design. Chapter 10 on office and commercial buildings can also be studied in a design course if preferred. The

problem of wind stresses in tier buildings, Chapter 11, is often reserved for a more specialized course in analysis. It forms a useful intermediate study between statically determinate and statically indeterminate stresses if time for its consideration is available.

In addition to the reorganization into eleven chapters the second edition has been revised in an attempt to clarify each statement. The value of digging for information that can be obtained more easily seems doubtful. The first chapter on the history of structural engineering was retained at the request of all reviewers. Similar unanimity led to considerable extension of the second chapter on the basic theories of stress analysis. Modern practices and procedures are reflected throughout the book. As one example, attention is given to flat roofed buildings in agreement with modern architectural practice.

Many individuals have contributed to the growth of this text. Among those engineers who have given detailed criticisms and suggestions are: Henry B. Alford, G. A. Beebe, H. B. Blodgett, J. H. Cissel, H. N. Cummings, D. A. du Plantier, Fred J. Evans, W. S. Evans, E. I. Fiesenheiser, A. T. Granger, James R. Griffith, Robert M. Holcomb, J. W. Hubler, Robert L. Lewis, Theodore Mavis, A. L. Miller, N. D. Morgan Jr., Lewis L. Palmer, Fred L. Plummer, J. J. Richey, Frank Randall, C. E. Sandstedt, C. R. Sandifer, R. L. Sanks, W. L. Shilts, F. C. Snow, Edward J. Squire, T. H. Terrell, I. A. Trively, J. B. Wells and D. M. Wilson. Others, including students, have contributed by pointing out errors and omissions. May this acknowledgment encourage others to suggest further improvements.

L. E. GRINTER

CHICAGO, ILLINOIS

FOREWORD FOR THE UNDERGRADUATE ENGINEER

As the young engineer passes from the sub-professional to the professional stage, the habits that he has formed in his first engineering studies go with him and determine his success or failure. One impression that your superior will form of your qualifications concerns your ability to produce neat, legible, and accurate calculations. The computation sheet that follows is typical of the calculations of professional engineers. All practicing engineers keep notebooks. Such notes pass back and forth between the engineer and his associates, including his superior. Hence you cannot afford to produce notes that are inaccurate, messy, or illegible.

Form. In this textbook you will find "pounds per square inch" written as *lb. per sq. in.* Trade papers and technical journals often accept *lb sq in.* or *p s i* as good form in their publications. But the designing engineer, always in a hurry, is more likely to write lb/in^2 , lb/sq" , $\text{\#/sq in.}$, $\text{\#/\square"}$, $\text{\#/in}^2$, $\text{\#in}^2$, or $\text{\#\square"}$. Now, insofar as we refer only to practical engineering calculations, you are fully justified in using any one of these forms. I would merely advise that you be consistent; adopt one form, and then stick by it. Here is one consistent form: $\text{\#/\square"}$, $\text{"\#}$, $\text{'\#}$, $\text{\#/'}$, $\text{\square"}$, etc. Also be consistent in using abbreviations, capitalization and punctuation.

Lettering. Engineering notes usually are lettered since they consist mainly of calculations anyway. There is no reason why you might not write or even typewrite your calculation sheets, but lettered notes look more professional. The engineer usually seems to develop a semi-lettered form that is faster than straight lettering and yet maintains a distinctive technical appearance. You will probably fall into this habit automatically. Just remember that orderly appearance always pays.

Sub-Heads. Headings are of utmost importance in computations. Remember that your computations will be read by the checker and perhaps by others as well. In every set of computations one expects to find headings that call his attention to all important steps. Headings should be short, and they must stand out clearly. You can letter them in caps, underscore them or set them out to the left of the computations. A broad arrow pointing to a very important result is also effective if not used too frequently. Notice the sub-heads on this page.

Completeness. Even when pressed for time, the engineer completes his notes and calculations for easy reference after they have become "cold." Also, your calculations will be checked by another person, and of course, they are "more than cold" to him. If you value his friendship or his good opinion,

add a few words of explanation on each page for his convenience. However, by completeness we do not mean the use of five or six significant figures. Simply read your slide rule carefully and record the result to three significant figures. Seldom is greater accuracy desirable.

Sketches. Finally, make your notes complete by the use of many sketches. The engineer rarely explains anything that he can clarify with a picture, a sketch, or a scale drawing. Free-hand sketches are useful if you can make presentable ones; otherwise, you had best use a straight edge. Always try to keep your sketches reasonably close to true scale. Moreover, place plenty of information on these sketches, for it is always very useful. The eye quickly picks up these data.

Calculation Sheet. The sheet of calculations that is presented herewith is offered as typical of good practice. The computations refer to a simple wood beam so that you can follow them without difficulty. If you are careful, you can make your calculations equally presentable. It is one chance to stand out from the crowd.

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Loading — 12" × 16" timber of long-leaf southern pine.
 Span 20' with 5000 # load at midspan.
 Find safe uniform loading to be added.

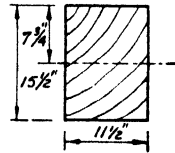
Allowable Stresses — 1600f dense southern pine.

Bending, extreme fiber 1600 #/sq"
 Horizontal shear 120
 Compression ⊥ grain 380

Cross-section — Surfaced one side and edge
 Reduce dimensions $\frac{1}{2}$ "

$$I = \frac{1}{12} \times 11.5 \times 15.5^3$$

$$= 3570 \text{ in}^4$$

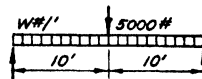


Fiber Stress — 5000 # load

$$M = (5000 \times 20) \div 2 = 300,000 \text{ in} \cdot \text{#}$$

$$f = M c / I = 300,000 \times 7.75 \div 3570$$

$$= 650 \text{ #/sq"}$$



Uniform Load —

$$f = 1600 - 650 = 950 \text{ #/sq"}$$

$$M = f I / c = 950 \times 3570 \div 7.75 = 438,000 \text{ in} \cdot \text{#}$$

$$\frac{1}{8} w L^2 = 438,000 \div 12$$

$$w = \frac{8}{12} \times 438,000 \div 20^2 = 730 \text{ #/'}'$$

$$\text{Wt of beam} = 60$$

$$\text{Safe uniform load} = 670 \text{ #/'}' \leftarrow$$

Check for shear —

$$R = 5000 \div 2 + 10 \times 670 = 9200 \text{ #}$$

$$v = \frac{3}{8} \times 9200 \div 11.5 \times 15.5 = 78 \text{ #/sq" } < 120 \checkmark$$

LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

A.A.S.H.O.	American Association of State Highway Officials.
A.I.S.C.	American Institute of Steel Construction.
A.R.E.A.	American Railway Engineering Association.
A.S.C.E.	American Society of Civil Engineers.
A.S.T.M.	American Society of Testing Materials.
C.G.	center of gravity.
E.U.L.	equivalent uniform loading.
k.	kip, 1000 lb.
N.A.	neutral axis.
W.F.	wide-flange beam.

Greek-Letter Symbols

α (alpha)	an angle such as the slope of one side of a triangle.
β (beta)	an angle usually contrasted to α .
θ (theta)	a particular rotation of a member; also, a general angle.
δ (delta)	a particular deflection of a truss or the side lurch of a building.
δL	change in length of a member from stress.
$\Delta M, \Delta V$	change in moment or shear due to a load movement.
π (pi)	3.14 in structural computations (actually 3.1416).
ϕ (phi)	a joint rotation of a truss; also, a general angle, usually contrasted to θ .
Σ (sigma)	Summation sign.

Arabic-Letter Symbols

A	total area of cross-section as of a truss member.
A_f	flange area in a plate girder.
A_w	web area in a plate girder.
A_n, A_g	net and gross areas.
$a-a$	refers to section a cut through a structure.
a, b	short lengths, usually parts of the span or width.
$A-B, a-b$	force designations in graphical constructions.
$A-I$	stress designation in graphical construction.
b	width of a member or of a structure.
C	compression force or total compressive stress.
c	distance from the neutral axis to the extreme fiber.
d	panel length in a bridge; depth of a beam or other structural member

Arabic-Letter Symbols—(continued)

d_c, d_v	depths of a column and of a girder of a tall building.
dA	differential area.
dx	differential length.
E	modulus of elasticity; 30,000,000 for steel.
e	eccentricity of load, force, or stress.
$E-A-1-E$	consecutive designation of points of a force polygon.
$E-60$	designation of a standardized locomotive loading.
F	total force (usually horizontal) acting on a structure.
f	unit stress.
f_c	unit compressive stress in a plate girder.
f_t	unit tensile stress in a plate girder.
G or G_1	weight of a group of wheels as opposed to the particular wheel P .
g	acceleration of gravity, 32.2 ft. per sec ² ; also, gage distance between lines of rivets.
H	pole distance in graphical constructions.
H or R_H	horizontal reaction, stress, or shear.
h	depth of a plate girder, often the depth of the web; also, height of a truss, rise of an arch, and story height.
$H\text{-comp } S_{EI}$	horizontal component of stress in the member EI .
HS_s, vS_s	horizontal and vertical components of the stress S_s .
$H-20$	designation of a standardized 20-ton truck loading.
I	moment of inertia, usually of the cross-section of a beam; also, impact percentage of the live load for bridge design.
I_0	moment of inertia about the centroidal axis.
i	height or maximum ordinate of a triangular influence line.
I_{x-z}	moment of inertia about the x axis.
J	polar moment of inertia.
K	stiffness, the moment of inertia divided by the length.
k	length measured as a part of the span.
L	span of a truss, length of a member, or loaded length.
L_0, L_1, L_2	lower chord joints of a truss, also designated as a, b, c .
l_1, l_2	fixed lengths, as the segments of an influence line.
M	bending moment or moment reaction; also, momentum as of moving air.
$M_{\max.}$	maximum moment to be considered.
M_w	wind moment.
m	number of members of a truss or a short dimension; also, number of panels to one side of a given panel.
$M-60$	designation of a conventional locomotive loading.
n	number of panels or number of joints in a truss.
O	designation of an intersection point.
$O-A$	designation of a ray in graphical construction.

Arabic-Letter Symbols—(continued)

<i>o-a</i>	designation of a string in graphical construction.
<i>P</i>	major load or unit pressure; wheel load in bridge design; also, a pull, as in the tie rod of an arch.
<i>Q</i>	statical moment of an area, usually about a neutral axis.
<i>q</i>	equivalent uniform load to replace an engine loading.
<i>R</i>	reaction; also, rivet value.
<i>R₀, ΣR</i>	resistance of wind bents in tall building analysis.
<i>R_v</i>	vertical reaction.
<i>r</i>	radius of gyration for a compression member.
<i>r₁, r₂</i>	radii or lever arms from moment center to truss members.
<i>S</i>	total stress as in a truss member; shear per rivet; also, total shear or direct stress in a building column.
<i>S_s</i>	shearing unit stress in the web of a girder.
<i>S_{cd}</i>	stress in the member <i>cd</i> .
<i>s</i>	rivet spacing; also, a length usually measured outside of a truss span.
<i>T</i>	a tension force or a total tension stress; also, a torque moment.
<i>t</i>	thickness of a member or of a piece of plate material.
<i>U₁, U₂, U₃</i>	upper chord truss joints, also designated as <i>A, B, C</i> .
<i>V</i>	vertical reaction or stress; total vertical shear; also, velocity of wind in miles per hour.
<i>V'</i>	modified shear in the web of a truss or girder.
<i>V-comp S_{EL}</i>	vertical component of the stress in the member <i>EL</i> .
<i>v</i>	velocity in feet per second; also, unit vertical shear in the web of a girder.
<i>W</i>	total load, usually a wind load or a locomotive; also, a total weight, as of air.
<i>w</i>	distributed or uniform load per unit length of span; also, weight of a truss per foot of length and weight of air per unit volume.
<i>W_c, W_q</i>	equivalent uniform load for moment at the center and at the quarter point of a simple span.
<i>x</i>	horizontal distance.
<i>x, y</i>	coordinates.
<i>y</i>	vertical distance from the neutral axis to any fiber of a horizontal beam; also, an intercept in graphical construction, usually vertical.
<i>z</i>	distance <i>x</i> or <i>y</i> when referring to either.

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THEORY OF MODERN STEEL STRUCTURES

STATICALLY DETERMINATE STRUCTURES

CHAPTER 1

INTRODUCTION

1. Scope of the Subject. Structural engineering is the broadest division of civil engineering. It includes the planning, the design, and the construction of buildings, of bridges, and of other structures. A few of the less usual structures are mine head-frames, coal bunkers, grain bins, crane trusses, water towers, gas receivers, ferris wheels, tunnel linings, quays, docks, barges, caissons, electrical distribution towers, dirigible frames, smokestacks, and oil fractionating towers. The bare mention of such varied types of structures pictures the interrelationship of structural engineering and industry, with all that the word *industry* suggests. Nevertheless, the two major fields of structural engineering continue to be those associated with the construction of buildings and bridges.

We sometimes hear the facetious remark that a good bridge engineer can design anything. A more truthful statement is that an experienced designer of both bridges and buildings is equipped to attack the design of any fixed structure. Our study of the methods of stress analysis, then, is applicable to structures of all types and to all building materials. Steel bridges and buildings are chosen for specific problems, rather than wood structures, for example, only because steel is so much more widely used. An occasional reference is made to structural aluminum, a strong material of light weight that should prove valuable for movable structures such as cranes and bascule bridges where weight reduction may be important. In airplanes weight saving is so important that the extra cost per pound of aluminum alloys is easily justified.

2. Analysis and Design. There is a reasonably clear dividing line between stress analysis and structural design. Structural design, in brief definition, is

the selection of the sizes of the members and the arrangement of their connections so that the frame will resist its internal stresses. The calculation of the values of these stresses as produced by the applied loads is called stress analysis. Hence we must be able to determine the loads acting on the structure as a preliminary step in the analysis of stresses. The loads acting on the structural frame of a tall building are dependent upon such general features as height, spacing of columns and materials used. *Height* is determined to provide the largest percentage return on the total investment in land and building. *Column spacing* may be influenced by the number and position of elevators. *Materials* must be chosen to provide the hours of fire resistance required of the structure. These features and many others must therefore be determined first and in such a manner that the structure will serve its intended function — whence the name, *functional design*, or preliminary design.

Stress analysis is essentially a mathematical process. At the same time, the estimate of the loads and all assumptions regarding the elastic action of the structure depend also upon the *judgment* of the engineer. Estimates, assumptions, and approximations should precede the actual calculation of the stresses, insofar as such early decisions become practicable. If we follow this procedure, we must conclude that meticulously exact mathematical calculations of stresses (for example, the use of more than three significant figures) cannot *improve* the standard of accuracy set by our basic assumptions. Judgment plays an even more important part in structural design. Specifications are seldom beyond misinterpretation, and the building code leaves a great deal to the intelligence and discretion of the engineer.

3. Mathematics, Mechanics, and Structural Analysis. Mathematics and mechanics are the important working tools in structural analysis. Structural design, in turn, is dependent upon mathematics and strength of materials. A good knowledge of *arithmetic* and of the use of the *slide rule* is essential. Most students will be encouraged to know that we will not go further into algebra than the solution of simultaneous linear equations. Trigonometry will be used, but less than in a course in surveying. The properties of the circle and of the parabola taken from analytical geometry will be found useful, while simple differentiation and integration as applied in mechanics and in strength of materials will limit our dependence upon calculus. Hence the mathematical tools for our work in structural analysis will be comparatively simple. This will be welcomed by those who have found theoretical mathematics difficult. Nevertheless, facility with these mathematical tools is completely essential.

4. The Structural Engineer's Work. A brief explanation of the work of the structural engineer should be of interest to those who intend to enter the profession. The structural engineer may work in the office as a draftsman,

designer, estimator, or purchasing agent, or in the field as an inspector, foreman, or construction superintendent. There are also opportunities for the structural engineer to enter into the promotion of engineering enterprises and into sales work. As a specialist, he may become a research investigator or a consultant on important projects such as bridges of long span. In nearly all his capacities, he will find opportunities to develop his executive ability and to become a manager. His further experience in handling *men, money, and materials* will fit him to hold a responsible administrative position if he prefers management to professional activity.

5. Theoretical Training and Practical Experience. The young graduate engineer must extend his knowledge of basic theory and become experienced in practice before he can develop a reputation as an expert in his particular field of engineering. A comparison of the relative importance of theoretical training and practical experience is difficult because each is *incomplete* without the other. However, we have come to realize that the value of theoretical training has been underemphasized in the past. Developments of the last few years have tended to place a continually increasing number of engineering problems upon a scientific basis. In structural engineering we can point to recent advances in the theory of elasticity and to those studies that have helped to clarify our knowledge of earth pressures. Practical experience serves its highest purpose when it helps us to extend our theoretical knowledge to the solution of problems that would otherwise remain unsolved.

Standardization has not influenced greatly the field of structural engineering. Mass production of office buildings, for example, is not possible because each client has his own ideas of appearance and arrangement. Hence the structural engineer cannot depend upon a knowledge of the solution of a few special cases. He must depend instead upon his understanding of *fundamental engineering theory*. Special cases are used in textbooks only to illustrate the application of the theory involved.



Courtesy American Bridge Co.

FIG. 1. DRIVING THE FIELD RIVETS. TOSSEr, CATCHER, DRIVER AND BUCKER ARE ALL VISIBLE.

HISTORY OF STRUCTURAL ENGINEERING

6. The Derivation of Engineering Terms. The history of structural engineering could be studied through the changes that have taken place in its professional words. A few of the most important technical words will be discussed briefly. The quotations given are from the *Oxford Dictionary*, where detailed references may be found. Preceding a quotation is the date of its first publication and the name of its author.

Bridge. Any structure that affords passage over low ground, water, or other obstruction. There are many obsolete forms of the word: *brycg*, *bregge*, *brudge*, *briggc*. Its derivation seems to be Germanic. It is closely related to the Danish *bro*, a pavement, and *brygge*, a pier. (1337, Trevisa, *There Xerxes the kyng made ouer a brigge of shippes*.)

Span. Originally a *span* was the distance that could be stretched between the thumb and the little finger. (1386, Chaucer, *She hadde a fair forheed. It was almost a spanne brood, I troue*.) Later, possibly from a connection with the span or stretch of a rainbow, the word became associated with arch bridges and then with bridges of all types. (1725, W. Halfpenny, *Set off the Spand of the Intersecting Arch from v. to t.*)

Pier. A support for a bridge. Possibly the word *pier* came from the Latin *petra* or the French *pierre*, meaning stone; in other words, a mass of stonework. (1440, *Pere or pyle of a brygge, or other fundament*.) (1866, Brande and Cox, *An abutment pier in a bridge is that next the shore; and, generally, this is made of greater mass than the intermediate piers*.)

Beam. Probably the derivation is from one of several Teutonic words meaning tree, hence, a squared tree or timber used to span an opening. A *purlin* is a roof beam which supports the *rafters*. A *joist* is a beam used in a building floor. A *girder* supports one or more beams or joists. A *girl* (small girder) is now used to designate a horizontal beam placed in the wall of a building. The *summer* (now obsolete) was originally the largest and most important beam in the structure. (1623, *At an instant the maine Summer or beame brake in sunder*.)

Cantilever. Any beam or projection extending outward more than about twice its depth without support at its outer end. It seems unlikely that the derivation is indicated by the unusual spelling *cant-a-lever*, *cant* meaning to tip. There is no explanation of the form *candiliver*. (1667, Primatt, *For Candilivers about eighteen inches deep and eight inches broad, handsomely carved with flowers*.)

Truss. A truss was first a bundle of things and later in building construction it became a framework of timber or iron used to span an opening. (1654, E. B. Jupp, *When any Chimney . . . shallbe sett upon a trusse of timber*.)

Web. That which is woven, from the Germanic *weban*, to weave. Probably its first structural usage was to the apparently interwoven interior members of a truss. Thence by analogy, the vertical plate of an iron girder, and even the horizontal plates in tubular construction. (1870, B. Baker, *The experiments on the model tube of the Britannia bridge indicated clearly that diagonal strains, both compressive and tensile, occurred in the webs of the tube*.)

Panel. The early meaning, a rectangular piece of cloth, later turned into a rectangular bordered area in building construction. Probably no better word appeared to designate the rectangular area bordered by the chords and two adjacent verticals of a truss.

Arch. The derivation of the word *arch* is not clear, but it might be either from the Latin *arca*, meaning a chest or coffer, or from *arcus*, a bow. (1425, Wynthoun, *An Arche of fayre werk and of fyne*.) (1637, B. Johnson, *'Tis the last keystone That makes the arch*.)

Dome. A rounded vault forming a part of the roof of a great building. It is a derivative from the Latin *domus*, a house or home. (1660, F. Brooke, *The Kings Palace . . . was built square with a Dosme.*) (1756, *The roof of the Pantheon is a round doom, without pillars or windows.*)

Spring. Literally the *spring* is the place of issuing from the ground. In technical usage, it is the point on the abutment or pier from which an arch or vault rises in an outward curve. The *spring line* for a dome would be the top of the supporting wall. (1726, Leoni, *Columns of height sufficient to reach to the spring of their Arches.*) An obsolete usage signified the height or rise of the arch itself. (1753, *The arch was fifty-five feet wide and had but eight feet of spring.*)

Tower. Originally a tower was a fortress or any high building. The meaning remains unchanged except to include the very tall American skyscrapers. (1387, Trevisa, *Men myste wade bytwene Temsebrugge and the toure of Londoun.*)

Rivet. The change from masonry to steel-frame structures may have been envisaged by Ruskin, 1851, *Bars and rivets instead of mortar for securin.g stones.*

Concrete. This word is a good example of one whose root meaning needs no reinterpretation. It comes directly from the Latin *concretus*, past participle of *concrecere* meaning to grow together. Hence, a mass of sand and stone that has solidified.

7. Ancient Structures. Structural engineering as an art had its beginning several thousand years before Christ as is suggested by the known high state that the art had reached as early as 1000 B.C. By that time all of the pyramids had been built, the palace of Knossos in Crete had been sacked and burned,

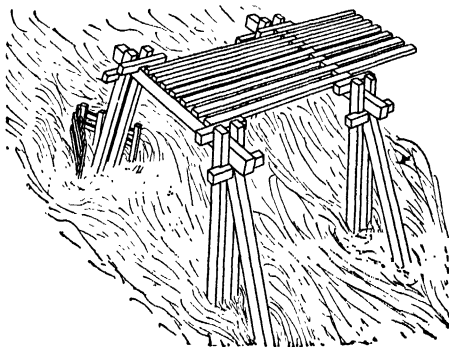


FIG. 2. CAESAR'S BRIDGE OVER THE RHINE.

Built by Apollodorus of Damascus near Turnu Severin in Rumania, Trajan's Bridge over the Danube consisted of 20 arches. Its length of 4000 ft. made it the largest in the Roman Empire.

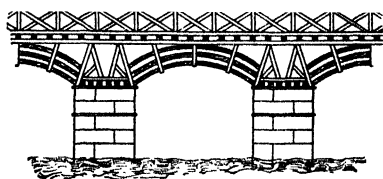
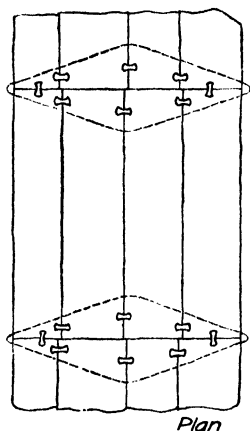


FIG. 3. TRAJAN'S BRIDGE (A.D. 104).

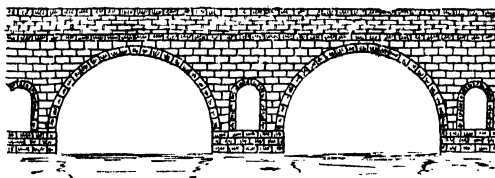
and the great temple of Amen at Karnak in Egypt was near the height of its grandeur. The great age of Greek architecture symbolized by the completion of the Parthenon, 438 B.C., was extended through the known world by Alexander (356-323 B.C.). Soon after this Hellenistic period the Romans, those renowned bridge builders, gained supremacy of civilization and constructed their roads and bridges from one end to the other of the Empire. One of their favorite bridge types was the pile structure, such as the bridge over the Tiber at Rome (in existence from 600 to 300 B.C.) and that famous bridge over the Rhine (Fig. 2) constructed by Julius Caesar during his invasion of

Germany. This latter structure, a third of a mile in length, was constructed in ten days. The other common type of Roman bridge was the semicircular masonry arch which also was much used in the construction of aqueducts. Many of these arch bridges remain today; to the engineer they are likely to be the most impressive of the remains from all ancient civilizations. See Fig. 4(b)

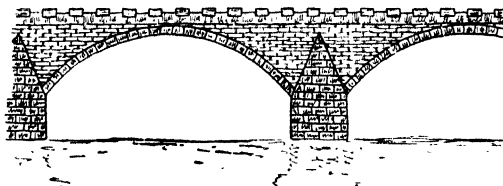
8. Bridge Building in the Middle Ages. Bridges continued to be built during the early Middle Ages, but from the few recorded accounts we conclude that these were inferior to the old Roman structures.* Certainly, there is



(a) Ancient Stone Slab Bridge.



(b) Semicircular Roman Arches B.C. 100—A.D. 100



(c) Fourteenth Century Italian Segmental Arch

FIG. 4. EARLY STONE BRIDGES.

no indication of an important advance in the art of bridge building until the construction of the Ponte Vecchio in Florence (1345) marked the introduction of the *flat segmental arch*, Fig. 4(c), a type well adapted to long spans. A famous bridge of the Middle Ages was the London Bridge completed in 1209 after a construction period of thirty-three years. It stood until 1824. Like most of the early English and Norman bridges, it was constructed by the clergy, who maintained a chapel on the center pier. It is possible that the art of the construction of great bridges might have been lost during the Middle Ages except for the existence of an order of monks known as THE BROTHERS OF THE BRIDGE. The members of this order built many fine structures in France. Their purpose was to aid travelers by the construction of bridges. Probably the original incentive developed from the religious fervor attending the Crusades.

* A. P. M. Fleming and H. J. Brocklehurst, *A History of Engineering*, Black, London, 1925.

Bridge construction dependent upon the masonry arch for its open spans had approached its height during the Roman Empire. Spans exceeding 100 feet were built as early as the beginning of the Christian Era. The use of the segmental type by Italian engineers had made possible the arch at Verona completed about 1354. Its span of 160 feet was for many years to remain the longest for a stone arch. A further development in bridge construction awaited the invention of the *truss*.

9. The First Wood Trusses in Europe. The idea of the truss was used by an Italian architect, Andrea Palladio (1518–1580), through the middle of the sixteenth century. Sketches of his trusses are remarkable in that they show structures practically of modern types. See Fig. 5. It seems clear that

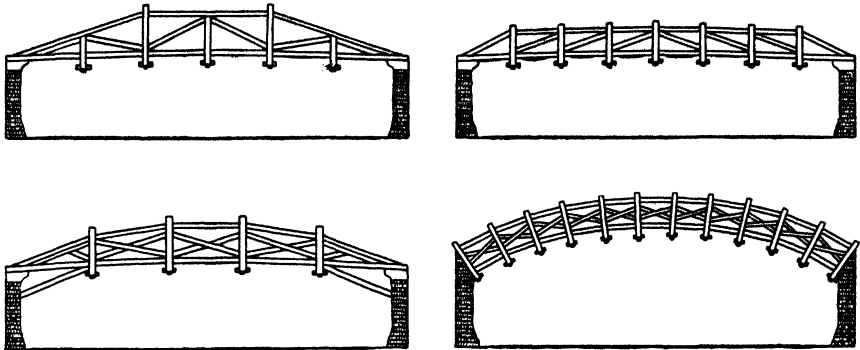


FIG. 5. TRUSS DESIGNS BY ANDREA PALLADIO (1518–1580).

Palladio's invention was not understood because we must bridge a gap of *over two hundred years* to find another man who was capable of realizing the importance of the truss. Ulric Grubenmann, a Swiss carpenter, constructed a wooden truss bridge of 170-foot span over the Rhine in 1758. This was followed by other structures and finally by a great wooden bridge of 390-foot span constructed by Ulric and Jean Grubenmann — which is said to be the longest wooden span ever built. Like the European and American bridges which followed, these structures combined the truss and arch principles. A favorite expedient was to support the truss partially by diagonal knee braces extending outward and upward from the pier. A horizontal thrust on the pier was inevitable with such construction.

10. Early American Bridge Builders. The earliest American bridges were pile structures. Theodore Cooper mentions "The Great Bridge," a pile structure, built across the Charles River at Cambridge in 1660.* From 1760 to 1800 numerous pile and wooden arch structures were completed. The

* Theodore Cooper, *American railway bridges*, Transactions, A.S.C.E., Vol. 21, p. 2.

names of the builders would make an interesting group for a biographer: Samuel Sewall, William Riddle, Enoch Hale, Timothy Palmer, and Rufus Graves. Palmer was the architect and engineer for "The Permanent Bridge," completed in 1806 over the Schuylkill River at Philadelphia. This structure, shown in Fig. 6, consisted of two arches of 150-foot span and one center arch of 195-foot span. Though this bridge was of wood construction, it remained

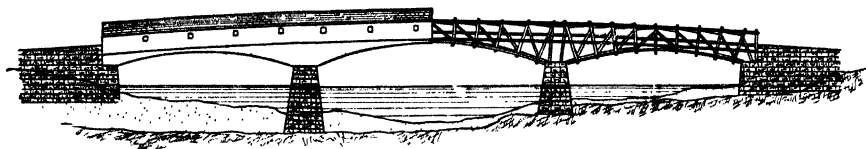


FIG. 6. "THE PERMANENT BRIDGE," PHILADELPHIA (1806).



FIG. 7. "COLOSSUS"—WOODEN ARCH SPAN OF 340 FEET (1812).

in use until 1850. Most of these old bridges were covered to protect the structural timbers from the weather, which fact probably accounts for their long lives.

After 1800 two other men became famous as bridge builders: Theodore Burr and Louis Wernwag. Burr's name was connected with the construction of numerous important structures. There is a tradition that Burr attempted to build a wood bridge 900 feet long which was to be divided into two equal spans. The failure of the first span is supposed to have led to a redesign with much shorter spans. Wernwag will always be remembered for his "Colossus" bridge over the Schuylkill River at Philadelphia. This arched truss, shown in Fig. 7, was singularly clean cut in appearance, and, because of its span of 340 feet, it was probably the longest wood bridge ever built in America. This bridge carried traffic from 1812 until it burned in 1838.

11. Town's Truss. As has been mentioned, the bridges built before 1820 were essentially trussed arches. The Town or lattice truss was patented in 1820 by Ithiel Town of New Haven. This structure, Fig. 8(b), was built of ordinary planks similar to the construction of one type of cheap roof truss used today. The Town bridge was the first truss since the trusses of Palladio which exerted *only vertical forces* on the abutments. However, its complicated web system was such that it cannot claim to be the first bridge truss of modern type.

12. Long's Truss. This truss patented in 1830 by Brevet Lieutenant-Colonel Long of the Corps of Engineers, U.S.A., Fig. 8(a), more nearly corresponds to the modern truss because of its simple type of web system. It is said, however, that Long's trusses were braced out from the abutments, and

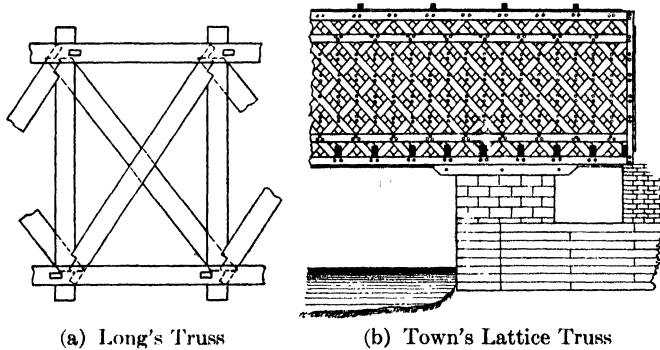


FIG. 8. EARLY WOOD TRUSSES.

therefore they exerted a horizontal thrust similar to the reaction of an arch. Long's truss, like Town's truss, was built with chords of two or more planks between which the web members were placed. These trusses were entirely of wood construction.

13. The Howe and Pratt Trusses. The immediate forerunner of the present simple-span truss with single web system was patented in 1840 by William Howe. Howe merely combined the simple supports used by Town

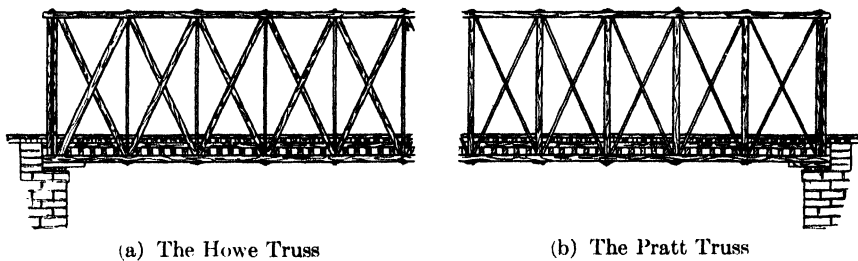


FIG. 9. WOOD TRUSSES WITH WROUGHT-IRON TIE RODS.

and the simple web system suggested by Long. However, his real improvement was in adopting the use of *wrought-iron tie rods* for vertical web members. Howe's patent was followed in 1844 by one issued to Thomas and Caleb Pratt. The only difference of importance between the two was that the Pratt truss used vertical posts of wood and *diagonal tie rods*. The Howe and Pratt trusses may be compared in Fig. 9.

14. Early Iron Bridges. The Howe and Pratt trusses were followed immediately by numerous iron bridges. Theodore Cooper credits the construction of the first iron truss to Earl Trumbull. The date was 1840.* This bridge was erected over the Erie Canal at Frankford, New York. Cast iron was used for compression members and wrought iron for tension members. It was a combination truss and suspension system.

It is not to be thought that 1840 is the date of the first iron bridge. This credit goes to the cast-iron arch at Coalbrookdale, England, built in 1776, which had a span of 100 feet 6 inches. This structure carried traffic for nearly a century and a quarter.† Several other cast-iron bridges of the arch type were constructed in Europe and America before 1840, but the Trumbull bridge probably represents the first exclusive use of iron for a bridge partially of the truss type. The construction of iron bridges in England was advanced during this period by four famous English engineers — John Rennie, Thomas Telford, and George and Robert Stephenson.

15. Squire Whipple (1804–1888). The first iron bridge constructed by this prominent American engineer was a bow-string truss with cast-iron compression members and wrought-iron tension members. The date also was

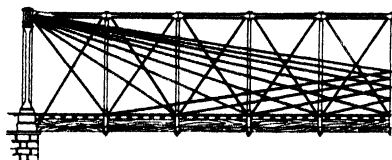


FIG. 10. THE BOLLMAN TRUSS.

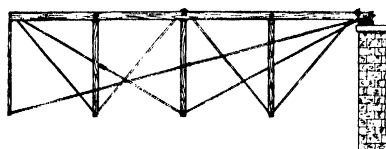


FIG. 11. THE FINK DECK TRUSS.

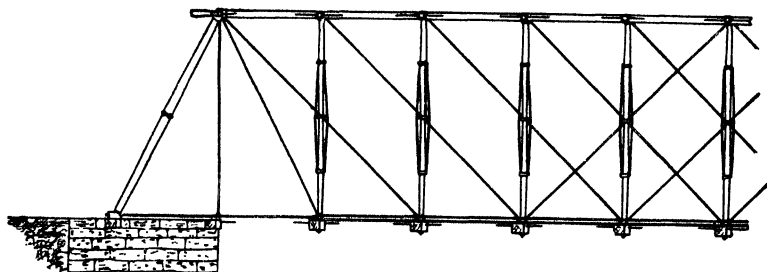


FIG. 12. THE WHIPPLE TRUSS.

1840. Other early iron structures were those of Bollman and Fink. Both of these bridges really were trussed beams in which the unloaded chord was unstressed under dead load and could even be omitted for deck structures. These trusses are illustrated in Fig. 10 and Fig. 11.

* Theodore Cooper, *American railroad bridges*, Transactions, A.S.C.E., Vol. 21, p. 12.

† Charles S. Whitney, *Bridges*, William Edwin Rudge, New York, 1929.

It remained for Squire Whipple to construct the *first essentially modern truss* entirely of iron. This bridge, a span of 146 feet, was completed in 1853. The outline of the Whipple truss with its double-intersection web system, is shown in Fig. 12. These early trusses were made of cast iron and wrought iron. They were connected by pins at all joints along the top chord.

16. Riveted and Pin-Connected Trusses. A discussion regarding the relative merits of pin-connected and riveted trusses began with the introduction of wrought-iron bridges and was carried on in the engineering journals for over a quarter of a century. The riveted truss was widely used in Europe while the pin-connected structure became the standard for American railway bridges. The advantage of *economy* was urged for the pin-connected truss while *rigidity* was put forward as the advantage of the riveted structure. A half century of use has shown that the two structures are approximately equal in cost. The riveted bridge has the advantage for short spans because of its inherent rigidity, but for long heavy spans the pin-connected truss is amply rigid and it may show a slight economy. A further argument arose when American engineers claimed that the pins in their trusses relieved the members of nearly all *bending stresses*. Investigation has shown that pins do not rotate freely in an old rusted structure; so it is now the common belief that pins are of little value for this purpose. One may find many interesting discussions about the relative merits of rivets and pins between eminent engineers of the nineteenth century in the publications of the technical societies during the latter half of that period. The reduced construction of railway bridges during the 1940's led to the abandonment of standard eye-bar production. Hence it seems probable that pin-connected trusses will be used infrequently in the future and only for great spans.

17. The Development of Structural Analysis. Although European engineers contributed greatly to the development of structural theory, an American engineer, Squire Whipple, was the originator of the present ideas regarding *truss analysis*. His book, published in 1847, made the scientific design of truss bridges possible. It seems odd that Coulomb, a French physicist, should have discussed correctly the theory of beams in 1776, that Navier, a French mathematician and engineer, should have completed his theory of flexure and even announced his analysis of arches and of continuous beams by 1826, and that the simpler problem of truss analysis should not have been understood until nearly 1850, but such is the case. Probably the fact that the first trusses were built with complicated web systems which would defy analysis even today (Fig. 10) accounts for the indifference shown by early engineers toward attempting a scientific calculation of stresses. As soon as a simpler arrangement of web members was adopted, the need for scientific methods of analysis and design soon was satisfied.

18. The Development of Long-Span Bridges. The scientific design of trusses reduced bridge costs since the old idea that a truss should be of constant section from end to end led to a great waste of material. This encouragement combined with the great demand for railroad bridges brought about the rapid development of bridge structures. Cast iron was replaced by wrought iron, and wrought iron was replaced by steel. Tubular bridges were developed, became popular for long spans, and then were superseded by steel arches,



FIG. 13. OLD ALFRED ROAD UNDERPASS 1947 AISC BRIDGE AWARD, CLASS III. The stark simplicity of this continuous beam bridge and its Tee-shaped supports provides an unusually attractive structure.

stiffened suspension bridges, and cantilever trusses. This development is typified by the Britannia tubular bridge over the Menai Straits, finished in 1850 by Robert Stephenson (two spans of 460 feet each), by the steel arch bridge at St. Louis completed by James B. Eads in 1874 (longest span 520 feet), by the Brooklyn suspension bridge completed in 1883 by John A. Roebling and Washington Roebling (clear span 1595 feet), and by the Firth of Forth cantilever bridge completed in 1890 by Sir John Fowler and Sir Benjamin Baker (two spans of 1710 feet each). Each became successively the world's greatest span.

For over half a century this group of bridges included the outstanding bridge structures of the world. They are now exceeded in importance by several bridges, the most recent of which are the Kill van Kull steel arch from Staten Island across to New Jersey, the Sydney Harbour arch at Sydney,

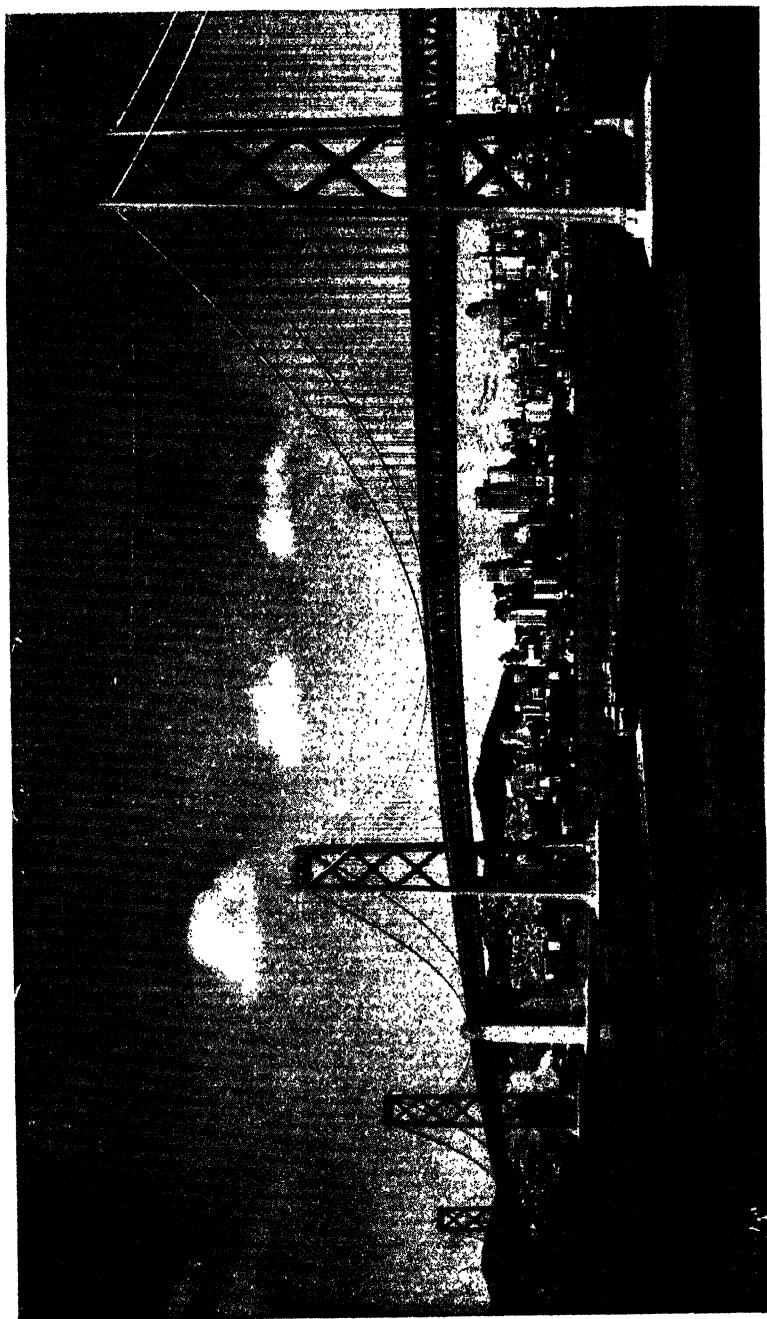


FIG. 14. SAN FRANCISCO-OAKLAND BAY BRIDGE.

The overall length of this bridge is eight and one half miles of which four and one half miles are over water. It is the world's longest bridge with great spans.

Courtesy Redwood Empire Association

Australia, the George Washington Bridge over the Hudson River at New York, and the Golden Gate Bridge at San Francisco. The Kill van Kull arch has a span of 1675 feet, and the Hudson River suspension bridge has a center span of 3500 feet. At the present time the 4200-foot suspended span of the Golden Gate Bridge is the longest clear span in the world. Other structures of equal or greater magnitude are under consideration.

When we consider the fund of information and experience behind our bridge engineers of today, we must marvel at the boldness of those pioneer builders of a half century or more ago — Fowler, Baker, Roebling, Eads, Stephenson. Perhaps our wonder should be even greater over the achievements of the Grubenmann brothers and of Riddle, Palmer, Burr, Wernwag, whose exceptional engineering judgment made it possible for them to design great wooden bridges without the aid of scientific analysis and to build such structures sometimes without even the use of nails or bolts. Perhaps, on the other hand, our greatest admiration should be reserved for the bridge builders of the Roman Empire whose marvelous stone arch bridges, frequently built without the aid of cement, have endured two thousand years and still are accepted as models of structural beauty and utility. Certainly the Roman arches would have fulfilled Palladio's classic definition of a satisfactory bridge more closely than most of our present structures, 'that it shall be *convenient, beautiful, and durable*.' Although in great contrast to each other, it is felt that the modern steel bridges of Figures 13 and 14 do fulfill Paladio's definition.

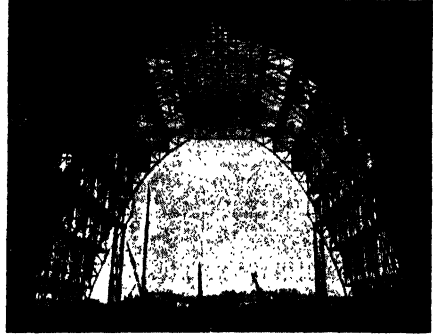
BUILDINGS OF THE ANCIENT AND MODERN WORLDS

19. The Desire for Great Buildings. We inevitably associate the conception of a building with the idea of *shelter*. However, all ancient monumental building structures were inspired by other motives. The Egyptian pyramids and mastabas were constructed as *tombs*. The great temple at Karnak was a *shrine* for the use of the King-Gods of the Egyptians. The Colosseum at Rome was a *stadium* for seating the spectators at exhibitions and games. America's skyscrapers are quite as far removed from the simple idea of shelter. Their inspiration has been primarily the desire for profit. It has been shown that a building over fifty stories high should produce the maximum return upon the total investment where the value of the land reaches above one hundred dollars a square foot.* Hence the upward extension of the skyscraper during the 1920-1930 period.

20. Ancient Monumental Structures. It is a mistaken notion that America's buildings are preeminently the world's most monumental structures. A citizen of ancient Egypt would have been impressed but certainly

* Based upon 1920-1930 costs and rentals. W. C. Clark and J. L. Kingston, *The Skyscraper*, American Institute of Steel Construction, New York, 1930.

not awed by their size. The Great Pyramid built by Khufu (Cheops) about 3700 B.C. was 482 feet in height and 768 feet along each side of the base. It was constructed of solid masonry and is estimated to have required the services of 360,000 men for twenty years during its construction, a waste of labor that may have been justified as unemployment relief. The Temple of Amen at Karnak completed about 980 B.C., but under construction for seven centuries, was 1200 feet in length and 350 feet across its greatest width. It may be compared in size with America's largest industrial building, the Goodyear airship dock at Akron, Ohio, which is 1175 feet long and 325 feet wide. In mass, the Temple of Amen exceeds anything in the nature of shelter ever conceived by man. The great central stone columns are 66 feet high and 10 feet 7 inches in diameter. The monolith stone slabs spanning the roof are 36 feet long and 4 feet thick. The mass of the Temple of Amen is in great contrast to the web-like steel frame of the Akron dock, Fig. 15.



Courtesy American Bridge Co.

FIG. 15. GOODYEAR AIRSHIP DOCK,
AKRON, OHIO.

This temple at Karnak has survived for 30 centuries. It is not to be expected that the Goodyear dock will be used for even one. Modern society needs to reconsider its plans particularly for the construction of public works which cannot be rebuilt for the use of each generation. Public works should be planned with such vision that they will serve for generations. Then their growth instead of replacement will raise our standard of living.

21. The Use of the Vault and Dome. Egyptian and Greek buildings were structurally dependent upon the column and the stone beam or lintel. Roman engineers made wide use of the arch and in one magnificent building, the Pantheon, of the masonry dome. The Pantheon at Rome is topped by a hemispherical dome 144 feet in diameter which rises from a circular wall whose height is the same as the diameter of the dome. This building, which carries the largest heavy masonry dome ever attempted, was completed about 120 A.D. Further development of the structural dome continued in the Eastern Roman Empire and reached its height in the construction of the church of Hagia Sophia in Constantinople, 563 A.D. This building has a nave 243 feet long by 107 feet wide covered by a central dome and two half domes at the ends partially supported upon masonry arches of about 100-foot span. Its interior is still said to be one of the supreme masterpieces of all time and to represent the highest development of the vault and dome.

22. The Colosseum at Rome. One of America's particularly interesting structures is the football stadium or bowl. We may compare the dimensions of the larger stadia with those of the Colosseum at Rome inaugurated by Titus, A.D. 80. The elliptical walls of the Colosseum measure 615 feet by 510 feet to their extreme faces, the arena being 281 feet by 177 feet. The structure consists of three arcaded stories with an upper gallery reaching a height of 160 feet to the top of the wall. It is only with modern structural materials, however, that a cantilever roof can be extended as much as 75 feet without support to shelter thirty or more rows of spectators.

23. The Great Towers. Even in the simple matter of height the American skyscraper is not entirely unique. The Washington Monument extends to a height of 555 feet. This white marble obelisk was constructed in several bursts of activity from 1848 to 1884. The Eiffel Tower built in Paris for an exposition in 1889 is 984 feet high. It is constructed of iron. Only a few skyscrapers rise to a greater height than the Eiffel Tower. The construction of great chimneys was common at an early date; for instance, the one at St. Rollox near Glasgow (435½ feet above ground) and Townsend's chimney at Glasgow completed in 1859 to a height of 468 feet. Rankine called attention to the St. Rollox chimney in 1858 as the third tallest structure in the world. The others mentioned by Rankine were the spire of Strasburg, the Great Pyramid, and the spire of St. Stephens at Vienna. Spires of other churches and incidentally of the town hall in Philadelphia were built to equal or greater heights long before the advent of the skyscraper.

24. Tall Buildings with Masonry Walls. The fact that structural steel shapes for use as beams and columns were not available until 1885 undoubtedly delayed the development of the skyscraper. The first buildings of iron-frame construction used cast-iron columns and wrought-iron beams. Rolled beams of *wrought iron* were first made available in 1854 at the ironworks in Trenton, New Jersey. The importance of rolled sections is tied up with the change from wall-bearing to skeleton-frame construction. Wall-bearing construction, where thick masonry walls carried the floor loads to the foundations, reached its maximum practical height in the 200-foot Monadnock Building in Chicago where the exterior walls were fifteen feet thick at the bottom. If similar interior walls had been used in this building, the walls would have consumed about one-half of the floor space at the basement level. This waste in floor space was greatly reduced by the substitution of iron columns and girders, with thin partitions resting on the floors, for the interior masonry walls.

25. Early Development of the Skyscraper. What might seem a comparatively simple change, to replace the heavy exterior walls by iron or steel columns and girders with curtain walls supported at each floor level on the structural frame, was in reality a great development, for it involved the design of the structural frame to resist not only the vertical loads but the

lateral wind forces as well. This development took place in Chicago, where it was pioneered by William Le Baron Jenny in the 10-story Home Insurance Building, 1885. Other pioneers connected with the introduction of the skyscraper were Leroy S. Buffingham, Bradford Lee Gilbert, Daniel W. Burnham, John W. Root, and William Holabird. Buffingham attempted to maintain patent rights on skeleton construction on the basis of a patent dated 1887, but he was unable to collect damages. Gilbert constructed the Tower Building in New York in 1889. This structure was 10 stories high and but $21\frac{1}{2}$ feet across the front; it was truly a skyscraper of skeleton construction. However, it was antedated by several Chicago buildings of less slender proportions, including at least the following: the 10-story Home Insurance Building, 1885; the 14-story Tacoma Building, 1887; and the Rand-McNally Building, 1889. These were followed within a year or so by the Leiter Building, the Fair Building, the Reliance Building, and the Masonic Temple, which group fully justified Chicago's claim to being the home of the skyscraper. Nevertheless, New York was soon to pass Chicago in the extension of the skyscraper with its 17-story Manhattan Life Insurance Building, its 26-story St. Paul Building, its 33-story Park Row Building, and its famous Flatiron Building, all completed at the beginning of the twentieth century.

26. The World's Tallest Buildings. Tall buildings had become so common by 1907 that the 41-story Singer Building constructed that year in New York to a height of 612 feet excited less comment than the construction ten years



FIG. 16. MANHATTAN'S FAMOUS SKYLINE.

previously of buildings but one-half that tall. The Woolworth Building, 58 stories and 760 feet to the top of its 22-foot flagpole, caught the popular interest again in 1913, and properly so, for it was to remain the world's tallest structure for fifteen years. From 1929 to 1931, New York celebrated in succession the completion of the Chrysler Building, the Bank of Manhattan Building, and the Empire State Building, all taller than the Woolworth Build-

ing. The Empire State Building, on its completion, became the tallest structure in the world, for it reached an elevation at the top of its mooring mast of 1250 feet above the level of the street. Its 102nd floor is at an elevation of 1224 feet above the street. The Empire State Building is shown at the extreme right of the sky-line view in Fig. 16.

27. The Importance of the Elevator. Skeleton construction has been mentioned as a necessary development leading to the possible erection of the tall building. Of equal importance was the introduction of the high-speed electric elevator, which made the tall building *useful*. When we realize that the modern elevator, rising 800 feet a minute, consumes at least $1\frac{1}{2}$ minutes in reaching the top of the Empire State Building, we understand that such a structure would be *impracticable* with appreciably slower elevator service. A speed of 1500 feet a minute is mechanically possible, and no doubt practical designs will gradually approach that figure.

28. Distribution of Skyscrapers. The skyscraper is by no means limited to New York and Chicago today. Almost every major city in the United States has several 40-story buildings, while 20-story structures are not

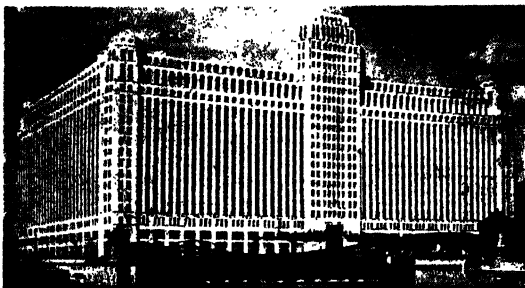


FIG. 17. MERCHANDISE MART, CHICAGO, ILLINOIS.

uncommon in smaller towns. Foreign countries have followed America, so that buildings of steel-frame construction are commonplace in Europe, Australia, and South America. It is of particular interest that the world's tallest reinforced concrete buildings are the 28-story Hotel Palacio Salva, Montevideo, Uruguay and the 34-story headquarters of the Banco do Estado de Sao Paulo in Brazil.* In the United States, concrete-frame buildings are seldom found to be economical above 20 stories. A development of recent years is that of arc-welded building frames. Arc welding is in common use for industrial structures and has been used for the 19-story office building of the Dallas Power and Light Company at Dallas, Texas, and the 24-story City National Bank at Houston, Texas. Incidentally, a large building need not be a tall one. Thus, the world's largest office building is actually but 18 stories high. It is the Merchandise Mart in Chicago, which covers a trapezoidal area 724 feet by 324 feet and contains 4,000,000 square feet of floor space, which is twice the floor area of the Empire State Building.

* See *World's tallest concrete building*, L. Compagna, Engineering News Record, September 2, 1948, p. 72.

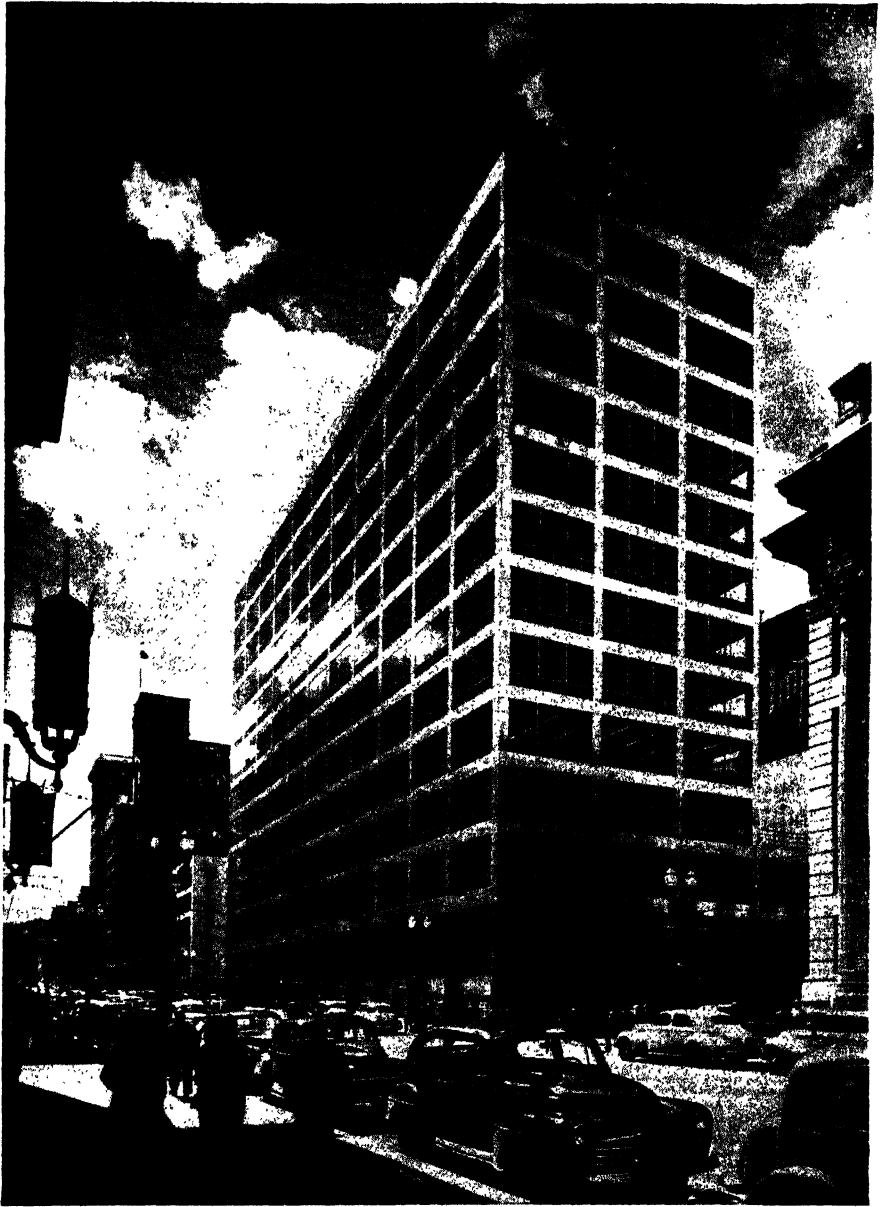


FIG. 18. THE EQUITABLE BUILDING, PORTLAND, OREGON. PIETRO BELLUSCHI, ARCHITECT.

Notice the strong emphasis upon structure as the main architectural feature. In effect this building is largely a structural cage with glass enclosure. The architect compromised with this simple concept in the lower story by changing the character of the columns but the structural plan remains unchanged. Photo by Sturtevant.

29. A New Development in Architecture. A great deal of interest attaches to the architectural features of America's modern buildings. Early office buildings were often criticized as esthetically unsatisfactory, the blame being placed by most critics upon the commercial motive. It was frequently pointed out that the most beautiful structures of the world had all been inspired by some influence other than the desire for a profitable investment. This argument has come back upon the critics, however, by the widespread acceptance of the belief that our industrial and office buildings constructed since 1925 have developed an entirely new type of monumental architecture, now called contemporary, which has taken its place with the other great schools of architecture. A glance at Figures 18 and 19 is convincing evidence that here is a new conception of building architecture that cannot die without a change in the life of America, because it mirrors the age which it serves.

30. Tall Buildings of the Future. We might look to a reduction in dead weight of our tall buildings as one important development of the future. Light-weight materials for walls, partitions, and floors might be combined with the joint use of steel and aluminum to reduce the enormous dead load on the columns and their footings. Whether this structural advantage will ever be used to extend our buildings farther skyward is beyond prediction. During the depression years of the 1930's, men predicted freely that we have passed the reasonable economic limit of height of our skyscrapers. Probably we have. but an engineer of high standing, Joseph K. Frietag, stated in 1901 that, "The Park Row Building is the highest office building ever erected, and it is very doubtful whether it will ever be found desirable or profitable to erect other buildings as high as this one." * It will be recalled that the Park Row Building had been built to the then unbelievable height of 390 feet above the street.

31. References. This brief outline of the history of structural engineering can be extended as far as the reader desires by the use of the engineering library. The following references indicate the various *kinds* of source material from which a complete history could be drawn.

- (1) Henry N. Maynard, *Bridges, Roofs, etc.*, Spon, London, 1868.
- (2) Charles B. Stuart, *Civil and Military Engineers of America*, Van Nostrand, New York, 1871.
- (3) Theodore Cooper, *American railroad bridges*, Transactions, A. S. C. E., Vol. 21, pp. 1-60.
- (4) *Memoir of Squire Whipple*, Transactions, A. S. C. E., Vol. 36, p. 527.
- (5) William H. Burr and Myron S. Falk, *Design and Construction of Metallic Bridges*, Wiley, New York, 1905.
- (6) William H. Burr, *Ancient and Modern Engineering*, Wiley, New York, 1907.
- (7) Henry G. Tyrrell, *The History of Bridge Engineering*, published by the author, Chicago, 1911.

* *Architectural Engineering . . . High Building Construction*, Wiley, 1901, p. 42.

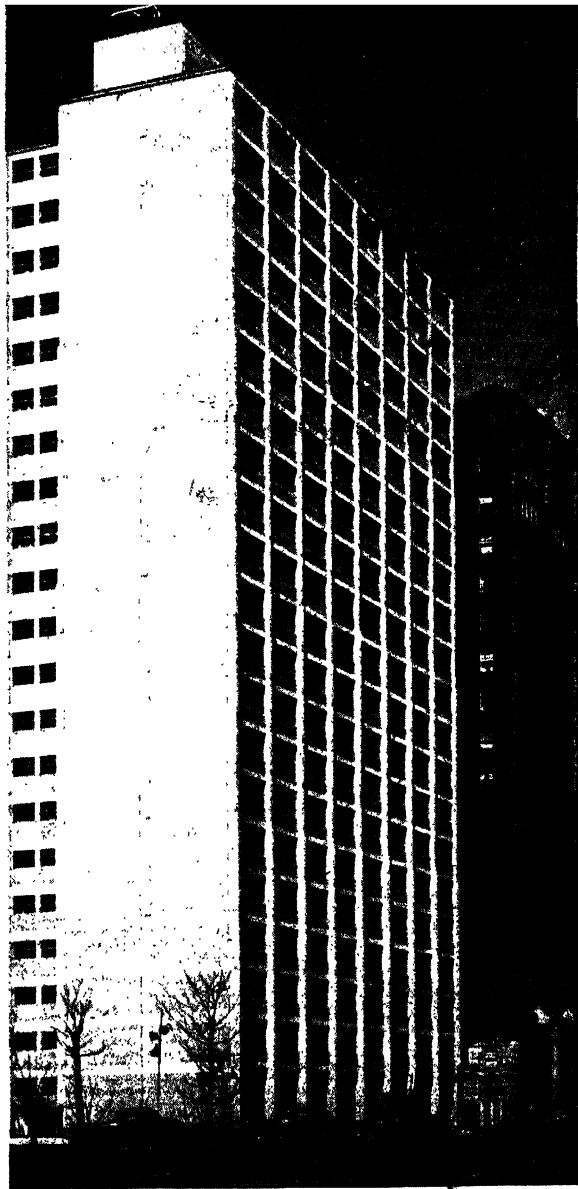


FIG. 19. APARTMENT HOUSE DESIGN BY MIES VAN DER ROHE.

Note here that the architect exposes the structure completely on the face of the building. Columns are seen to taper upwards and beams grow shallower toward the top as the wind stresses are reduced. The structure extends to the ground without attempt at refinement of the lower story. The entire architectural effect is therefore associated with the structure. Photo by Edgar F. Zobel.

- (8) Charles S. Whitney, *Bridges*, William Edwin Rudge, New York, 1929.
- (9) Phillip G. Laurson, *The development of bridge construction*, Journal of Engineering Education, April, 1931.
- (10) Robert Fletcher and J. P. Snow, *A history of the development of wooden bridges*, Transactions, A.S.C.E., Vol. 99, pp. 314-408.
- (11) William A. Starrett, *Skyscrapers and the Men Who Build Them*, Scribners, New York, 1928.
- (12) Robins Fleming, *Whence the skyscraper*, Civil Engineering, October, 1934, pp. 505-509; also December, 1934, pp. 634-638.
- (13) Alfred C. Blossom, *Building to the Skies*, Studio Publications, New York, 1934.
- (14) A. P. M. Fleming and H. J. Brocklehurst, *A History of Engineering*, Black, London, 1925.
- (15) Richard S. Kirby, *Etymology of some common engineering terms*, Civil Engineering, October, 1934, pp. 529-532.
- (16) Richard S. Kirby, *Specifications and plans of ancient times*, Civil Engineering, Vol. 11, 1941, pp. 393-396.

LOADS, REACTIONS AND STRUCTURAL MEMBERS

32. Loads on Beams and Trusses. Loads on structures will be discussed very briefly here because the loads acting on bridges and buildings will be

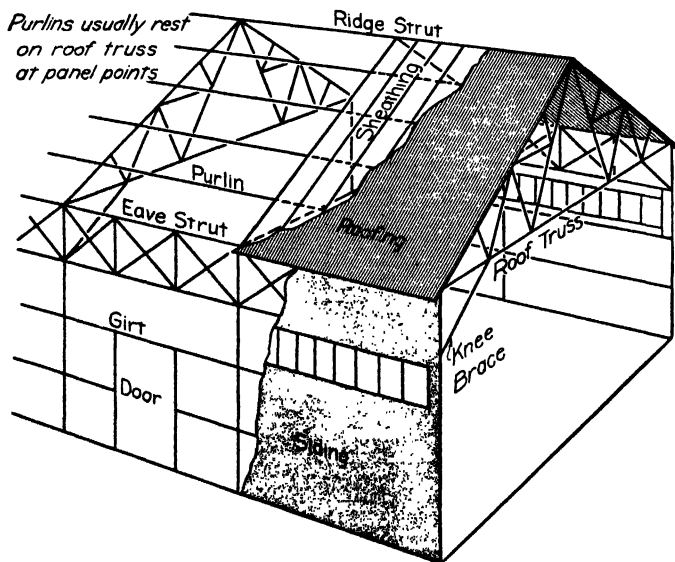


FIG. 20. STRUCTURAL STEEL FRAME OF AN INDUSTRIAL BUILDING.

considered in detail in the following chapters. However, even to begin a study of the mechanics of stress analysis it must be understood that the roof system of an industrial building and the floor system of a bridge serve the

purpose of carrying the applied loads and the weight of the roof or floor itself to a series of supporting beams, columns or truss panel points. In Fig. 20, for example, the roofing material is in continuous contact with the *sheathing*

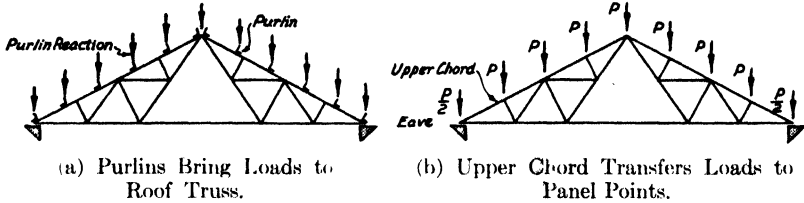


FIG. 21. TRANSFER OF PURLIN LOADS TO PANEL POINTS BY UPPER CHORD.

which supports it. In turn the sheathing, which may be of wood, corrugated metal or pre-cast concrete, is of sufficient strength to span from one roof beam to the next up the roof. Such roof beams are called *purlins*. The weight of roofing and sheathing produces a distributed or continuous load along the entire length of the typical roof beam or purlin which must span from one roof truss to the next.

At the point where each purlin rests upon the roof truss there is a concentrated load applied to the upper chord of the truss of Fig. 20. These purlin

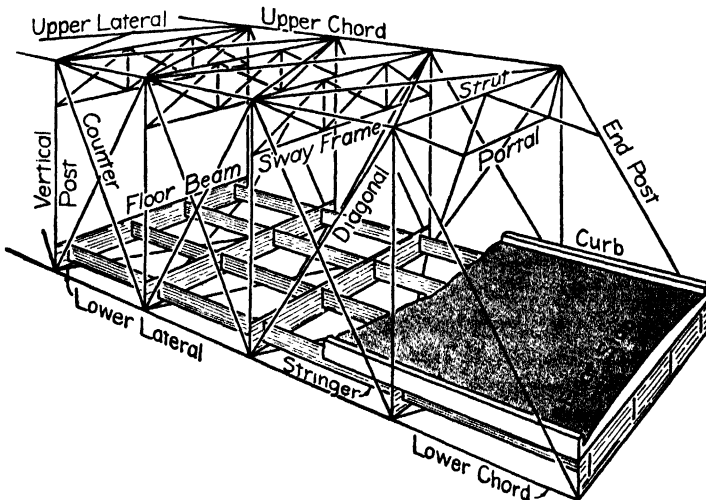


FIG. 22. TRUSSES, FLOOR SYSTEM AND BRACING OF A THROUGH-TRUSS HIGHWAY BRIDGE.

reactions on the upper chord may be represented as in Fig. 21(a). Immediately, however, the upper chord of the truss acting as a lateral beam brings each purlin reaction to the adjacent panel points of the roof truss. Hence,

in truss diagrams we commonly show the dead load of the roof itself as a series of downward loads at the upper panel points of the roof truss as in Fig. 21(b). If the distance between roof trusses is 20 ft. and the distance along the slope of the roof between panel points is 6 ft. while the combined weight of roofing, sheathing and an allowance for the purlins is 25 lb. per sq. ft. of roof surface, a single panel point load concentration as in Fig. 21(b) would be $20 \times 6 \times 25 = 3000$ lb. or 3 kips. To this figure would be added in Fig. 21(b) one-eighth of the estimated weight of the roof truss since there are seven full load concentrations and two half load concentrations at the eaves.

For the through-truss highway bridge of Fig. 22 it is seen that the floor slab rests upon the longitudinal beams or *stringers* which in turn rest upon (or frame into) the transverse *floor beams*. Each floor beam frames into a

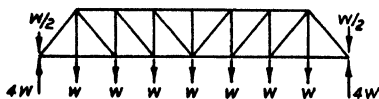


FIG. 23. DEAD PANEL LOADS —
TRUSS OF FIG. 22.

truss joint at either end. Hence, the dead load of the entire floor system is transferred through the stringers and floor beams to the lower panel points of the trusses. In addition, the weight of the steel trusses and the upper and lower chord bracing is transferred by

the truss members to the upper and lower chord panel points of the trusses. However, to simplify analysis, all of this dead load is usually added in with the dead load of the floor system at the lower chord panel points of the trusses as is indicated by Fig. 23.

33. Live Loads for Buildings and Bridges. Each major city has a Building Code which specifies the live or movable load (ordinarily to be uniformly distributed) for which the floors in each kind of building are to be designed. Such live loads may range from a low value of 40 lb. per sq. ft. for a school building floor to perhaps 250 lb. per sq. ft. for a warehouse. In setting such specified uniform live loads in the Chicago Building Code the committee of engineers considered the following points: (1) safety of the public particularly wherever crowds may gather (2) probability of change of use of a building with consequent increase or decrease of loading, (3) weakening of the structure from a fire of short duration (4) effect of heavy equipment such as a steel safe that may be moved into a building at any time during its life (5) shifting of partitions to meet the needs of new tenants (6) impossibility of perfect control by the City Building Department over the quality of design and construction and of inspection during construction and thereafter. For these reasons, specified live loads are considerably greater than the actual loads that normal operations bring onto a building floor. There is a tendency for such specified design loads to reflect overly conservative thinking.

Recognized regulatory bodies such as the American Association of State Highway Officials (A.A.S.H.O.) specify the truck or truck-train loading for

which a highway bridge must be analyzed and designed. The American Railway Engineering Association (A.R.E.A.) gives locomotive and train loads for railway bridge design. Such loadings are illustrated in Fig. 24. It will be understood that there are difficult problems to be solved in placing such moving loads on bridges in position to produce the most severe stress in a given member. The heaviest truck of the truck train or the railway locomotive

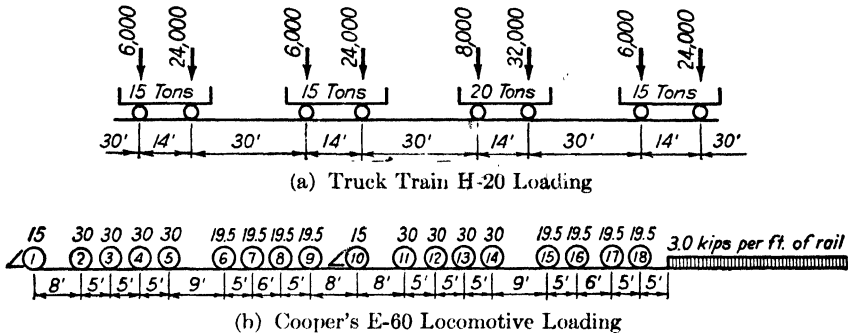


FIG. 24. LIVE LOADINGS FOR THE DESIGN OF BRIDGES.

may be placed anywhere along the length of the bridge. The question to be answered is "exactly where should it be placed to cause the greatest stress in a particular member?" This question will be answered in later chapters.

34. Special Loadings. Besides normal dead and live loads there are a number of special load conditions whose effects on the structure should be investigated. The following are the most serious: impact, load deacceleration, sway, wind, snow, centrifugal force and earthquake effects.

Impact is taken as a percentage increase in the live load. If a live load could be applied instantaneously, the value of its static load value would have to be doubled for the purpose of stress calculation. This condition would represent 100% impact. Hence, the shorter the time increment or the shorter the length of bridge which the live loading must roll onto and completely cover to produce the stress desired, the higher will be the percentage of impact. The impact formula $I = 50/(L + 125)$ for highway bridges is typical. The percentage of impact I will be only 50/500 or 10% when $L = 375$ ft. For $L = 40$ ft., which is about the loaded length of bridge that contributes to stress in a floor beam, $I = 50/165 = 30\%$.

Deacceleration, sway and centrifugal force are other dynamic effects. When the brakes are applied on a locomotive we usually assume that a longitudinal sliding force of 20% of the weight of the locomotive is produced in the direction of motion due to friction of wheel on rail. Sway is a transverse force that tends to produce overturning of the locomotive. Its value is covered by specification since it depends upon the unknown alignment of

the track and, therefore, cannot be calculated. However, a similarly acting force caused by track curvature can be calculated. Centrifugal force is equal to mass times radial acceleration or $(W/32.2)(v^2/r)$.

Earthquake acceleration in a known earthquake area is usually specified at not less than $0.1 g$ acting in any horizontal direction. This simply means that the equivalent earthquake acceleration produces a horizontal force on every part of the structure equal to 10% of the normal weight of that part plus any superimposed live loading that it may be carrying. In borderline areas the acceleration force for use in design may be reduced to $0.05 g$ or even less.

Wind and snow act on every structure. The snow load varies from 5 to 10 lb. per sq. ft. of horizontal surface in the southern states to as much as 40 lb. per sq. ft. in some northern localities. As will be discussed later, wind pressure varies with the shape of the structure, with elevation above the ground, with exposure and with geographical location. However, wind load can seldom with any degree of safety be taken at less than 20 lb. per sq. ft. of exposed area. It has probably reached 100 lb. per sq. ft. in storm centers.

35. Supports or Reactions. The manner in which a structure is supported has much to do with its stresses. Since we will deal with planar structures the maximum number of reaction components that can be calculated by the equations of statics is three, and only two reaction forces can be determined if they are either concurrent or parallel. Suppose a post is set in concrete at its lower end. This one support as shown in Fig. 25(a) has three reaction components; a vertical force, a horizontal force, and a moment. Since common structures usually have two or more points of support, it becomes evident that simpler reactions than the fixed end of Fig. 25(a) must be devised.

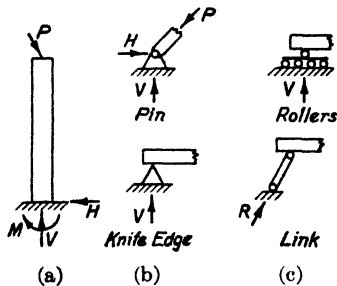


FIG. 25. KINDS OF REACTIONS.

The easiest part of the reaction to eliminate is the moment. In machines and in movable bridges an actual lubricated pin may be provided which eliminates nearly all moment resistance at the pin. For cruder structures we may merely set the structure on a narrow support and let its natural tendency to tip eliminate any moment restraint. See Fig. 25(b). In either case a small open circle on the drawing indicates the fact that moment resistance will be essentially eliminated at that point.

If two of the three components of reaction are to be eliminated at a point of support, free movement along a plane must be permitted as well as free rotation. This freedom for translation and rotation is provided effectively

by rollers or by a double-pin link. In Fig. 25(c) the horizontal bed of rollers is practically frictionless. Therefore the reaction must act normal to the plane of the rollers. The sloping link with two pins can resist a force only if it is exactly in line with the axis of the link. A reaction in any other direction would force the link to rotate into line with the direction of the reaction.

36. Stability from Reactions. Based upon our knowledge that three and only three directed reactions in the plane can be determined from the equations of statics we conclude that any one of the reaction arrangements shown in Fig. 26(a) would be statically stable. However, the parallel reaction links

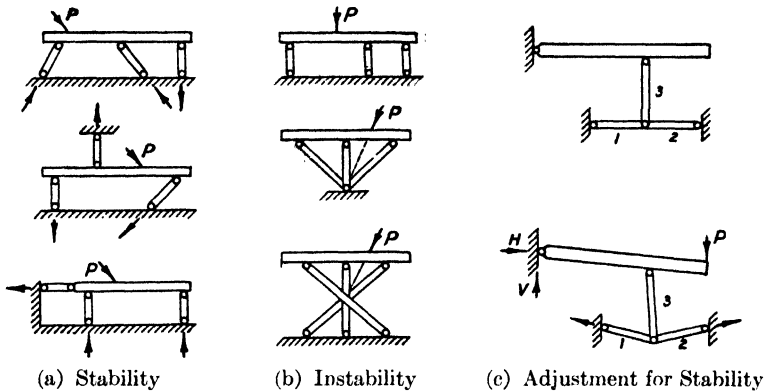


FIG. 26. STABLE, UNSTABLE AND STATICALLY DETERMINATE REACTIONS.

shown in the upper sketch of Fig. 26(b) would be unable to resist a load acting other than vertically. If the three reaction links intersect on a pin as in the middle illustration of Fig. 26(b), the structure can fall down by simple rotation about the common pin. However, in the lower illustration where the reaction links intersect at a point at which there is no pin, rotation of the structure about the point of intersection can still occur by bending the links. Such an arrangement of *hourglass reactions* is always to be avoided. For the particular loads P shown in Fig. 26(b) each of these structures would be theoretically stable. Actually, they would not be structurally practical because a slight change in the direction of the load P would produce collapse.

An unusual form of instability is illustrated by Fig. 26(c). In the upper sketch the unloaded horizontal beam is shown supported by a pin at the left and by three numbered reaction links. The reaction links (1) and (2) are horizontal but they must resist a vertical load from the link (3). For such a double link to resist a vertical force it must sag at the center as shown in the lower sketch. Then its direct stresses will provide an upward component of reaction to resist a part of the downward load P . Such a structural system in whatever form it may occur is inevitably statically indeterminate. The

reason is that stability is achieved through change of shape which obviously depends upon the relative cross-sectional areas of the bars or members and upon their relative moduli of elasticity. For instance, in Fig. 26(c) if link (1) is made of aluminum and link (2) of steel, the final slope of bar (3) will be different from its slope if all links are of steel. Evidently the reaction H at the left-hand end of the main beam depends directly upon the slope taken by the 'vertical' reaction link (3). Only in a structure with statically indeterminate reactions do the magnitudes of the reactions depend upon the deflections and, therefore, upon moduli of elasticity and cross-sectional areas of the bars.

37. When Does Superposition Work? For the structure of Fig. 26(c) the reactions and link stresses due to a load $2P$ would not be exactly twice the corresponding forces for the load P . Again the reason is found in the fact that the structure actually changes shape under the loading. Since the new shape for the load $2P$ is quite different from the deformed shape due to P

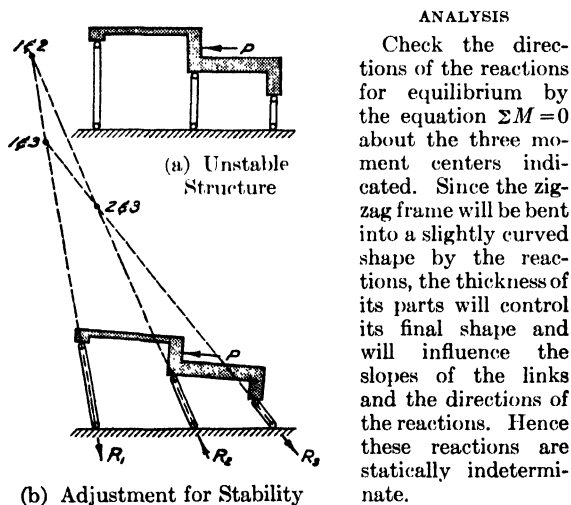


FIG. 27. VERTICAL LINKS RESIST A HORIZONTAL FORCE.

there is no reason why we should expect the stresses and reactions to be directly proportional to the loads. This is true even though the material has a straight-line relationship between stress and strain according to Hook's law. The nonlinearity that makes superposition of loads, stresses and reactions unacceptable for Fig. 26(c) is a geometrical nonlinearity.

We may also have to recognize stress-strain nonlinearity of such materials as concrete under sustained loading, cast iron, and many of the plastics for which superposition is always an approximation. Only with perfect linearity

of all relationships are exact reactions and stresses obtainable by superposition or addition of the reactions and stresses caused by individual loads. The minor shape changes of statically determinate structures and also of most statically indeterminate structures are usually neglected, but this procedure is not acceptable for such unique structures as Fig. 26(c) or Fig. 27. Fortunately, such structures are seldom encountered in practice although watch springs, leaf springs, suspension bridges and rubber shock mounts are among those structures for which change of shape influences stresses in an important manner.

38. Structural Members and Connections. The main structures that are of interest fall into four classifications (1) beam and column construction such as the ordinary building frame (2) triangulated truss construction (3) the use of curved members such as those of cables, arches and rings (4) flat and curved plate construction including tanks and the stressed skin of airplanes. The parts of steel structures are connected by rivets, pins or welds. Throughout the study of structural analysis it is necessary to decide whether a connection between two members or parts of members is to permit realignment of the parts as would be true at a lubricated pin, or whether we want the alignment to remain unchanged as if the two members or parts were frozen together. Welding and riveting may be used in the latter case although both welded and riveted connections can be designed either to permit reasonably free rotation or to maintain nearly rigid continuity. Such structural details will be shown in the following chapters but their actual design will be reserved for the author's "Design of Modern Steel Structures." In analysis we will usually assume one of two ideal conditions (a) that perfect 'pin action' will occur at a joint or (b) that an undeformable 'rigid joint' will be constructed.

CHAPTER 2

THE THEORY OF STRESS ANALYSIS

39. Review of Mechanics. Analytical mechanics investigates forces. The theory of structures uses analytical mechanics to determine the reaction forces and the internal forces of useful structures. An outline of the principles developed in the mechanics of *coplanar forces* follows:

Force is a *vector quantity* having magnitude, direction or sense, and a fixed line of action with a point of application on this line of action.

The principal of *transmissibility* states that the point of application of a force along its fixed line of action has no influence upon the equilibrium (or state of motion) of the entire body but will change the forces acting within the body.

Concurrent forces being vector quantities may be added and subtracted graphically by use of the *parallelogram law* or the *triangle law*. When two forces are added as vectors the third force obtained is their *resultant*. Then the original forces become *components* of the resultant force.

Algebraically, the resultant of two concurrent forces may be obtained by use of the *cosine law*

$$R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2 \cos \theta}$$

where θ is the angle within the triangle of forces opposite R . Then the direction of the resultant is obtained from the *law of sines*,

$$\frac{\sin \theta}{R} = \frac{\sin \alpha}{F_1} = \frac{\sin \beta}{F_2}$$

where each angle is opposite the corresponding force in the triangle of forces.

The resultant of *several* concurrent (coplanar) forces may be obtained by use of the *force polygon*. The resultant acts through the point where the forces intersect.

A resultant of concurrent forces not only replaces these forces in their effect upon possible translation of the body, but, by Varignon's theorem, it also replaces their *moment* or rotational effect.

A *couple* may be turned and translated within its plane or moved into a parallel plane without changing the overall turning or twisting effect on the body. If its moment is constant, the size or direction of its forces (within its plane) does not influence its tendency to produce rotation.

Parallel forces acting in a plane may have a resultant that is a force or a couple or both a force and a couple.

A force with a given line of action may be replaced by a parallel force and a couple or *vice versa*.

The resultant of parallel forces or of any other nonconcurrent group of forces can be obtained in magnitude and direction from a force polygon and in line of action by passing the resultant through the intersection of the first and last strings of the *funicular polygon*. If the first and last strings are parallel, the resultant is a couple.

Algebraically the resultant of coplanar forces that are neither parallel nor concurrent is found from

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}.$$

Then $\tan \theta = \Sigma F_y / \Sigma F_x$ where θ is the angle between R and the x -axis. The line of action of the resultant follows from the requirement that it produce the same turning moment about any point in the plane as the original forces.

A single force can never produce equilibrium of a body except when it has a zero magnitude.

Two forces acting alone on a body may be in equilibrium only if they have the same line of action, identical magnitudes and if they act in opposite directions.

Three coplanar forces acting alone on a body may produce equilibrium only if they are concurrent or parallel.

Four coplanar forces acting alone on a body may produce equilibrium whether they be concurrent, parallel, or of no specialized group.

Any group of forces and any number of forces from one to infinity may, however, fail to be in equilibrium.

For equilibrium the arrows of the forces in the triangle of forces or in the force polygon follow consecutively (*tip to tail*) and close the polygon.

The closure of the force polygon guarantees a zero resultant force which is expressed algebraically in mechanics by the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. The closure of the funicular or equilibrium polygon guarantees a zero resultant couple which may be expressed algebraically by the equation $\Sigma M = 0$.

EQUILIBRIUM OF STRUCTURES

40. The Laws of Statics. The statement that a body is in *static equilibrium* simply means that it is not moving. A structure may move as a unit in the way that an airplane frame or a lift bridge moves; but the individual members of a single structural unit do not move relatively to each other, for the structure would then become a *machine*. All stationary structures, and indeed all moving structures when acceleration or inertia forces are considered, must be acted upon by a set of forces (loads and reactions) that are in equilibrium. The laws of statics state that, for any set of coplanar forces in equilibrium, the algebraic sum of the components of these forces along any axis in this plane is equal to zero; and further, that the moment of these forces about any point in this plane is equal to zero. It is unnecessary to consider here the general case of forces in space, because the loads, reactions, and stresses of beams and trusses usually may be considered to act in a *single plane*. The study of space structures will be reserved for later study.

The Equations of Statics. For convenience, structural engineers are accustomed to write the laws of statics for coplanar forces in shorthand as follows:

- | | |
|-----|--|
| (1) | $\Sigma H = 0$ (often written $\Sigma F_x = 0$ in mechanics textbooks) |
| (2) | $\Sigma V = 0$ (" " " " " " ") |
| (3) | $\Sigma M = 0$ (" " " " " " ") |
- $\Sigma M_c = 0$ for center of moments C)

The equations $\Sigma H=0$ and $\Sigma V=0$ refer to the summation of the force components along horizontal and vertical axes. Any pair of rectangular axes may be used with equal confidence. Axes chosen parallel and perpendicular to main truss members are sometimes found convenient in the analysis of roof trusses. The equation of moments, $\Sigma M=0$, has no such limitation, since it applies to the moment of the forces about *any point in the plane of the loads*.

41. Stress Analysis by Statics, *Free Body Diagrams*. Any structure, part of a structure, truss joint, truss member, part of a member, or even any infinitesimal particle of a beam or truss must be acted upon by a set of forces

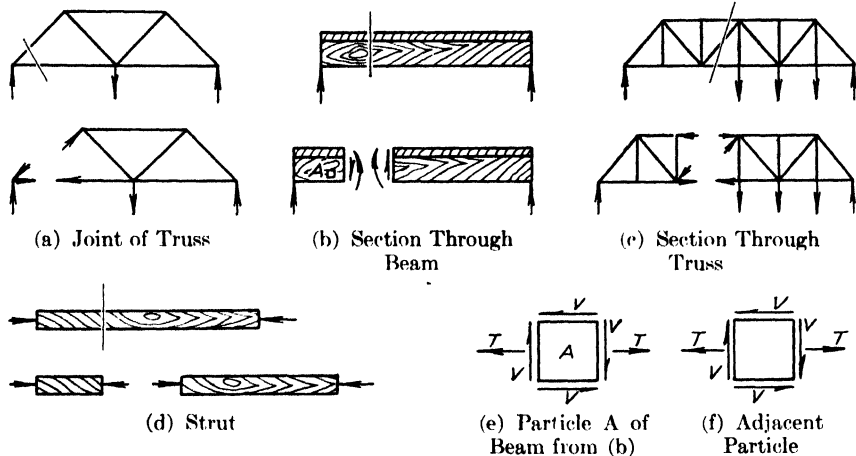


FIG. 28. ISOLATED PARTS OF STRUCTURES.

that are in *equilibrium*. In order to compute the values of the reactions, we can apply the three equations of statics to the structure as a whole. Thus, in general, three reaction components can be determined. When stresses are to be calculated, a part of the structure is cut away or isolated from the rest, and the internal stresses at the cut section are shown as external forces on the isolated part of the structure. *The forces pictured should represent the effect of the remainder of the structure upon the isolated part.* Accordingly, a compressive stress is shown as a *push* and a tensile stress as a *pull*. A few structures are cut by sections and the isolated parts are shown in Fig. 28. Such isolated parts of structures with all contact and body forces shown are called "*free body diagrams*." Notice that the sum of the forces placed at the cut section will always be zero, since no actual *external* forces exist there.

The Number of Stresses Determinable by Statics. In general, three unknown forces can be calculated for an isolated body by use of the three equations of statics. Only two unknowns can be determined from the equations $\Sigma V=0$ and $\Sigma M=0$ for *parallel coplanar forces*, such as the vertical reactions of a truss or beam. The equation $\Sigma H=0$ merely shows

that no horizontal reaction is needed when all loads are vertical. Two unknowns can be found for *concurrent coplanar forces* by use of the equations $\Sigma H = 0$ and $\Sigma V = 0$. The equation $\Sigma M = 0$ is not needed because any group of concurrent forces that obeys the equations $\Sigma H = 0$ and $\Sigma V = 0$ must have a zero moment about any point in its plane. All three of the equations of statics are independent for *nonconcurrent non-parallel forces in a plane*, and three unknown forces can be determined. Such forces exist at a cross-section of a beam and at most sections that are cut through trusses. See Fig. 29.

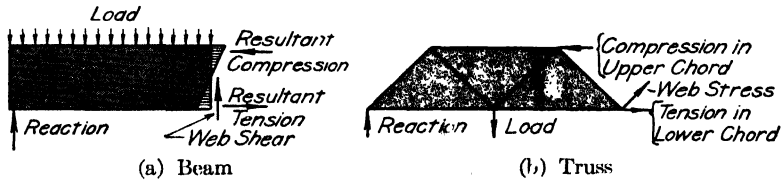


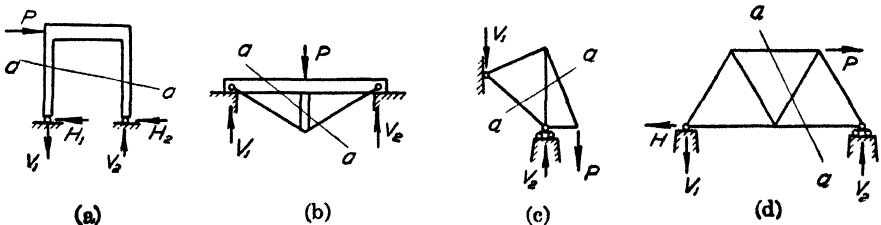
FIG. 29. COMPARISON OF INTERNAL STRESSES.

It is of interest to observe in Fig. 29 the correspondence of loads, reactions, flange stress and chord stress, web shear and web stress between the beam and the truss. This similarity should be studied because it is the basis of a simplified method of truss analysis.

Use of Resultants. An error is made frequently by the novice in attempting to replace several loads on a structure by their resultant. It is permissible to replace all loads on a structure by their resultant when calculating reactions by use of the entire structure as a free body. Only those forces acting to the right or left of a cut section may be replaced by their respective resultants when the shears or moments in a beam, or the stresses in a truss, are computed. This limitation will be clear if we recall that only those forces to the left of a given section would *even be shown* on a free-body sketch of that portion of the structure to the left of this section. A resultant that included some forces to the right of the section certainly should not be used. The ability to draw free-body diagrams is of great importance in structural analysis.

PROBLEMS

1. Draw neat sketches showing the isolated parts of each structure as cut by the section *a-a*. Study the directions of the loads and reactions in order to show the internal forces and moments acting in their correct directions on each free body diagram.



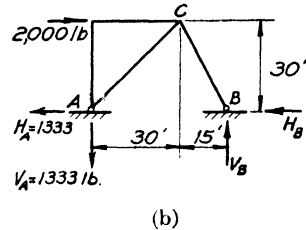
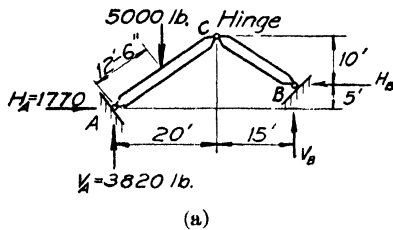
PROBLEM 1.

2. Isolate each bar of these structures and show the forces acting on each. Determine the direct stresses in the bars.

HINT. Start by isolating the entire structure as a free body to find the reactions at B.

Ans. to (a), $S_{BC} = 2130$ lb. C.

Ans. to (b), $S_{BC} = 1490$ lb. C.



PROBLEM 2.

42. Graphical Conditions of Equilibrium — Concurrent Coplanar Forces.

Corresponding to the mathematical requirements that the equations $\Sigma H = 0$ and $\Sigma V = 0$ must be satisfied for a set of concurrent forces in static equilibrium, there is the necessary and sufficient condition of graphical analysis that *the force polygon must close*. Consider the four concurrent forces acting at the left-hand end of the truss of Fig. 30(a). These forces are drawn to scale in (b) with the arrows pointed continuously around the polygon. The polygon (c) is drawn to show that closure is obtained when the forces are taken in a different order. *The order chosen is immaterial*, but the arrows must point *continuously* around the polygon.

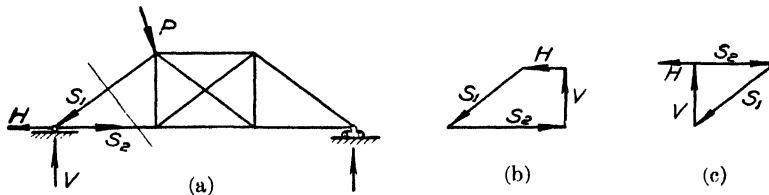


FIG. 30. FORCE POLYGONS FOR THE LEFT-HAND REACTION JOINT.

43. Graphical Conditions of Equilibrium — Non-Concurrent Coplanar Forces. Non-concurrent forces in equilibrium must satisfy the same conditions as concurrent forces plus the requirement expressed by the equation of statics, $\Sigma M = 0$. The mathematical requirement expressed by this equation is satisfied graphically by the necessary condition that *the equilibrium polygon shall close*. The procedure for drawing the equilibrium polygon or funicular*

* This is also called the *string* polygon. *Funicular* simply means string-like. We will define the *equilibrium* polygon to mean the special case of the string or funicular polygon when the forces considered are in equilibrium.

polygon will be explained in the following sections. The equilibrium polygon is a useful device, but it is not possible to give it quite the physical significance that is so evident in the force polygon.

44. Bow's Notation. In order to obtain the greatest saving of time by graphical constructions, it is necessary to follow a standardized system of

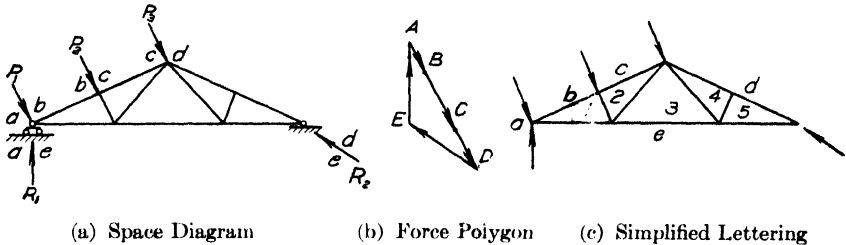


FIG. 31. BOW'S NOTATION.

notation. Bow's notation (Robert Bow — England — 1873) consists of the use of two letters or figures placed at the two ends of a vector in the force polygon to designate the force represented by it. Then the same letters or figures are written on the two sides of the action line of the same force. Figure 31 represents the use of Bow's notation. In (a) and (b) the load P_1 is properly termed *force A-B with action line a-b*; the reaction R_2 is *force D-E with action line d-e*.

The sketch (c) shows how *lettering can be economized* by placing a single letter between each pair of forces. It also shows how internal stresses can be designated by numbers. The force 3-E would represent the stress in the center member of the lower chord while the force 2-3 would represent the stress in the adjacent web member.

45. The Equilibrium Polygon. The forces in Fig. 32 are lettered with lower-case letters in the space diagram and with upper-case letters in the force

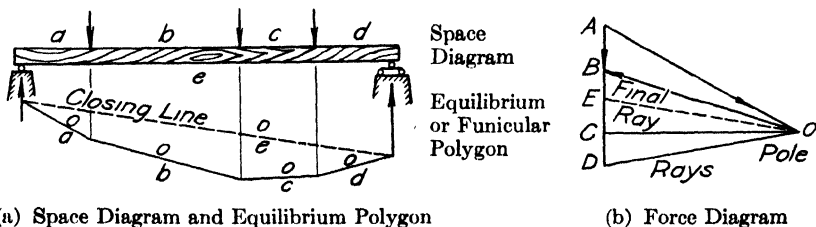


FIG. 32. GRAPHICAL CONSTRUCTION OF THE EQUILIBRIUM POLYGON.

polygon. We will assume that the forces shown (loads and reactions) are in equilibrium. A pole O is chosen conveniently to either side of the force polygon and the rays $O-A$, $O-B$, etc. are drawn. Clearly $A-O$ and $O-B$ are

components of the force $A-B$. (Law of the triangle of forces from mechanics.) Similarly, $B-O$ and $O-C$ are components of $B-C$. Note then that the force $O-B$ acting as a component of $A-B$ is equal but oppositely directed to the force $B-O$ acting as a component of $B-C$. When each ray is thus looked upon as representing two equal and opposite force components, the entire set of rays forms a substitute force system that will replace the effect of the original force system on the structure. Hence, if these equal and opposite pairs of forces can be shown to be collinear, both the original force system and the substitute force system must be in equilibrium. In the construction of the equilibrium polygon, as will be shown, we will place each pair of equal and opposite force components along the same line of action so that the conditions of equilibrium will be fulfilled.

PROCEDURE. The equilibrium polygon in Fig. 32(a) is drawn by replacing each force in the space diagram by its two components from the force diagram. Naturally the two components of each force must intersect on that force. Hence the reaction acting along the line $e-a$ in the space diagram is replaced by the strings $e-o$ and $o-a$ drawn parallel to the rays $O-E$ and $O-A$; the load with action line $a-b$ is replaced by the strings $a-o$ and $o-b$; etc. Note that we arrange the replacement so that the action line of $o-a$ when we replace $E-A$ is the same as the action line of $a-o$ when we replace $A-B$. Therefore, the force components $o-a$ and $a-o$ which have equal magnitudes, opposite directions, and the same line of action, cancel each other completely. In the same manner, each string (from $o-a$ to $o-e$ inclusive) of the equilibrium polygon in Fig. 32(a) represents the action line of two equal and opposite force components which cancel each other. But this will only be true for all strings if the string or funicular polygon is actually closed. It follows that the original forces which form a closed force polygon must be in equilibrium when the string polygon also closes. If the string polygon does not close, one pair of equal and opposite

force components will not have the same line of action but will have parallel lines of action. Hence the original forces cannot be in moment equilibrium. Instead, their resultant will be a couple.

46. The Equilibrium Polygon and the Equation $\Sigma M = 0$. Consider the body shown in Fig. 33 acted upon by the force P or $A-B$, which may properly be considered to represent the resultant of several forces, such as loads and reactions to the left of a section. The rays $O-A$ and $O-B$ are

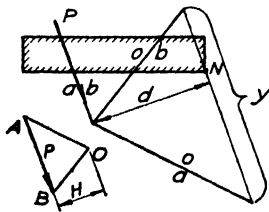


FIG. 33. MOMENT BY GRAPHICS.

drawn from a pole O in the force diagram, and these rays are paralleled by the strings $o-a$ and $o-b$ on the space diagram. The intercept between these strings on a line drawn through the point N parallel to the resultant force P is called y . The pole distance H in the force diagram is taken perpendicular

to the force P , as is the distance d in the space diagram. Then, from similar triangles, we find

$$(4) \quad \frac{y}{d} = \frac{P}{H}, \quad \text{or} \quad Hy = Pd = M_N,$$

the moment of P about the point N . Observe that the intercept y must be measured on a line through the moment center parallel to the resultant load, and that the pole distance H must be measured normal to the resultant load.

Moment Diagram by Graphics. In Fig. 34, the intercept of the string polygon under the point N , or y_1 , lies between the strings $o-b$ and $o-d$. Accordingly, y_1H represents the moment of the force $D-B$ (whose components are $D-O$ and $O-B$ in the force diagram) about the point N . Note that DB is also the resultant of the load and reaction to the left of section N .

By definition the moment about the section of the resultant force to the left of the section is the bending moment at the section. Hence y_1H is the

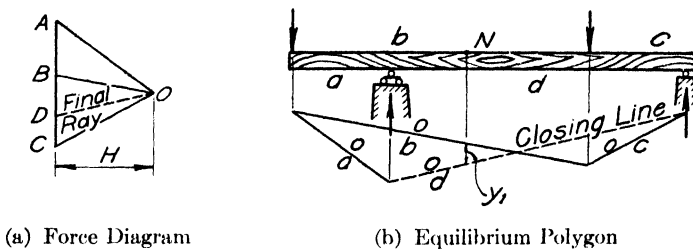


FIG. 34. BENDING MOMENT OBTAINED GRAPHICALLY.

value of the bending moment at the section N in Fig. 34. In general we may state that the intercepts of the equilibrium polygon for a beam when multiplied by the pole distance are equal to the corresponding bending moments.

Units for Graphical Moments. The pole distance furnishes the scale for measuring intercepts on the equilibrium polygon. More specifically, intercepts from the equilibrium polygon measured to the scale of the space diagram (ft.) can be multiplied by the pole distance measured to the scale of the force diagram (lb.) to obtain bending moments in corresponding units (ft.-lb.). Hence the equilibrium polygon is the bending moment diagram to scale when the pole distance in the force diagram is chosen as unity. It is more common to choose a pole distance of 1000 lb. or 10,000 lb. Then the intercept on the equilibrium polygon measured in feet is simply multiplied by 1000 or 10,000 to obtain the bending moment in foot-pounds.

Graphical Criterion for Rotational Equilibrium. The force $D-B$ in Fig. 34 is either the resultant of $D-A$ and $A-B$, the forces to the left of N , or, with sign reversed, it may represent the resultant of $B-C$ and $C-D$, the forces to the right of N . The product y_1H may therefore represent the moment about N

either of the forces to the right or left of the point N . Since these equal moments have opposite signs, we conclude that the total moment of all forces about N is zero, or that the structure is in rotational equilibrium about this point. But, as N was taken to represent any point on the structure, the general statement may be made that closure of the equilibrium polygon is proof that the structure is in rotational equilibrium about all points in its plane, that is, the loads and reactions satisfy the equation $\Sigma M = 0$.

PROBLEMS

NOTE TO THE INSTRUCTOR. These problems involve some propositions from mechanics that may require classroom review.

3. Show how to obtain graphically the resultant of a series of concurrent forces that are not in equilibrium. Set up a special case and draw the force polygon.

4. Four vertical forces of 1500 lb. each act at intervals of 10 ft. alternately up and down so that each pair of adjacent forces forms a clockwise couple of 15,000 ft.-lb. value. Draw a string polygon and note that the initial and final strings are not collinear. Scale the values of these strings in pounds and the distance between them in feet and compare the moment obtained with the magnitudes of the two couples.

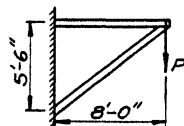
5. Determine the dead-load stresses for the highway bridge truss shown, by drawing a separate force polygon for each joint. Each of the two trusses weighs 38,800 lb. The total weight of steel in the stringers, floor beams, laterals, sway frames, portals, and hand-rails is 62,300 lb. The 18-ft. roadway requires a concrete floor slab $19\frac{1}{2}$ ft. wide and 8 in. thick. Each curb is 9 in. high and 9 in. wide. The concrete weighs 150 lb. per cu. ft. Place the entire dead load at the lower-chord panel points.

SUGGESTION. Two unknown stresses can be found at each joint from the condition that the force polygon must close.

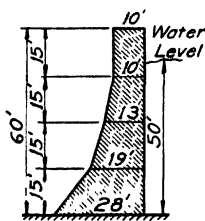
Ans. $S_{\max} = 119$ kips.



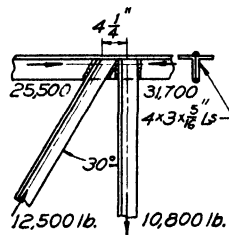
PROBLEM 5



PROBLEM 6



PROBLEM 8



PROBLEM 10

6. The welded cantilever crane has a horizontal tension member composed of two steel bars 2 in. by $\frac{5}{16}$ in. If the crane was designed equally strong in all parts, what load would it lift at a working stress of 20,000 lb. per sq. in. in the horizontal member?

Ans. 17,200 lb.

7. Develop a graphical procedure for determining the resultant of a series of nonconcurrent forces that are not in equilibrium. Consider the use of partial resultants.

SUGGESTION. Where intersections of the forces do not occur on the sheet, it is necessary to use the uncanceled strings of the funicular polygon. The strings are components of the resultant and the resultant must therefore pass through their intersection.

8. Determine the resultant base pressure per lineal foot for the dam shown. Estimate the weight of masonry at 145 lb. per cu. ft. For stability, the resultant pressure must be within the middle third of the base. Is this dam stable against overturning? Check your graphical work by algebraic calculations. In practice a wider base would be used.

Ans. R per lineal ft. = 154 kips acting at 18.5 ft. from heel.

9. Graphically determine the moment diagram for a simple beam or cantilever uniformly loaded. Treat the uniform load as a series of ten concentrations. Show that the polygon obtained approximates a parabola.

SUGGESTION. By proper choice of the pole distance in the force diagram, an equilibrium polygon may become a bending moment diagram to any desired scale.

10. By drawing a force polygon and an equilibrium polygon, show that the resultant of the stresses meeting at this joint is a couple whose value is 45,900 in.-lb. Assume that each top-chord member resists one-third of this moment. Determine its flexural stress. Suggest an improved detail.

Ans. 6200 lb. per sq. in.

REACTIONS OF STRUCTURES

47. Supports for Beams and Trusses. The function of the reactions of a structure is to resist the tendency of the loads to move the structure in any direction. Frequently it is necessary at the same time to arrange the end supports of a structure so that certain motions are possible. For instance, a

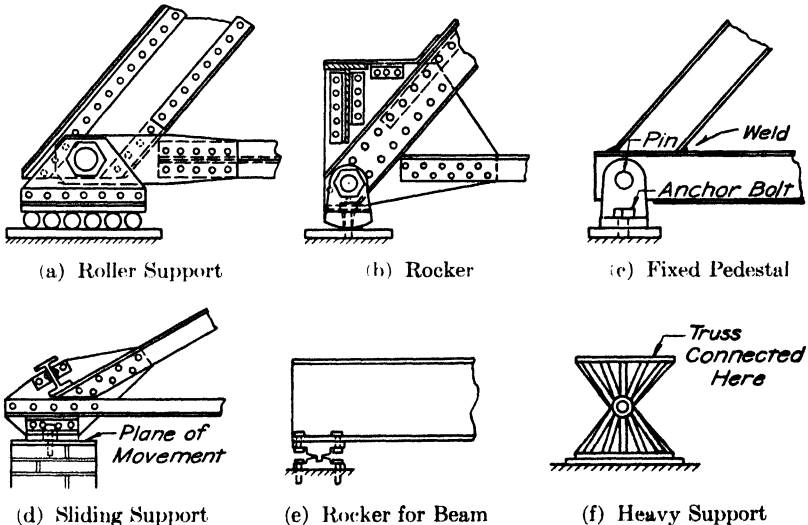


FIG. 35. REACTION SUPPORTS FOR BEAMS AND TRUSSES.

bridge must have one end supported on rollers or rockers to allow for the movement which occurs from increase or decrease in the span length because of temperature change. A fixed support with a rotating pin is desirable at the other end to take care of the end rotation that accompanies vertical deflection of the structure. A roller, a rocker, a sliding shoe, and a fixed

support are shown in Fig. 35. Each of these types of support is used in bridges or buildings. It is assumed that the reader will familiarize himself with the appearance and use of such elementary structural devices as rollers, pins, hinges, rockers, and links. A reaction pin and a roller box are seen in Fig. 36.



Courtesy American Bridge Co.

FIG. 36. EXPANSION SHOE.

Symbols for Reactions. We must agree on the meaning of the symbols that will be needed for use on sketches to designate the kind of reaction support to be provided. At the end of a simple beam a knife-edge support $\triangle$ is sometimes designated. This symbol means that the reaction is normal to the support and that rollers are not needed. If freedom of end rotation is considered very important as at the end of a bridge truss, this symbol will usually be changed to $\triangle$ or $\triangle$ where a pin is to be provided.

See Fig. 35(c), (e) or (f). In important structures it is often necessary to provide free movement normal to a reaction. In such structures the symbol $\triangle$ or $\triangle$ with three circles represents a roller or rocker support similar

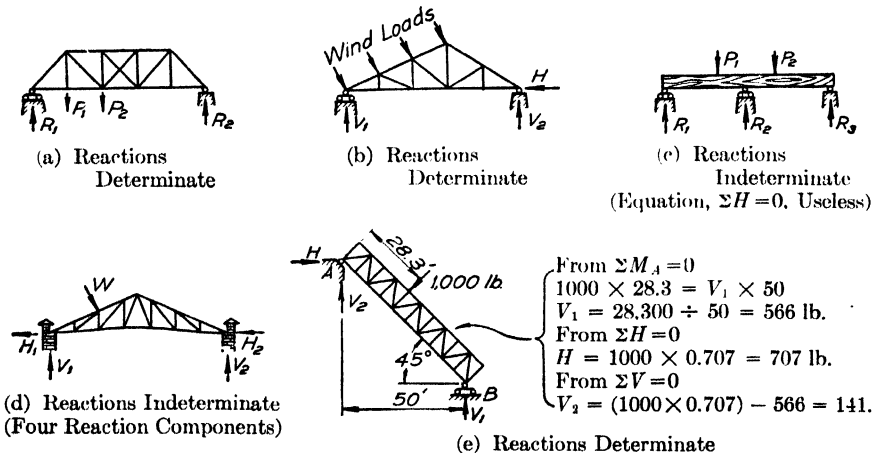


FIG. 37. STATICALLY DETERMINATE AND INDETERMINATE REACTIONS.

to (a) or (b) of Fig. 35. At such a support the reaction must always be normal to the plane of the rollers. When the support symbol shows only a single pin we understand that the reaction may have any direction or slope.

Calculation of Reactions. It should be understood by this time that the three equations of statics are sufficient for calculating three reaction unknowns and *no more than three*. Three things may be unknown about a single force: its magnitude, its slope or direction, and its point of application. This information can be obtained if its H component, its V component, and its point of application are known. Ordinarily the point of application of the reaction is known, and one or both of its two components are to be determined. Since two sloping reactions (unknown in value and direction) have four components, which could not be calculated by the three equations of statics, it is usually necessary to arrange the manner of support so that one component of one reaction will be zero. This requirement is met by placing one end of the truss on rollers in order to make the horizontal component of reaction equal to zero at that end. As has been explained, this arrangement also allows for free temperature expansion or contraction. Trusses that have more than three reaction components are called *statically indeterminate* in regard to the external forces. Structures of both types are shown in Fig. 37.

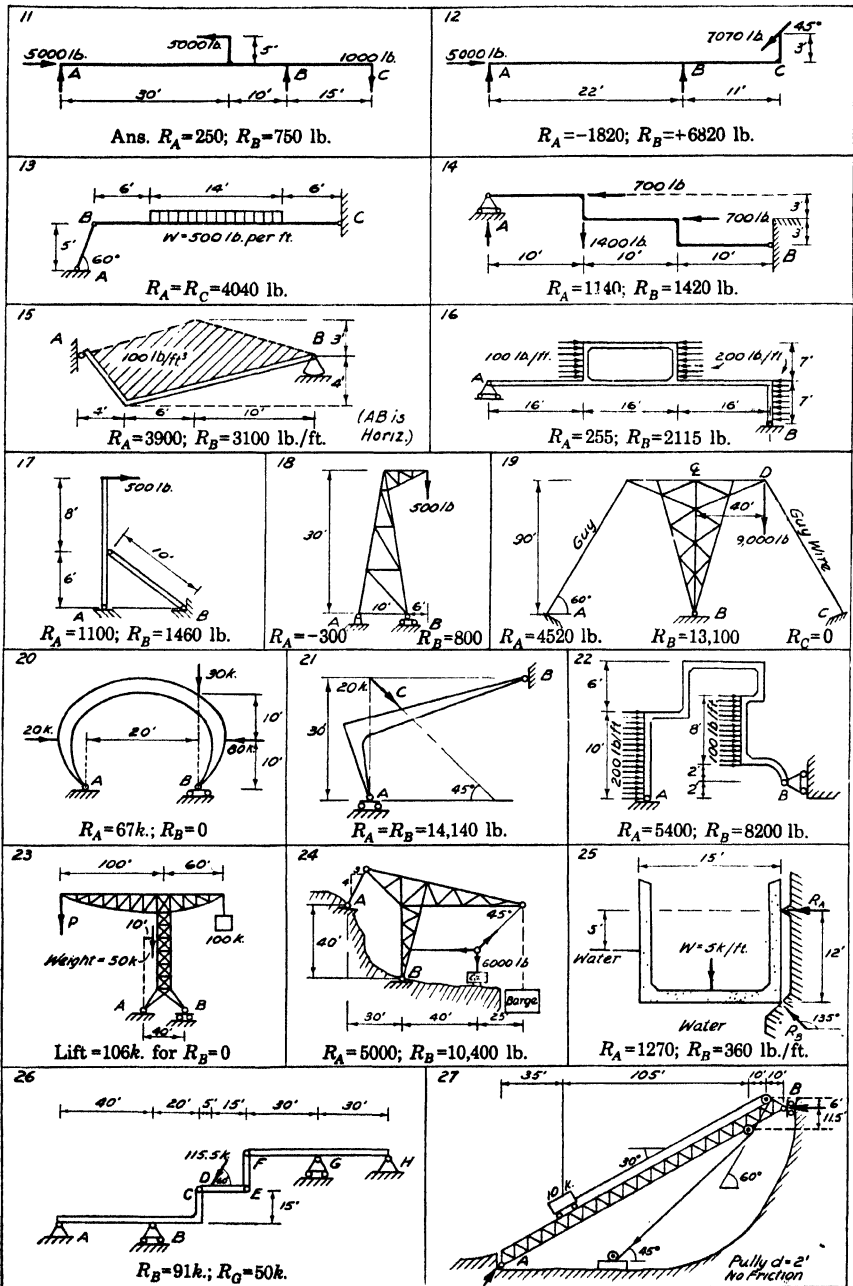
PROBLEMS

11-27. These problems are set up for practice in the calculation of reactions that are in some degree unusual. The equations of statics are nevertheless adequate for all calculations. Answers are in pounds unless otherwise designated. See p. 42 for illustrations.

- | | |
|--|--------------------------------|
| 11. $R_A = 250, R_B = 750$ | 12. $R_A = -1820, R_B = +6820$ |
| 13. $R_A = R_C = 4040$ | 14. $R_A = 1140, R_B = 1420$ |
| 15. $R_A = 3900, R_B = 3100$ lb. per ft. | 16. $R_A = 255, R_B = 2115$ |
| 17. $R_A = 1100, R_B = 1460$ | 18. $R_A = 300, R_B = 800$ |
| 19. $R_C = 0, R_A = 4520, R_B = 13,100$ | 20. $R_A = 67k., R_B = 0$ |
| 21. $R_A = R_B = 14,140$ | 22. $R_A = 5400, R_B = 8200$ |
| 23. Lift = 106k. for $R_B = 0$ | 24. $R_A = 5000, R_B = 10,400$ |
| 25. $R_A = 1270, R_B = 360$ lb. per ft. | 26. $R_B = 91k., R_G = 50k.$ |
| 27. No answer given. | |

48. Reactions under a Simple Span — Vertical Loads. *The algebraic solution* of this problem is quite elementary. The equation of statics, $\Sigma M = 0$, when written for a center of moments at the right-hand reaction, can be solved for the one unknown which is the left-hand reaction. We then determine the right-hand reaction from the equation $\Sigma V = 0$. The values of the reactions can be checked by use of an equation of moments about the left-hand reaction.

The graphical procedure for determining the vertical reactions under a horizontal span is in exact correspondence with the algebraic method. We first draw a partial force polygon (reactions as yet unknown) and a partial



PROBLEMS 11-27.

funicular or string polygon. Figure 38 shows these constructions in solid lines. Then the string $o-a$ is extended (broken line) until it intersects the reaction line $d-e$ at r . Note that the intercept $q-r$ or y between the strings $o-a$ and $o-d$, when multiplied by the pole distance H , is equal to the moment of the three loads about R_2 . Hence, $R_1 = yH \div s$. However, if the closing

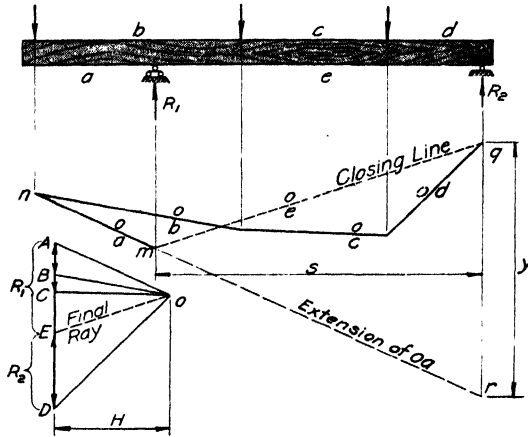


FIG. 38. REACTIONS FOR A SIMPLE OVERHANGING BEAM.

line $m-q$ (string $o-e$) is drawn and paralleled by the ray $O-E$ in the force polygon, it is noticed that the triangle $A-O-E$ is similar to the triangle $r-m-q$ since corresponding sides are parallel; therefore we may write

$$(5) \quad \frac{y}{s} = \frac{E-A}{H}.$$

But since $R_1 = yH \div s$ we may write

$$(6) \quad R_1 = \frac{yH}{s} = \frac{(E-A)H}{H} = E-A,$$

and it follows that $R_2 = D-E$. Hence we conclude that a ray drawn parallel to the closing line of the equilibrium polygon will divide the load line into the two reactions which may then be scaled. The extension of the string $o-a$ to r was made in Fig. 38 to illustrate the proof given above; it is not needed in the solution of problems.

49. Reactions of a Roof Truss — Wind Loads. This is the general problem of determining two reactions, one known in direction and point of application and the other known in point of application only. *Algebraically*, the equation $\Sigma H = 0$ shows that the H component of R_1 (see Fig. 39) is equal and opposite to the H component of the loads when R_2 acts vertically. Then an

equation of moments about the left-hand pin contains but the one unknown, R_2 , which is determined. Finally, the equation $\Sigma V = 0$ makes possible the calculation of the V component of R_1 , after which the reaction R_1 can be found both in magnitude and direction from its two known components.

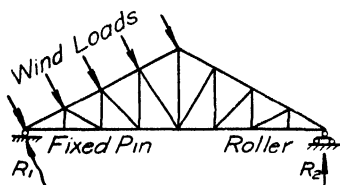


FIG. 39. WIND REACTIONS.

the point F . ($F-G = R_2$.) Observe that the loads and reactions are laid out in a clockwise order.

The funicular polygon must be started at the one known point on the action line of the sloping reaction, which is the point m in Fig. 40(a). The funicular polygon is then constructed to an intersection at r on the action line of R_2 . Note that the string $o-a$ parallel to the ray $O-A$ connects two intersecting forces at their point of intersection and hence disappears. The closing string $o-g$ (line $r-m$) is paralleled in the force diagram by the ray $O-G$, which is extended to an intersection with a vertical line through F . The force $F-G$ is scaled to obtain the magnitude of the reaction R_2 , while the force $G-A$ represents both the magnitude and direction of R_1 .

Graphically, the procedure is almost identical with the method already discussed for determining the vertical reactions under a simple span. Lay out the force diagram $A-B-C-D-E-F$, Fig. 40 (b). Choose a pole O and draw in the rays $O-A$ to $O-F$ inclusive. The location of G is unknown, but, since R_2 is vertical, G must lie on a vertical line above

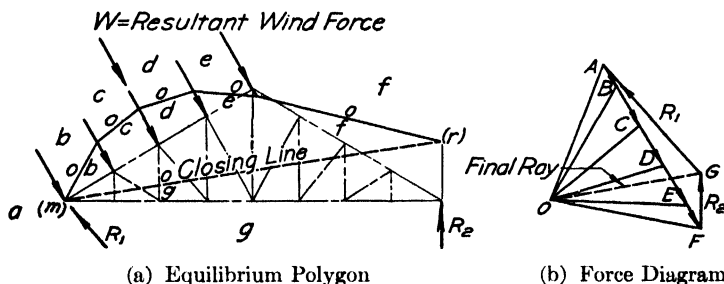


FIG. 40. TRUSS REACTIONS FROM WIND.

Simplification. This problem can be much simplified in most cases by replacing all wind loads by their resultant force [see W in Fig. 40(a)] before starting the force polygon. However the use of a resultant load is recommended only when the line of action of W may be determined by inspection. This simplification is also useful in the algebraic method. Use is made of this simplification in Fig. 42.

50. Reactions for a Three-Hinged Arch. The structure of Fig. 41 has two sloping reactions or four reaction components. However, it is not statically indeterminate, since the hinge at the crown furnishes an extra equation, $\Sigma M_{\text{crown}} = 0$, which makes the algebraic determination of the reactions possible by statics.* In Fig. 41, the two reaction points are shown on the same level. Hence the equation $\Sigma M_B = 0$ contains only the one unknown, V_1 , which can be determined. Then the equation of moments for the forces to the left of C , $\Sigma M_C = 0$, will contain but one unknown, H_1 . This equation can be solved for the value of H_1 . If the supports are at different levels,

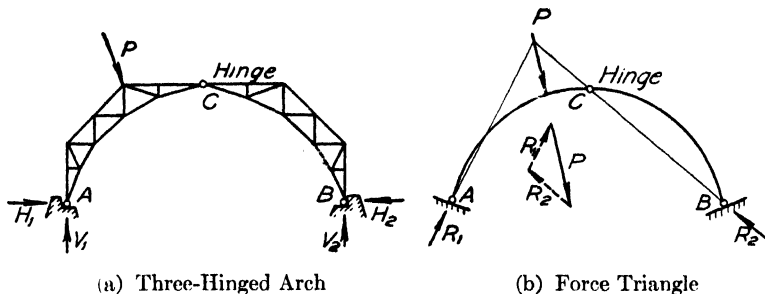


FIG. 41. REACTIONS FOR A THREE-HINGED ARCH.

these two equations will each contain the two unknowns, H_1 and V_1 . Such equations must be solved simultaneously. As soon as H_1 and V_1 are known, H_2 and V_2 can be determined from the equations $\Sigma H = 0$ and $\Sigma V = 0$ applied to the entire structure.

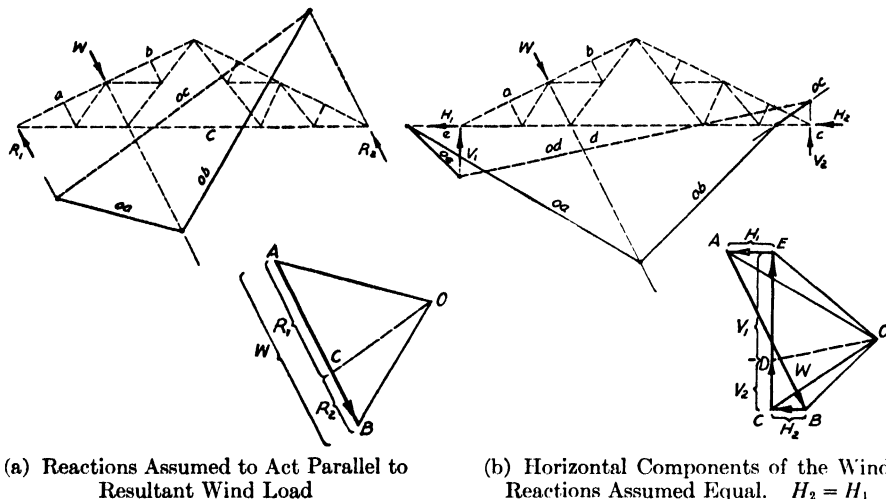
The graphical determination of the reactions is illustrated in (b) of Fig. 41. The action line of R_2 (reaction on the unloaded side of the arch) must pass through the center hinge, because the bending moment there (moment of R_2 , the only force to the right of C) must be zero. The two reactions, being components of the load, must intersect somewhere on the action line of the load. When their action lines have been determined in the space diagram, the values of the two reactions may be found from the force diagram. This method can be used to determine the reactions for each of several forces separately and then these partial reactions can be combined into a resultant reaction.† It is more convenient to make use of two resultant loads, one for each half of the arch. A single resultant for loads on both halves of the arch

* A frictionless pin cannot resist moment. However, we do not obtain an independent equation by writing $\Sigma M_C = 0$ for the entire arch as a free body. This equation must hold true for equilibrium even without a pin at the crown. Instead, we isolate as a free body the part of the structure to one side of the crown and obtain an entirely new and independent equation, $\Sigma M_C = 0$ for the forces acting on this free body.

† A direct method of computing the reactions of a three-hinged arch graphically introduces the problem of passing an equilibrium polygon through the three hinges. See Johnson, Bryan, and Turneaure, *Modern Framed Structures*, Part I, 1926 Edition, p. 59.

can not be used conveniently because neither reaction for this case will pass through the central pin. The moment of a reaction alone is not zero about the center hinge when a load exists between that reaction and the hinge.

51. Two Sloping Reactions for a Roof Truss. Unlike the three-hinged arch, the roof truss becomes statically indeterminate externally if there are two sloping reactions, both unknown in direction. Small roof trusses may be constructed cheaply by bolting both ends down to pillars or masonry walls. This arrangement produces four unknown components of reaction.



(a) Reactions Assumed to Act Parallel to Resultant Wind Load

(b) Horizontal Components of the Wind Reactions Assumed Equal. $H_2 = H_1$

FIG. 42. REACTIONS DETERMINED GRAPHICALLY BY USE OF ASSUMPTIONS.

Since the stresses in such trusses are small, it is considered satisfactory to assume the direction of one or both of the reactions. For instance, in Fig. 42(a), the assumption is made that the line of action of each reaction is parallel to the resultant of the wind loads.

Algebraically, the two reactions in Fig. 42(a) can be computed from two equations of moments about the reaction points. Another assumption that is commonly used is that each of the two horizontal components of reaction is equal to one-half of the horizontal component of the entire wind load. See Fig. 42(b). Each of these assumptions gives reactions that are only approximations. These approximate methods of computation should not be used for large trusses until a careful study has shown that the greatest possible error in the reactions will not *endanger* the structure.

Graphically, the determination of two sloping reactions assumed to act parallel to the wind loads needs no explanation. The procedure becomes self-evident from Fig. 42(a). It is almost identical with the determination of the vertical reactions under a simple span. Figure 42(b) shows how the reactions

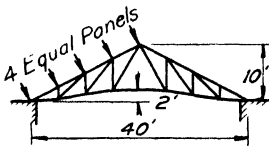
must be divided into components when the H components of the reactions are taken as equal. One must observe a *regular order*, either clockwise or counterclockwise, in laying out the force polygon.

52. Directions or Signs of Reactions. Since the reactions hold the loads in equilibrium, their directions can usually be determined by inspection. When we are not certain of the direction of a reaction, we may assume its direction. Then its *algebraic* sign, when determined by the equations of statics, will be positive if the assumed direction is correct, and negative if the assumed direction must be reversed. The sign or direction of the reaction can be found from the *graphical* requirement that *all arrows must point continuously around the closed force polygon of the loads and reactions*.

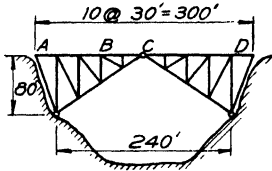
PROBLEMS

28. Compute the values of the reactions for the roof truss as caused by a wind pressure of 25 lb. per sq. ft. of roof surface acting normal to the roof. Trusses are spaced 16 ft. apart. Assume that the right-hand end of the truss is supported on rollers. Check your calculations by graphical determination of the reactions.

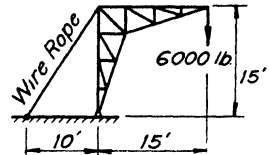
Ans. Right-hand reaction = 2500 lb. Left-hand reaction = 6800 lb.



PROBLEM 28.



PROBLEM 31.



PROBLEM 32.

29. Same as Problem 28, but assume that the reactions act parallel to the wind load.

Ans. Right-hand reaction = 2810 lb.

30. Same as Problem 28, but assume that the horizontal components of the two reactions are equal.

Ans. Right-hand reaction = 3200 lb.

31. Compute the values of the reactions for the three-hinged arch caused by the dead load of 3000 lb. per ft. acting from A to D. Then place a single vertical load of 30,000 lb. (representing a road roller) at point B and determine the reactions both algebraically and graphically.

Ans. D. L. reaction = 516,000 lb.

32. Graphically determine the values of the reactions for the cantilever truss illustrated.

Ans. Rope stress = 10,800 lb

Hint: Start a string polygon at the one known point on the action line of the right reaction.

STRESS ANALYSIS OF TRUSSES

53. Definition of a Truss. A truss is any framework of bars* placed together to form triangles. An exterior bar forms a side of one triangle only,

* The word *bar* commonly is used to signify any truss member. Actually, an *eye-bar* is a tension member of rectangular cross-section drilled at each end for connection to a pin.

but an interior bar forms a side of each of two triangles. A truss of this simple type is called statically determinate *internally* because all of the stresses in the bars can be calculated by use of the three equations of statics. Figure 43(a) represents a simple type of statically determinate truss, and (b) represents

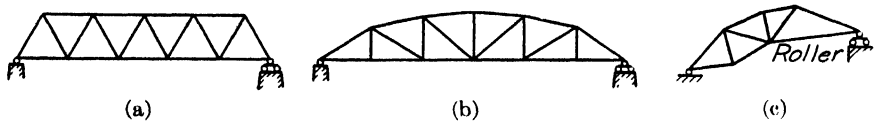


FIG. 43. STATICALLY DETERMINATE TRUSSES.

another common type of bridge truss which is also determinate. The highly irregular structure shown in (c) fulfills the preceding definition of a statically determinate truss.

54. Statically Indeterminate Trusses. Wherever truss members apparently form sides of more than two triangles, and in most cases where members cross each other, the truss is statically indeterminate *internally*. This statement signifies that there are too many interior members to be analyzed by the equations of statics alone. If a truss becomes statically determinate when a single member is removed (called the redundant member), the truss is statically indeterminate internally to the *first degree*. A truss

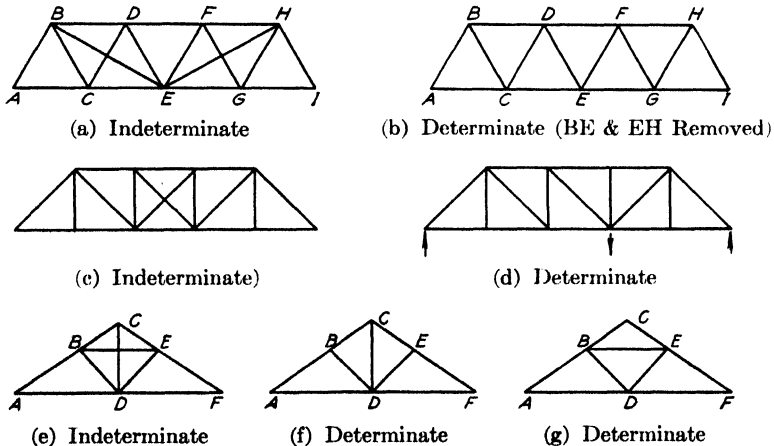


FIG. 44. STATICALLY INDETERMINATE TRUSSES.

may have two, three, or more redundant members. The truss of Fig. 44(a) is indeterminate in the second degree since the removal of two members in (b) results in a standard type of truss.

Double Diagonals. The truss shown in Fig. 44(c), with double diagonals in the center panel, is a common type of bridge truss. Usually this structure

is treated as statically determinate. The explanation is that the double diagonals are designed as tension members that are *incapable* of resisting compression. We conclude that only the diagonal stressed in tension actually is in operation, as shown in (d). When both diagonals are stiff compression members, the truss is indeterminate to the first degree. The common procedure of analysis for stiff diagonals is to assume that they are stressed equally, one in tension and the other in compression. Hence each diagonal resists one-half of the stress that it would receive if it acted alone. This assumption has been justified for normal conditions by an analysis based upon the deformations of the members.*

55. Test for Static Indeterminateness. Since the bar stresses at a joint form a group of concurrent forces, each joint provides two equations of statics, $\Sigma H=0$ and $\Sigma V=0$. It follows that two unknown bar stresses can be determined at each joint provided that two unknown stresses exist at that joint

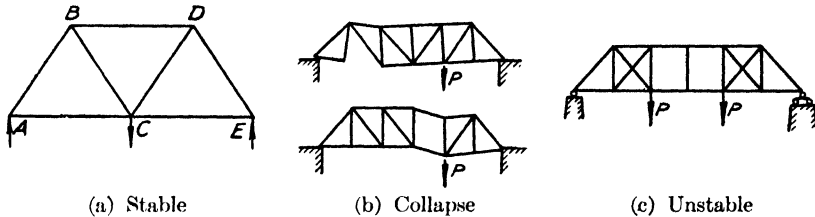


FIG. 45. STABLE AND UNSTABLE EQUILIBRIUM.

which have not been determined at a preceding joint. For the simple truss of Fig. 45(a), it is seen that the stresses in the members AB and AC are determined from the joint A , that the stresses BC and BD are determined from the joint B , and that the stresses CD and CE are determined from the joint C . Only one bar DE remains, and its stress can be found from the joint D . The other equation from the joint D and the two equations from the joint E are used merely as a check. Hence three of the possible equations of statics are not available here for determining bar stresses. (Actually, they have been used in the calculations of the reactions.) The following rule extends these observations to the general case. *The number of bar stresses in a planar truss that can be found by statics is twice the number of joints minus three.* Expressed as an equation, this rule is

$$(7) \quad m = 2n - 3,$$

where m is the number of members and n is the number of joints.

* See Johnson, Bryan, and Turneaure, *Modern Framed Structures*, Part I. For the special case of a Pratt bridge truss, calculations show the live load stress in one diagonal to be $-21,800$ lb. and in the other diagonal to be $+23,200$ lb. The approximate stresses are $22,500$ lb. each. However, a greater variation is possible for unusual conditions. In fact, in Vol. II *Theory of Modern Steel Structures*, Rev. Ed., p. 91, a problem is given for which the two stresses are $-12,300$ and $+30,100$ lb.

USE OF THE RULE. If the number of bars exceeds $2n - 3$, the truss is over-determined internally, or, as it is more commonly expressed, the truss is internally statically indeterminate. No true truss can be statically under-determined internally because a truss has been defined as a structure that is made up of triangles placed side by side. Each joint is thus fixed in position by being tied to two other joints, which in turn are fixed to the supports by means of a system of such rigid triangles. However, if one of the bars is omitted, so that the remaining number of members is less than $2n - 3$, the truss becomes statically under-determined or unstable. See Fig. 45(b).

Common sense must be used in the application of the rule for counting the number of members and the number of joints. Figure 45(c) shows a truss which is over-determined in two panels and under-determined in two other panels. There are 12 joints providing $(2 \times 12) - 3 = 21$ equations of statics. A count of the bars gives us 21 also. Therefore, $m = 2n - 3$. The rule shows that the structure should be statically determinate and stable. Actually, it is in *unstable equilibrium* under the loads shown and it would collapse under unsymmetrical loading.

56. Fundamental Assumptions. Certain fundamental assumptions are made in truss analysis with which the reader must become familiar.

- (1) The gravity lines of all bars meeting at a joint intersect at a single point.
- (2) There is a frictionless pin at each joint of the truss.
- (3) There is no bending in any member of the truss. Assumption (3) also summarizes (1) and (2).

It is possible to make the first assumption practically true by detailing the members and joints properly. The correct procedure is to lay out the truss members by placing their gravity axes on the center lines of the truss, which

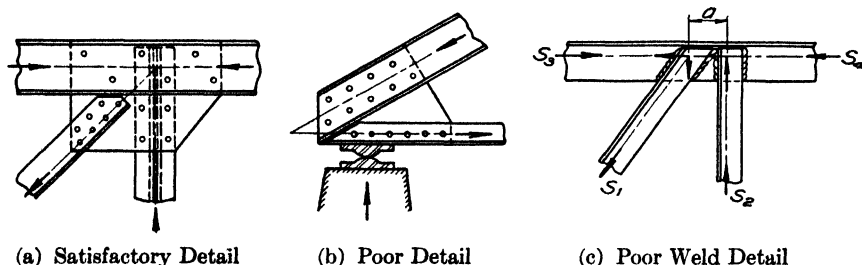


FIG. 46. STRUCTURAL JOINTS.

in turn intersect at a single point at each joint. The only approximation usually allowed is that the gage line of an angle is taken as its gravity axis. A joint detail where the members meet at a point is shown in Fig. 46(a). Joint details where these assumptions are incorrect are shown in (b) and (c).

EXAMPLE. Flexure of Truss Members. If the action lines of all forces at a joint do not meet at a single point, flexure is produced in the members. Consider the joint shown in (c). The stress S_1 is resolved into its H and V components at its intersection with S_3 and S_4 . We can see that there must be a bending moment equal to $(S_2 \times a)$ resisted by the members

at this joint because the V component of S_1 is equal to S_2 and these two parallel forces form a couple of this value. It follows that this joint does not fulfill assumption (3) above.

The assumption that there is no bending in any truss member is incorrect, of course, because any bar which is not vertical must act as a beam to carry its own weight to the adjacent joints. In long heavy members this bending stress may be of real importance. However, the specifications for design usually require such deep sections that the flexure caused by the weight of the member may be neglected.

57. Secondary Stresses in Trusses. If pins become rusted and refuse to rotate, or if the joints are riveted or welded, the truss deflection will produce bending in all members. A deflected truss with frictionless pin joints is

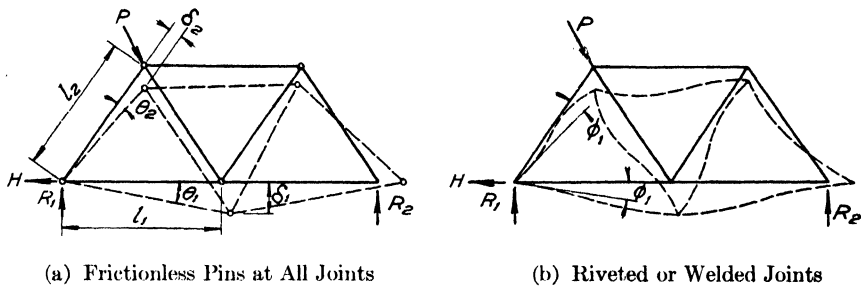


FIG. 47. SECONDARY FLEXURAL STRESSES.

shown in Fig. 47(a). It is very unlikely that the deflections δ_1 and δ_2 would bear such relationship to each other that the values of θ_1 and θ_2 would be equal. But the end joint must rotate as a unit through the angle ϕ_1 for the welded or riveted structure shown in (b). The result is that most of the *members are forced to bend into an S-shape* or reversed curve, as shown in the illustration.

The moments caused by bending of the members from deflection of the truss are known as secondary moments. The corresponding fiber stresses in the bars are called secondary stresses, in contrast to the direct or primary stresses acting in line with the bars. Experiment has shown that the secondary stress ordinarily does not exceed 30% of the primary stress for a given member. Since the computation of secondary moments is rather tedious, it is customary to reduce the working stress to allow for an assumed secondary stress of about one-third of the primary stress. This method is not economical of material because it provides an excessive allowance for most members, but it is reasonably safe. In order to take advantage of possible economy in the design of large important structures, the secondary moments should be calculated and used in the design with increased working stresses.

58. Rules of Stress Components. In the study of mechanics, components of stress are found by use of functions of angles. In stress analysis, the engineer prefers to use the known lengths of the members to represent these

functions of angles. In Fig. 48 it is seen that the force and space triangles are similar. Hence, from the proportionality of sides we see that

$$(a) \quad \frac{\text{Stress}}{\text{Component of stress}} = \frac{\text{Length of member}}{\text{Corresponding component of length}}$$

By transposition we may write two other useful rules:

$$(b) \quad \text{Stress} = \frac{\text{Component of stress} \times \text{Length of member}}{\text{Component of length}}$$

or,

$$(c) \quad \text{Component of stress} = \frac{\text{Stress} \times \text{Component of length}}{\text{Length of member}}$$

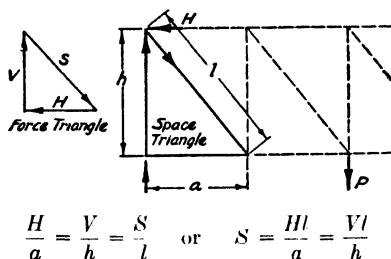


FIG. 48. RULES OF FORCE COMPONENTS

59. Truss Analysis by the Method of Joints. The external loads and internal bar stresses acting at an isolated joint of a truss form a set of forces that intersect at a single point and are in equilibrium. If the equations $\Sigma H = 0$ and $\Sigma V = 0$ are used to calculate the unknown bar stresses, the equation $\Sigma M = 0$ gives no additional information. Accordingly, *only two unknown bar stresses can be found from each isolated joint*. In order to find a starting point we locate a joint at which only two bars meet, for instance the end joint at the reaction. We can then proceed in succession to joints at which all but two bar stresses have already been determined. An example of the analysis of a roof truss by the method of joints is given in Fig. 49.

60. The Graphical Method of Joints—The Stress Diagram. The graphical counterpart of the algebraic method of joints is the stress diagram. In reality, the stress diagram is a group of superimposed force polygons, one for each joint of the truss. This correspondence is illustrated by Fig. 50 where the separate force polygons are drawn for each joint. These force polygons are superimposed upon one another in (f) to form the stress diagram. Note that in the separate force polygons all vectors corresponding to the bar stresses are drawn twice, but that this duplication is avoided in the stress diagram by superposition of equal but opposite vectors. Hence each of the dotted lines in the stress diagram (f) represents two equal and opposite

vectors. Arrows would be confusing and are omitted. Instead, the arrows are placed directly upon the truss diagram where they will show the *sign of the stress*.*

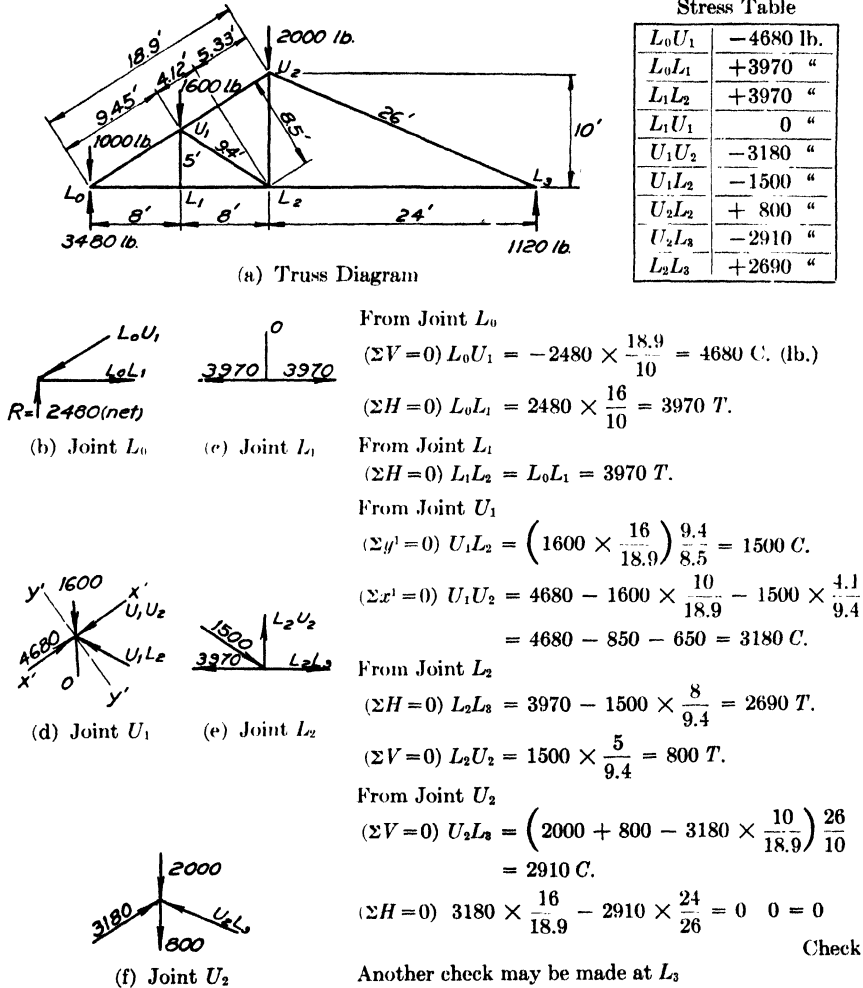
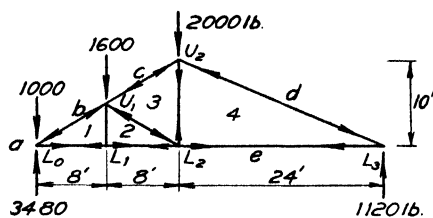


FIG. 49. TRUSS ANALYZED ALGEBRAICALLY BY THE METHOD OF JOINTS.

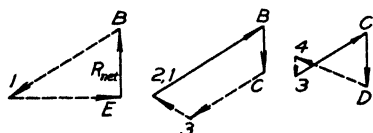
61. Procedure for Drawing the Stress Diagram. The first step in drawing the stress diagram is to lay out a complete force polygon for the loads and

* A member that is resisting compression will lengthen when the stress is released. Such a member must therefore be pushing outward against its end joints. The arrows, which picture the action of the member on its two end joints, must be directed *outward* for a compression member and *inward* for a tension member.

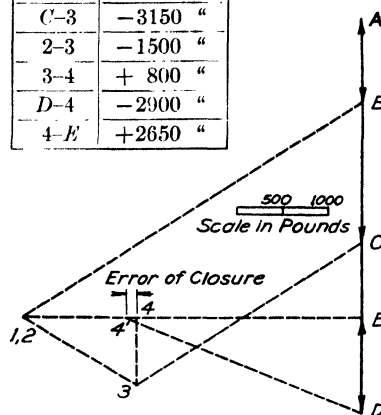
reactions. Naturally, this force polygon must close. The order used can be clockwise or counterclockwise, but the *forces must be drawn consecutively* as they are encountered as a pencil is moved around the truss. The order is clockwise in Fig. 50(f). The force polygon $A-B$, $B-C$, $C-D$, $D-E$, $E-A$ forms a straight line because there are only vertical forces acting on this truss.



(a) Truss Diagram

(b) Joint L_0 (c) Joint U_1 (d) Joint U_2 (e) Joint L_2

Stress Table	
$B-1$	-4650 lb.
$1-E$	$+3950$ "
$2-E$	$+3950$ "
$2-1$	0 "
$C-3$	-3150 "
$2-3$	-1500 "
$3-4$	$+800$ "
$D-4$	-2900 "
$4-E$	$+2650$ "



(f) Stress Diagram

FIG. 50. GRAPHICAL ANALYSIS OF THE TRUSS OF FIG. 49.

The next step is to draw a force polygon for the reaction joint L_0 . This force polygon is $E-A-B-1-E$ of Fig. 50(f). Observe that the order in which the forces are taken around the joint is clockwise. The order used in drawing the force polygon for the external loads was clockwise, and *these two orders must agree*. The direction arrows for the stresses are directed from B to 1 and from 1 to E as in (b). Thus we establish the directions of the arrows shown at the joint L_0 in (a).

The second force polygon drawn is for the joint U_1 , that is, $1-B-C-3-2-1$. (The stress in L_1U_1 obviously is zero which makes the point 2 coincide with point 1 .) Here the vector $1-B$ is re-used from the previous polygon for L_0 , but its arrow is reversed. Force polygons may now be drawn for the joints L_2 and U_2 , which completes the stress diagram.

Error of Closure. The work may be checked by observing whether or not the stress vectors for $E-4$ and $4-D$ form a triangle with the right-hand reaction $D-E$. This triangle $D-E-4$ is the force polygon for L_3 . Unless it is a perfect triangle, there is an error in the

drafting, or in the scaled value of the reaction, or in both. This error may be expressed in terms of an "error of closure" since the location of the point 4 has been obtained twice. See (d) and (e) of Fig. 50. The error of closure is the scaled value in pounds of the distance between the two locations of the point 4. Careful work will reduce the error of closure so that no important stress will be in error more than one or at most two per cent. A small unimportant stress may show a considerably larger percentage of error, and this can not be avoided in graphical work. However, by comparing the stresses obtained graphically in Fig. 50 with those obtained algebraically in Fig. 49 we find a maximum variation of only 1.5 per cent.

62. The Method of Sections. A fundamental objection to the method of joints and to the stress diagram is that they are not easily applied to the determination of the stress in a single member near the center of a truss. By these methods it is necessary to compute the stresses from one end of the truss up to the bar in question. On the other hand, the method of sections permits us to cut a section *anywhere* through the structure, and to compute the values of *three unknown non-concurrent stresses* cut by this section. The section chosen may cut any number of bars, but all except three must have known stresses. These three stresses must be non-concurrent if we are to calculate their values by statics.

EXAMPLE. Two sections are cut through the truss of Fig. 51 and several stresses are calculated. The other stresses are given, so that they may be checked by the reader for

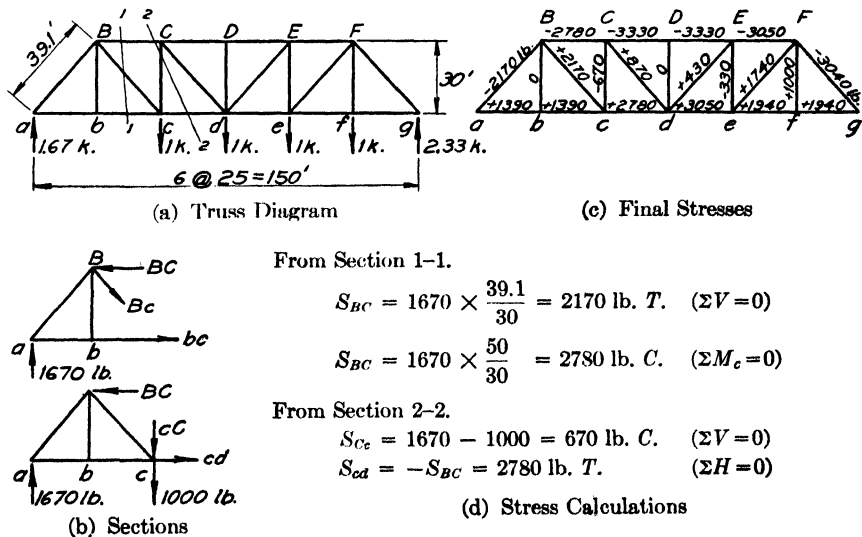


FIG. 51. TRUSS ANALYZED BY METHOD OF SECTIONS.

practice. The section 1-1 illustrates the calculation of the stress in a diagonal web member from the shear in the panel. The equation $\Sigma V = 0$ shows us that the V component of the stress in a diagonal is equal to the vertical shear in the panel, which is the left end reaction

minus any loads to the left of the section. This conception is valid only for a *truss with horizontal chords*. It may be stated as a rule which applies both to diagonals and verticals.

$$(a) \quad \text{Stress in web member} = \text{Panel shear} \times \frac{\text{Length of member}}{\text{Height of truss}}$$

Chord members are analyzed by means of the equation $\Sigma M=0$. The stress in the member *BC* is determined from the section 1-1 by an equation of moments about *c**. The point *c* is chosen because it is the intersection of the bars *Bc* and *bc*, the other two members cut by the section 1-1. Hence, for a *truss with parallel chords*,

$$(b) \quad \text{Stress in chord member} = \frac{\text{Moment about intersection of other members cut}}{\text{Height of truss}}$$

We find the equation $\Sigma H=0$ to be useful for the determination of the chord stresses in a truss with vertical web members. The section 2-2 clearly illustrates the use of this equation to show that the horizontal members *BC* and *cd* must have equal and opposite stresses since the web member cut is vertical.

63. The Method of Moments. All stresses in a truss with sloping chords can be found quite easily by the method of moments, a variation of the method of sections. Three equations of moments involving the three

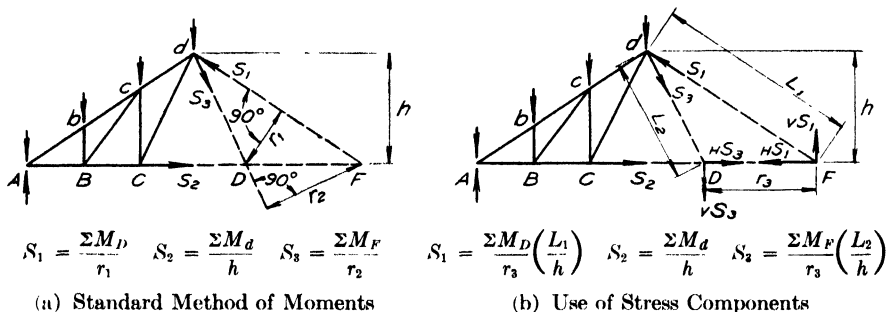


FIG. 52. STRESS ANALYSIS BY THE METHOD OF MOMENTS.

unknown stresses acting on any cut section can always be obtained. We commonly choose the moment centers† at the intersections of pairs of unknown stresses, so that each of the moment equations will contain only a single unknown. See Fig. 52(a).

Use of Stress Components. A variation of the method of moments is obtained by the use of components. The need for simultaneous equations can be eliminated by selecting the moment center at the intersection of two of the unknowns. Then the third unknown is broken up into components at

* When a truss is lettered with capital and lower-case letters we read *Bc* as *B major, c minor* and *bc* as *b minor, c minor*, etc.

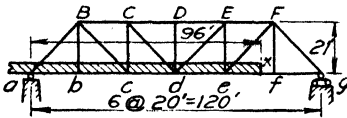
† If the three moment centers chosen are on a straight line, an unknown force of any magnitude might act along this line without affecting the moment equations. In general, two of the equations of moments will be *dependent* if the three moment centers lie on a straight line.

a point selected so that one of these components *also passes through the moment center*. The other component is the only unknown in the moment equation. Figure 52(b) illustrates the method of moments by the use of components of stress. The horizontal component of S_3 , marked uS_3 , for example, passes through the moment center F .

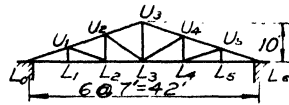
PROBLEMS

33. Analyze for the stresses that would be produced in this Pratt bridge truss by a line of motor cars extending from a to x . Consider the line of cars to be represented by a uniform load determined on the assumption that the average car weighs 3300 lb. and takes up a space of 20 ft. including the space between cars. Remember that the loads come onto the floor system which produces panel concentrations on the truss.

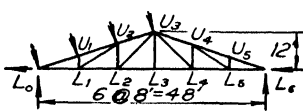
Ans. $EF = -11,100$; $eF = +7400$; $ef = +6000$ lb.



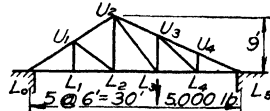
PROBLEM 33.



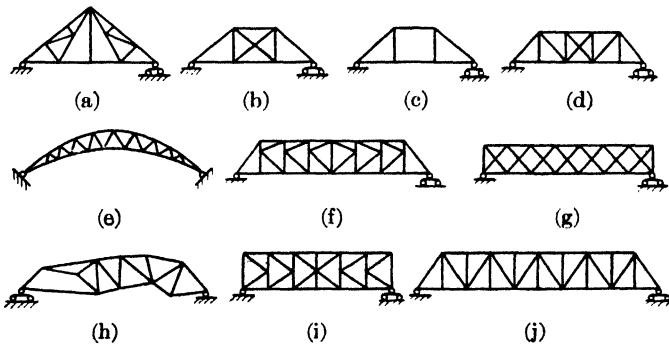
PROBLEM 35.



PROBLEM 36.



PROBLEM 38.



PROBLEM 39.

34. Same as Problem 33 except that the live load extends from a to d .

35. Compute the snow-load stresses in the Howe roof truss. The trusses are spaced 15 ft.-5 in. apart and the snow load is 20 lb. per sq. ft. of roof surface. Use the method of joints and check by drawing a stress diagram.

Ans. $U_3U_4 = -8300$; $L_3U_4 = -3500$; $L_2L_4 = +10,000$ lb.

36. Compute the wind-load stresses in the Pratt roof truss. Assume that the reactions have equal horizontal components. The trusses are spaced 13.4 ft. apart. The wind

pressure is 25 lb. per sq. ft. normal to the roof. Use the method of joints and check by calculating the stresses in U_2U_3 , L_2U_2 , and L_1L_2 by the method of moments.

Ans. $U_2U_3 = -8700$; $L_2U_2 = -5000$; $L_1L_2 = +6400$ lb

37. Same as Problem 36 except use the graphical method of analysis. Assume that the reactions are parallel to the wind loads.

38. Determine the stresses in this roof truss caused by the crane load of 5000 lb. at L_3 . Use any method you prefer and check at least three stresses by one other method.

Ans. $U_2U_3 = -6700$; $U_3L_3 = U_3L_4 = 0$; $L_3L_4 = U_2L_3 = +6000$ lb.

39. Count the number of members and the number of joints in each of these trusses and decide whether each is statically determinate, statically indeterminate, or unstable. Change the trusses where necessary to make them statically determinate.

COMPOUND TRUSSES

64. Simple Trusses Joined Together. It is possible to make up an unlimited number of trusses by joining two simple trusses together. If two component trusses are joined as in Fig. 53(a) with three non-parallel non-concurrent bars so that the section $a-a$ will cut three non-parallel non-concurrent stresses, the structure will be statically determinate and stable. The

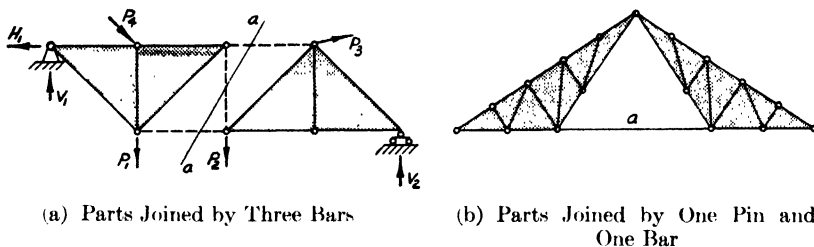


FIG. 53. SIMPLE TRUSSES JOINED TO PRODUCE A COMPOUND TRUSS.

compound truss has the same number of joints (n), the same number of available equations of statics ($2n$) but three more members (m) than the two individual trusses. But remember that each of the two individual trusses required three reactions for stability or six in total while the compounded truss has only three reactions H_1 , V_1 and V_2 . Hence, the elimination of three reactions by compounding provides the three equations needed to analyze for the stresses in the three additional members, and the equation $m = 2n - 3$ is still satisfied.

EXAMPLES. In Fig. 53(b) a roof truss is compounded by joining two unsymmetrical trusses on one pin at the top and by connecting them together with a single tension bar (a) at the bottom. In this process three of the six required reactions of the two individual trusses are eliminated which releases three equations of statics for bar stress analysis. However, at the same time one end joint of each truss becomes a common center joint of the compounded truss with the loss of two equations. Therefore, we have a net gain of one equation which is adequate for the analysis of the stress in the single additional member a .

One of the simplest compound trusses would be composed of two elementary triangles joined by three nonparallel non-concurrent members. See Fig. 54. Since three members meet at every joint of this truss, it is impossible to start an analysis by the method of joints; and since no section can be drawn that cuts only three members which are non-concurrent, the method of sections or of moments also breaks down. However, by considering the rigid triangle $U_1U_2L_2$ as a free body, the external forces are merely the loads P_1 , P_2 , the known reaction R_2 and the three unknown bar stresses (a at U_1 , b at U_2 and c at L_2). Since these three unknown bar stresses are non-parallel and non-concurrent they can be determined from this one free body. This same procedure is also effective with unsymmetrical loads acting in any direction if the reactions are determinate.

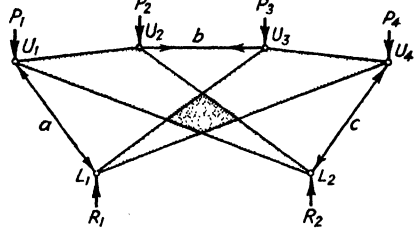


FIG. 54. COMPOUND TRUSS WHICH IS NOT SUBJECT TO ANALYSIS BY THE METHODS OF JOINTS AND SECTIONS.

PROBLEMS

40. Analyze for the stresses in the compound truss of Fig. 54 for the following loads and dimensions: P_1 , P_2 , P_3 , $P_4 = 5,000$ lb. each. Each panel along the upper chord is $20' - 0''$ horizontally and the height vertically from L_1 to U_2 is $17' - 6''$. Point U_1 is $2' - 6''$ below L_1 . The distance from R_1 to R_2 is $30' - 0''$. The member U_1L_1 has a slope of 45° .

Ans. Stress in $U_2U_3 = + 10,800$ lb.

41. Repeat Problem 40 but with load P_1 acting alone. Refer to Fig. 54.

Ans. Stress in $U_2U_3 = + 3,470$ lb.

42. Repeat Problem 40 but with a single 5,000-lb. force acting horizontally to the right at U_1 . Place the horizontal reaction first at L_2 and then at L_1 . Refer to Fig. 54.

Ans. Stress in $U_2U_3 = - 1150$ or $- 3470$ lb.

SHEAR AND MOMENT DIAGRAMS FOR BEAMS

65. Shear Diagrams. When the loads upon a beam are fixed in position, a complete analysis of stresses may be made from the information contained in the shear diagram and the moment diagram. The method of drawing these diagrams will be reviewed particularly in regard to *signs*. If the resultant force to the left of a section is upward, the shear is considered positive. This upward force must be accompanied by a downward resultant force to the right of the section, a condition that also defines positive shear.

PROCEDURE. In drawing a shear diagram, start with the left-hand reaction and draw a vertical ordinate equal to the value and in the direction of the reaction, as $a-a'$ in Fig. 55. Extend the horizontal line $a'-b$ to indicate constant shear. (The weight of the beam is being neglected.) Draw $b-b'$ vertically downward and equal to P_1 . Extend $b'-c$ horizontally to indicate constant shear. Draw $c-c'$ vertically downward and equal to P_2 . Extend $c'-d$ horizontally to indicate constant shear. Draw $d-e$ sloping downward by an increment of w for each foot horizontally. Finally, draw the line $e-e'$ vertically upward and equal to R_2 . There is a check here since the ordinate $e-e'$ should bring the shear diagram back to the horizontal base line. This procedure may be used either as an *algebraic* or *graphical* method.

66. Moment Diagrams. The moment diagram is of great importance in beam and girder analysis, for it is in effect a diagram of extreme fiber stresses. Usually, a sketch of the moment diagram will be as useful as a scale drawing, provided that two or three numerical values of critical ordinates are written upon it. These critical ordinates exist at concentrated loads, at the ends of pieces of distributed load, and always at points of zero shear, points which will be shown to correspond to points of maximum moment.

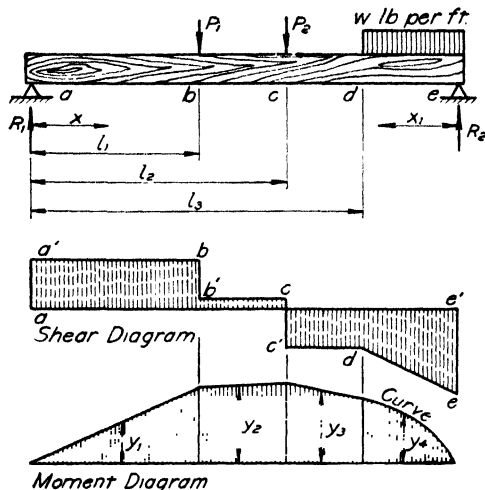


FIG. 55. SHEAR AND MOMENT DIAGRAMS.

The bending moment at any section may be computed *algebraically* by taking the moment about the section of all forces either to the right or to the left of the section. The determination of the moment diagram *graphically* follows from the discussion of § 46. The equilibrium polygon for the loads and reactions is in effect a bending-moment diagram. The scale of the resultant moment diagram involves the combined scales for forces and distances, as explained in § 46.

SIGN OF THE BENDING MOMENT. The usual rule of signs is that the bending moment is positive when the moment of the forces to the left of the section is clockwise or when the moment of the forces to the right of the section is counterclockwise. An easier way to remember this rule of signs is to recall that positive moment causes tension in the bottom fibers, as in a simple beam, and that negative moment causes tension in the top fibers, as in a cantilever beam.

67. Shape of the Moment Diagram. We will show that the moment diagram is a straight line between points of concentrated loads when no uniform load exists and that it becomes a parabola when there is a uniform

load. Actually, a piece of beam without a uniform load can exist only when the beam is vertical. Any sloping or horizontal beam must carry its own weight. However, when the weight of the beam is small as compared to the values of the concentrated loads, its weight is frequently neglected in drawing the shape of the moment diagram.

PROOF. The bending moment for each section of the beam shown in Fig. 55 may be expressed as follows:

$$(8) \quad \text{From } a \text{ to } b: \quad y_1 = R_1x$$

$$(9) \quad \text{From } b \text{ to } c: \quad y_2 = R_1x - P_1(x - l_1)$$

$$(10) \quad \text{From } c \text{ to } d: \quad y_3 = R_1x - P_1(x - l_1) - P_2(x - l_2)$$

$$(11) \quad \text{From } d \text{ to } e: \quad y_4 = R_1x - P_1(x - l_1) - F_2(x - l_2) - \frac{w(x - l_2)^2}{2},$$

or, by using the right-hand end of the beam, we obtain

$$(12) \quad y_4 = R_2x_1 - \frac{wx_1^2}{2}.$$

Equations (8), (9), and (10) are linear in x and must represent straight lines. Equation (11) looks rather complicated, but it is identical with equation (12), which is clearly a first degree curve or a straight line minus a second degree curve or a parabola. The resultant curve is therefore parabolic.

68. Moments Computed from Areas under the Shear Curve. The area of the shear diagram between two concentrated loads that act normal to a beam as in Fig. 56 (where the weight of the beam is neglected) is equal to the constant value of the shear times the distance between the loads. Evidently, then, this area is equal numerically to the change in moment in the beam between the two load points. Note that any area from a shear diagram is obtained as a product of a shear ordinate (pounds) multiplied by a horizontal length (feet) and is therefore obtained directly in the proper units for bending moment (foot-pounds). We can also show that these statements still apply when there are intermediate concentrated loads.

DEMONSTRATION. The area (1) of Fig. 56 represents the change of moment from a to b , and the area (2) represents the change in moment from b to c . Therefore, the area (1) plus the area (2) represents the change in moment from a to c . No further proof is needed for the case of a uniform or distributed load because any distributed load can be treated as a large number of small concentrated loads. Accordingly, we may state that the area of the shear diagram between any two points in a beam is identical (both numerically and in actual units) with the change in bending moment between the same two points. Of course, this statement applies only to change in moment due to normal loads. A moment may be applied at one point through a bracket producing a sudden change or break in the moment diagram which may not be expressible as a shear area.

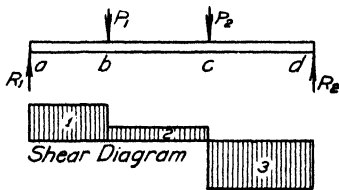


FIG. 56. SHEAR AREAS.

CALCULATION SHEET — MOMENT AND SHEAR DIAGRAMS FOR AN UNLOADING FRAME

Loading —

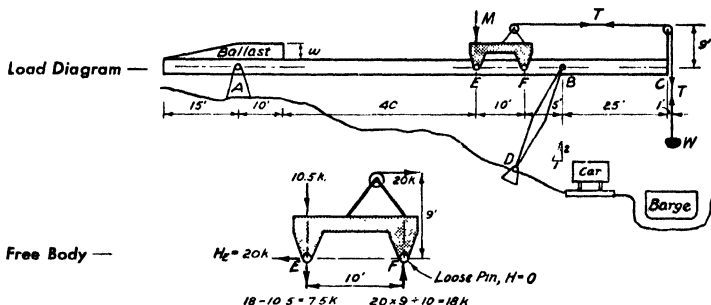
Lift, $W = 10 \text{ tons} = 20 \text{ k.}$

Tension in cable, $T = 20 \text{ k.}$

Neglect the weight of the girder.

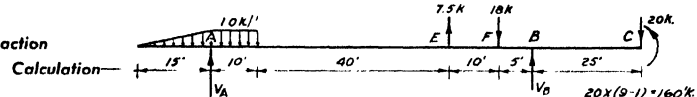
Wt. of machinery, $M = 10.5 \text{ k.}$

Ballast = $1.0 \text{ k./ft.} = w$



Reaction

Calculation —



$$\Sigma M_A = - \left(\frac{1.0 \times 15}{2} \right) 5 + (1.0 \times 10) 5 - 7.5 \times 50 + 18 \times 60 - V_B \times 65 + 20 \times 90 - 160 = 0$$

$$V_B = \frac{-37.5 + 50 - 375 + 1080 + 1800 - 160}{65} = \frac{2357}{65} = +36.3 \text{ k (up)}$$

$$\Sigma M_B = - \left(\frac{1.0 \times 15}{2} \right) 70 + V_A \times 65 - (1.0 \times 10) 60 + 7.5 \times 15 - 18 \times 5 + 20 \times 25 - 160 = 0$$

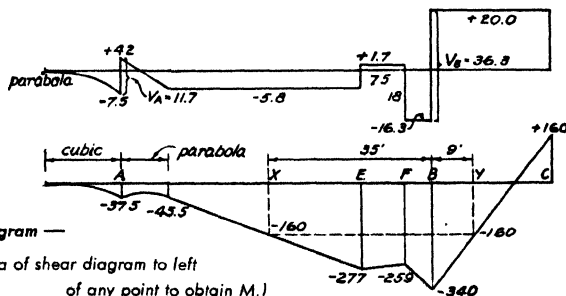
$$V_A = - \left(\frac{-525 - 600 + 112.5 - 90 + 500 - 160}{65} \right) = + \frac{762.5}{65} = +11.7 \text{ k. (up)}$$

Check —

$$\Sigma V = 7.5 - 11.7 + 10 - 7.5 + 18 - 36.3 + 20 = 0 \quad 0 = 0$$

Shear Diagram —

(Sum forces to left of any point to obtain shear.)



Moment Diagram —

(Sum area of shear diagram to left of any point to obtain M.)

CALCULATION SHEET FOR AN UNLOADING FRAME *Continued*

Design — $M_{\max} = 340$ kip-ft A section adequate to resist 160 kip-ft. could be strengthened with extra flange material from X to Y over 44 ft. First, however, the dead weight of the girder would be estimated and the DL moments added.

Comments — Because of the slope of the stiff leg BD the reaction V_B is accompanied by a force H_B not shown. Also $H_E = 20$ k. The necessary H-reaction for equilibrium occurs at A. Unless all pins are attached to the girder on its center line, these horizontal forces would change the M-diagram.

Check on M_E — (Hold until after a study of § 72). It is desirable to check one moment by another method which will serve as an adequate check on the shear and moment diagrams both of which were involved in the calculation of M_E by the cumulative process of summing shear areas

Virtual Work — The procedure used will be to produce an imaginary unit angle change ϕ_E such that the work performed by M_E will be $M_E \times \phi_E = M_E \times 1 = M_E$. Since the total virtual work is zero, we may then equate M_E to the remainder of the virtual work which will be produced by the external loads. The displacement $\phi_E = 1$ will produce no displacement of the reactions. The load 3 (below) occurs at E.

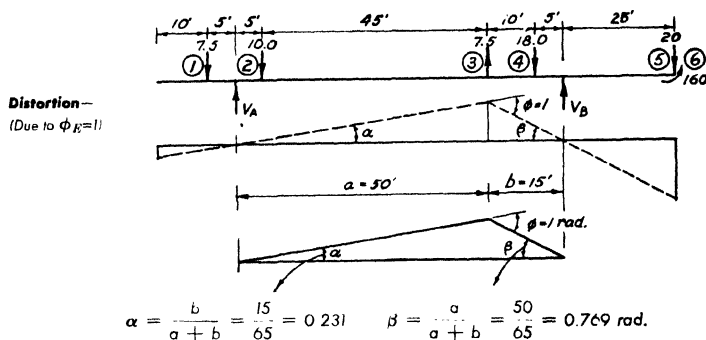


Table of Virtual Work

Load in kips	M in kip-ft.	$\theta \times \text{arm} = \text{defl.}$	Work = $Fd = M\theta$
(1) + 7.5		$0.231 \times 5 = 1.15$ down	+ 8.7
(2) + 10.0		$0.231 \times 5 = 1.15$ up	-11.5
(3) - 7.5		$0.231 \times 50 = 11.55$ up	+86.5
(4) + 18.0		$0.769 \times 5 = 3.84$ up	-69.0
(5) +20.0		$0.769 \times 25 = 19.22$ down	+384.5
(6)	-160	0.769 (clockwise ϕ)	- 123.0

$$M_E \times (\phi = 1) = \Sigma W = +276 \text{ (pos. work)}$$

Hence, $M_E = 276$ kip-ft (negative bending moment acting in the direction of $\phi = 1$)

Again, by referring to Fig. 56, we conclude that the area (1) plus the area (2) equals the area (3) because the area from either end to the point c represents the bending moment at the point c when the end moments are zero. Evidently the positive and negative shear areas are always equal for a *simple beam*. It is possible to use the area of the shear diagram in the computation of bending moments. For instance, the bending moment in a cantilever beam at any point is equal to the area of the shear diagram between that point and the free end. However, a horizontal load applied through a vertical bracket attached to the free end of a horizontal cantilever beam will produce a constant moment throughout the length of the beam but a zero shear. Hence the bending moment for loads that are not normal to the beam may not be expressible in terms of a shear area.

69. Points of Maximum Moment. It will be shown that the point of zero shear in a beam is also a point of maximum positive or negative moment. If we start to sum areas of the shear diagram for a horizontal beam by moving

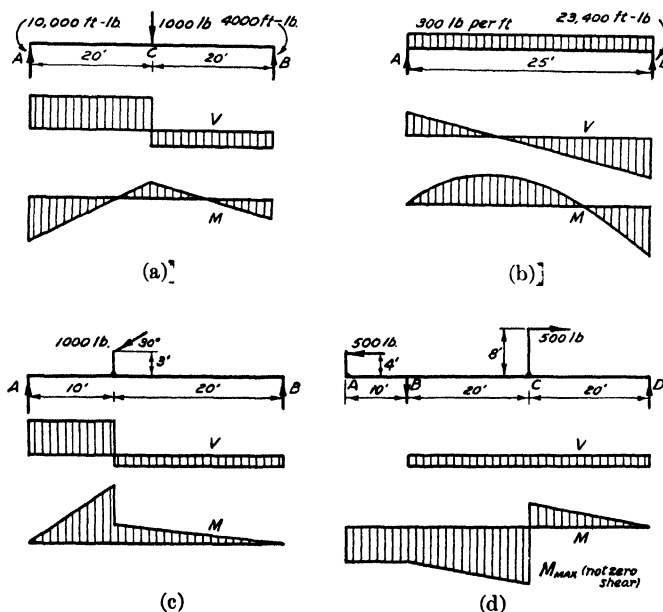


FIG. 57. MAXIMUM MOMENTS IN RELATION TO THE SHEAR DIAGRAM.

From (a) and (b) it is seen that the addition of one or more end moments to a beam does not change the fact that a point of zero shear is also a point of maximum positive or negative moment. However, when loads are applied through brackets, a point of zero shear may occur at a point of maximum moment as in (c) or there may be no correspondence between these points as in (d). Note that there is no vertical component of load in (d).

inward from an end where the moment is zero, the total summation of area or change in moment will *increase* either negatively or positively with increase of the distance x from the end until the shear diagram crosses the base line. At this point we must start numerically to *subtract* succeeding areas, and the

moment will start to decrease. A point of maximum positive or negative moment must have existed where the shear diagram crossed the base line. The quickest way to obtain the maximum negative and positive moments for selecting the size of a beam is first to locate the points of zero shear and then to compute the bending moments at these points. It will be clear from Fig. 57(a) or (b) that the addition of one or more end moments to the beam does not change the fact that a point of zero shear is also a point of maximum or minimum moment.

Loads Applied to Brackets. Whenever a load is applied to a beam through an interior bracket as in Fig. 57(c) or (d) the point of maximum moment does not necessarily occur at the point of zero shear. See Fig. 57(d). The relationship of maximum moment to zero shear was derived from a consideration of normal loads on a beam and does not apply to other possible loadings.

PROBLEMS

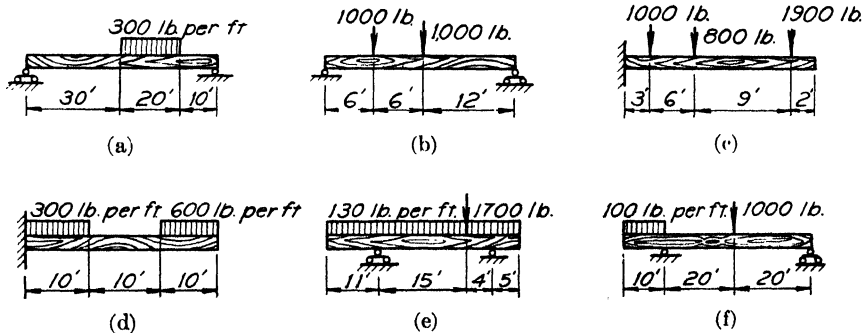
43. Calculate the values of all critical ordinates on the shear and moment diagrams illustrated in Fig. 57. Start by computing the reactions.

44. The entire weight of one 20-ton truck rests directly on one girder of a 60-ft. girder bridge. The truck has axles spaced 14 ft. apart, and 80% of the weight is on the rear axle. From the knowledge that the shear must be zero at the point of maximum moment, locate the wheels along the length of the girder for maximum moment at the center of the span. Compute the maximum moment at the center allowing for a dead load of 1400 lb. per ft. of girder.

Ans. 1,174,000 ft-lb

45. Repeat Problem 44 for maximum moment at the quarter point and for a point 22 ft. from one end.

Ans. 894,000 and 1,100,000 ft-lb.

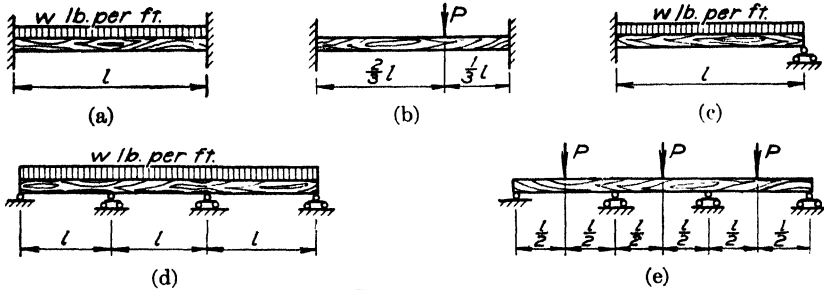


PROBLEM 46.

46. Plot shear and moment diagrams to scale for each of the beams shown. Write values of all critical ordinates on the diagrams. Answers given are for maximum moments.

Ans. (a) 66,700; (b) 9,000; (c) 44,400; (d) 165,000; (e) 6,350; (f) 12,500 ft-lb.

47. Sketch the general shape of the shear and moment diagrams for each of the indeterminate beams shown. Do not attempt to calculate ordinates.



PROBLEM 47.

COMPUTING STRESSES BY MEANS OF DISPLACEMENTS

70. **Method of Virtual Work or of Unit Displacement.** In Volume II the principle of virtual work (virtual meaning imaginary) is developed in detail and used for the calculation of the deflections of structures. However, in a very simple and convenient manner we can develop the principle in elementary form for use in computing bar stresses, reactions, shears or moments of simple statically determinate structures such as those already dealt with in this chapter. The advantage of this method over the equations of statics is that one can compute a bar stress in a truss or a bending moment in a beam directly without knowledge of any other stress and often without knowing the values of the reactions. Hence, we will call the method *direct use of virtual work*. It is as simple to use and sometimes more convenient than the equations of statics.

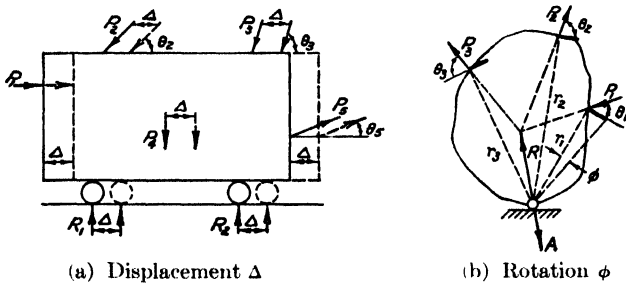


FIG. 58. VIRTUAL DISPLACEMENT AND VIRTUAL ROTATION.

Consider the body shown in Fig. 58(a). Give this body a horizontal displacement Δ . The work performed by each force will be obtained by multiplying the displacement Δ by the component of the force in the direction of the displacement. Since the horizontal displacement is perpendicular to the vertical load P_4 and the reactions R_1 and R_2 , these three forces do no work.

Hence,

$$\text{Total virtual work} = P_1 \Delta - P_2 (\cos \theta_2) \Delta - (P_3 \cos \theta_3) \Delta + (P_4 \cos \theta_4) \Delta$$

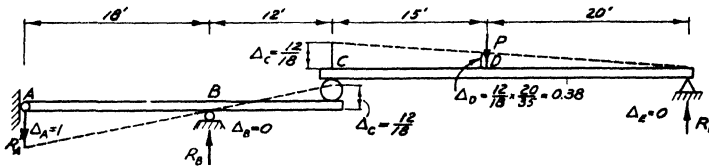
$$(13) \quad W_v = \Delta \sum_1^n P \cos \theta \quad (\text{where } \Sigma \text{ includes all forces performing work})$$

$$(14) \quad W_v = (\Delta)(\text{Comp. of resultant in direction of } \Delta)$$

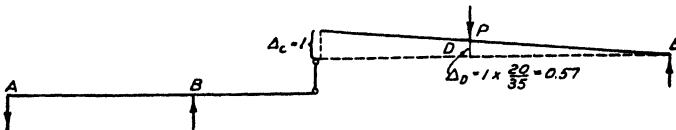
$$(15) \quad W_v = \text{zero when resultant is zero for forces in equilibrium}$$

Two important facts are clear from this derivation: (1) when one is computing virtual work a resultant force may be substituted for a group of forces and, therefore, for a distributed loading, (2) the total virtual work of any group of forces in equilibrium is zero as caused by a lineal displacement of any nature. It can likewise be shown that the total virtual work due to a unit virtual rotation (ϕ in Fig. 58(b)) is zero for a group of forces in equilibrium. In fact, the expression obtained is $W_v = \phi \Sigma M$ which clearly is zero when $\Sigma M = 0$.

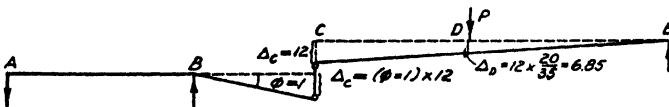
71. Reaction and Shear Calculation by Virtual Work. The compound beam of Fig. 59(a) may be analyzed by statics without difficulty. However, for practice in the use of virtual work computations its reaction at A and its shear at C will be determined by the work method.



(a) Computation of Reaction R_A by Virtual Work



(b) Computation of a Vertical Shear V_C by Virtual Work



(c) Computation of a Bending Moment M_B by Virtual Work

FIG. 59. VIRTUAL DISPLACEMENTS FOR COMPUTING REACTION, SHEAR AND MOMENT.

EXAMPLES. Consider the deflected structure in Fig. 59(a) when the reaction point is moved vertically a unit distance ($\Delta_A = 1$) in the direction of the reaction R_A . Since the structure is statically determinate, all members remain straight lines under this virtual displacement. See the dotted lines of Fig. 59(a). A simple set of proportions shows that the

corresponding upward displacement of D is $\Delta_D = (12/18)(20/35) = 0.38$. Since the load P at D and the reaction R_A are the only forces that would be moved or would do work under the virtual displacement considered, and since all forces remain in equilibrium, we may write

$$(16) \quad W_v = +[R_A(\Delta = 1)] - [P(\Delta = 0.38)] = 0, \quad \text{or} \quad R_A = 0.38P.$$

Here the signs merely mean positive or negative work, i.e., movement *with* or *against* the force.

In Fig. 59(b) a unit shearing displacement is shown just to the right of the pin at C . The corresponding position of the beam is shown by the solid sloping line. For unit displacement at C the movement of the load P at D is $\Delta_D = 1 \times 20/35$ or 0.57. Hence we may write

$$(17) \quad W_v = V_c \times 1 - P \times 0.57, \quad \text{or} \quad V_c = 0.57P.$$

The same result follows by statics from a moment equation about E .

72. Bending Moment Calculation by Virtual Work. Since work is expressed either as force times distance (Fd) or moment times angular rotation ($M\theta$), it seems reasonable to expect that a unit angle change introduced into a beam would make possible the calculation of the bending moment at that point by setting the total virtual work equal to zero.

EXAMPLE. In Fig. 59(c) a unit angular discontinuity ($\theta = 1$) is introduced at B . The corresponding deflections at C and D are obtained by drawing the deflection diagram which is made up of straight solid lines as shown. The work performed is concentrated at B and D . Since the moment M_B is in equilibrium with the loads and reactions acting on that part of the structure to the right of B which is displaced in Fig. 59(c), we may set the expression for the total virtual work equal to zero.

$$(18) \quad W_v = -M_B(\theta = 1) + P(\Delta = 6.85) = 0, \quad \text{or} \quad M_B = 6.85P.$$

Hence if $P = 1,000$ lb., $M_B = 6,850$ ft.-lb. It will be noticed that the value of M_B has been computed by virtual work without use of the reactions. This is not possible by use of the equations of statics. As a second example of the calculation of a bending moment in a more complex structure see the second computation sheet for the analysis of an unloading crane, p. 63. Here again the moment desired is calculated by virtual work without use of any reaction.

73. Bar Stress of a Truss Computed by Virtual Work. It is also possible to compute the stresses in the members of a truss by the principal of virtual work. The trick is to be able to determine the displacements of the loads due

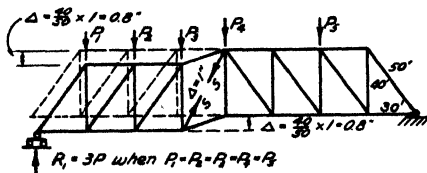


FIG. 60. BAR STRESS COMPUTED BY VIRTUAL WORK.

to a unit virtual change in length of the bar under consideration. If this can be done, the stress in the bar can be obtained easily by equating the total virtual work of displacement to zero. A simple example will be considered first.

EXAMPLE. For the simple truss of Fig. 60, one diagonal is lengthened by 1.0 in.

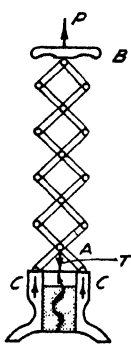
This diagonal displacement results in a vertical displacement of the vertical loads P_1, P_2, P_3

and also the vertical reaction R_1 of 0.8 in. Note that these forces are the ones which are in equilibrium with the bar stress S . The loads P_1 and P_3 do not move. Again the total virtual work for any set of forces in equilibrium may be equated to zero. Thus we obtain

$$(19) \quad W_v = (S \times 1) - R_1 \times 0.8 + (P_1 + P_2 + P_3) \times 0.8 = 0$$

$$S = 0.8(R_1 - P_1 - P_2 - P_3).$$

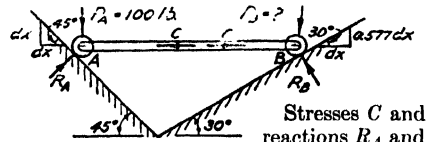
This same expression is obtained from statics since $R_1 - (P_1 + P_2 + P_3)$ is the vertical shear in the panel or the sum of the vertical forces to the left of the section. In this case the use of statics is more convenient than the use of virtual work. The method of displacements will prove useful for more complicated trusses, however. Two problems for which a simple solution by virtual work eliminates the need for consideration of internal forces are illustrated by Fig. 61.



This cork puller has a geometrical arrangement producing nine times as much pull at A as at B for there are four diamonds above A but only a half diamond below A . Since the compressive forces C acting on the bottle neck do no work, the pull P needs to be only one-ninth of the pull on the cork. Consider the virtual work due to a unit movement at point A .

$$W_v = T (\Delta_A = 1) - P (\Delta_B = 9) = 0$$

$$P = T/9.$$



Stresses C and reactions R_A and R_B do no work.

For a horizontal translation dx of the stick the vertical movement at A will also be dx and the wheel will roll 1.414 dx down the 45° slope. Wheel B will move 0.577 dx vertically as it rolls up the 30° slope. For the total virtual work of the vertical loads we may write for equilibrium

$$W_v = 100 dx - P_B(0.577 dx) = 0$$

Hence,

$$P_B = 100/0.577 = 173 \text{ lb.}$$

FIG. 61. PROBLEMS SOLVED CONVENIENTLY BY VIRTUAL WORK.

PROBLEMS

48. Check the reactions H and V_1 in Fig. 37(e) by virtual work.
49. Check the stresses in members L_1L_2 and U_1U_2 from Fig. 50 by virtual work.
50. Check the stresses in BC , de and Ee from Fig. 51 by virtual work.
51. On the calculation sheet p. 62 use virtual work to check the value of the shear 20 ft. to the right of A and the moment 35 ft. to the left of B .

UNUSUAL TRUSS ARRANGEMENTS

74. Methods of Analysis. Up to this point we have considered mainly the more common types of trusses. One structure, the compound truss of Fig. 54, presented the difficulty that no starting joint could be found at which only two unknown forces intersected and no section could be located cutting

but three non-concurrent, non-parallel unknown forces. This difficulty was circumvented by subdivision of the compound truss into two elementary trusses, but such a simple solution is not always feasible. For complex trusses two methods (1) the use of *virtual work* and (2) the use of *substitute members* will be presented. Virtual work will always be preferred if the joint movements due to a unit virtual displacement can be determined readily.

75. Usefulness of Virtual Work for a Complex Truss. We will consider the complex truss of Fig. 62. By starting at L_0 we can compute the stresses by the method of joints for L_0L_1 , L_0U_1 , U_1U_2 and U_1L_1 . Then we find that three unknown stresses occur at L_1 and U_2 which makes further progress by the method of joints impossible. By use of joints L_4 and U_6 we can progress backward from the right-hand reaction, but we cannot get past the joints U_5 and L_3 . Neither can a section be passed between the joints L_1 and L_3 that makes possible the calculation of the stress in any member of the truss of Fig. 62. The normal procedures of statics therefore break down since there is no obvious subdivision of this truss into parts that appears helpful. Nevertheless, its 11 joints provide 22 equations which should be adequate to solve for the stresses in its 19 members with three permissible reactions. The only difficulty would be in solving the simultaneous equations involved.

If a single interior stress such as U_3U_4 of Fig. 62 could be determined by virtual work, the remainder of the stresses could then be computed by the method of joints. Hence, we will shorten the member U_3U_4 as a virtual displacement and try to determine the corresponding movements of the loads and reactions. Only the vertical components of these movements are needed because all external forces are vertical, and no work is performed from horizontal movements of vertical forces.

76. Joint Displacement Diagram for a Unit Movement. Later when the deflections of trusses due to stress is being considered, the full Williot displacement diagram will be developed. At this point, however, we need only a very simple version of this construction since all members except one remain of unchanged lengths. The diagram to be drawn depends upon these observations:

(1) The movement of any chosen corner of a triangle may be located from known positions of the other two corners by striking intersecting arcs from the other two corners with the lengths of the connecting members as radii.

(2) When movements are small, such as those due to a virtual unit displacement, perpendiculars to the directions of the members may be substituted for arcs without loss of appreciable accuracy.

(3) The "relative displacements" of truss joints may be obtained *step by step* by use of successive triangles starting from two joints that are assumed to be fixed in position.

(4) For use with the principle of virtual work "relative displacements" based upon any starting pair of immovable joints are adequate because the total virtual work of a set of forces in equilibrium due to *any consistent set of displacements is zero*. Note that a bar stress is necessarily in equilibrium with the loads and reactions.

PROCEDURE. In Fig. 62(b) is drawn the displacement diagram for the complex truss of (a) due to a virtual shortening of U_3U_4 . This diagram is started with L_0L_1 in a fixed horizontal position. Since we are interested only in the "relative displacements" of the joints and not in their actual positions in space, point L_1 becomes the same point as L_0 in the displacement diagram. Also, U_1 being connected by bars of unchanging length to L_0 and L_1 has no displacement relative to L_0 or L_1 and must be shown coincident with L_0 and L_1 on the displacement diagram. Step by step we find that U_2 and L_2 are also coincident with L_0 , L_1 and U_1 since the left part of the truss from L_0 to U_2 and L_2 (shown shaded in Fig. 62(a)) remains fixed in position so that there are no "relative displacements" between any of these five joints.

A shortening of the member U_3U_4 in Fig. 62(a) produces rotation of the right hand unshaded triangle $L_2U_3L_4$ about L_2 . Since the amount of the shortening of U_3U_4 is not important as long as it can be determined, we have fixed the vertical displacement of L_3 at unity due to the aforementioned rotation of $L_2U_3L_4$ about L_2 . Then, because L_4 is at twice the distance of L_3 from the center of rotation L_2 , we conclude that L_4 moves vertically two units. Hence, in the displacement diagram Fig. 62(b), L_3 is placed one unit and L_4 is placed two units above L_2 . Next locate U_5 in Fig. 62(b) by the intersection of its horizontal line of possible movement from L_3 (perpendicular to L_3U_5) and its diagonal line of possible movement from L_2 (perpendicular to L_2U_5).

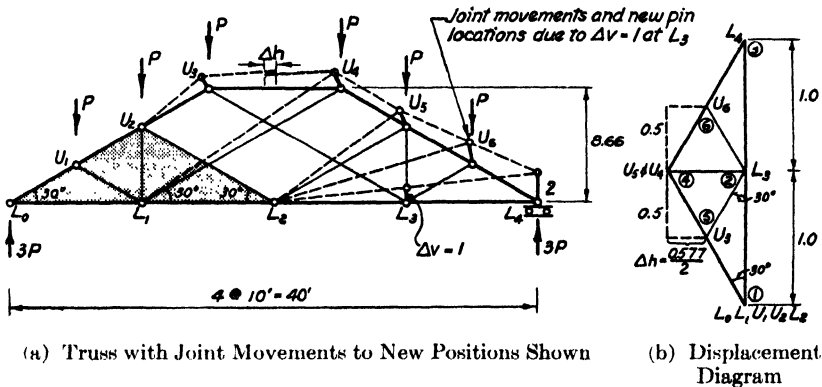


FIG. 62. EXAMPLE OF DEFLECTIONS OF TRUSS FROM VIRTUAL DISPLACEMENT OBTAINED BY A GRAPHICAL DIAGRAM.

From U_5 and L_1 in Fig. 62(b) we locate the new position of U_4 which is found to be coincident with U_5 . Then U_3 is located by the intersection of its diagonal line of movement from U_2 (perpendicular to U_2U_3) and its diagonal line of movement from L_3 (perpendicular to L_3U_3). Finally U_6 in Fig. 62(b) is located from L_3 and U_5 . This completes the displacement diagram. Our interest was in the vertical components of the deflections of the joints

produced by a horizontal displacement between U_3 and U_4 . These movements can be scaled directly from Fig. 62(b). The important movements of the joints are

Δv at L_4	Δv at U_5	Δv at U_3	Δv at U_4	Δv at U_3	Δh from U_4 to U_3
2.0 up	1.5 up	1.0 up	1.0 up	0.5 up	0.288

77. Chord Stress Computed by Virtual Work. By equating to zero the total virtual work due to the displacements of Fig. 62(b) (as given in the table above) we have

$$(20) \quad W_v = -S_{U_3U_4} \times 0.288 - P(0.5 + 1.0 + 1.0 + 1.5) + 3P \times 2.0 = 0$$

where the negative sign refers to work performed *against* a load or a stress.

$$(21) \quad S_{U_3U_4} = \frac{-4P + 6P}{0.288} = 6.94P. \quad (\text{Compression as assumed})$$

With the stress in U_3U_4 known, the remaining stresses can be computed by the method of joints. In fact, by noting that symmetry requires the stresses in U_2L_2 and U_5L_2 to be zero ($\Sigma V=0$ at L_2), the use of virtual work could have been avoided, but with unsymmetrical loading it would be needed.

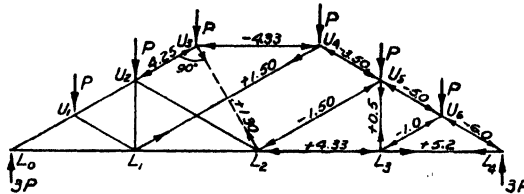
78. Method of Substitute Members.* When a statically determinate truss such as the one of Fig. 62 is encountered for which neither the method of joints nor the method of sections provides a starting point for an analysis, recourse may be made to the use of substitute members. The principle behind this method is really very simple. If there is no joint at which only two unknown bar stresses intersect, one can provide such a joint by removing one member and substituting another member for it. This interchange will be a correct one if it is then possible for you to compute all stresses in the modified truss by the methods of joints and sections.†

EXAMPLE. For the truss of Fig. 62 the member U_3L_3 will be removed and the member U_3L_2 substituted for it as shown in Fig. 63(a). By starting at the joint L_4 the stresses shown in the illustration may be computed by the method of joints, and by continuing toward the left the stresses in all members will be obtained for the modified truss. Then in (c) the exterior loads are removed and two 1-lb. forces are applied in line with the member removed (U_3L_3). Again no difficulty will be found in starting with the joint L_4 and in computing all stresses by the method of joints. Then, as is explained below Fig. 63, one determines the number of pounds of force (n) required to be applied as in (c) so that the stress in the substitute member will be reduced to zero. With this pair of forces (n) applied along with the loads, the substitute member becomes inactive and may be removed. The stress

* S. Timoshenko credits this procedure to L. Henneberg, 1886. Since complex trusses were much used before 1890 by American Railroads, it seems likely that a variation of this method was also used in this country. See "Theory of Structures," Timoshenko and Young, for further use of this method.

† The reason why some trusses are complex while others with the same number of joints and members are simple structures is due to the fact that each joint of the complex truss has three or more connecting members while the simple truss has a more uneven distribution of members with only two meeting at some joints and four or even five members meeting at other joints. A two-member joint is a necessary starting point for the method of joints.

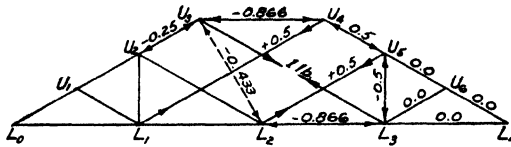
in any member is then the sum of its stress from (a) and n times its stress from (c). It is found in Fig. 63, that the value of n is 3.0 for this truss. The stress already computed by virtual work is checked and all others may be obtained quite easily.



(a) Stresses in Modified Truss Due to Loads



(b) Unmodified Truss



(c) Stresses due to 1-lb. Forces in Line with Member Removed

The stress +1.30 in member U_3L_2 could be reduced to zero by introducing a tension stress in U_3L_3 as shown in (c). Since a 1-lb. tension in U_3L_3 produces a compression of 0.433 lb. in U_3L_2 , we would need to apply $S = 1.30/0.433$ or 3.0 lb. in U_3L_3 of (c) to reduce the stress of +1.30 to zero in the substitute member (U_3L_2). Hence 3.0 times the stresses in (c) when added to the stresses of (a) give the correct stresses in the unmodified truss. For example, the correct stress for U_3U_4 is $-4.33 - (3 \times 0.866) = -6.93$ lb. which checks the value obtained in § 77 by virtual work when $P = 1$ lb.

FIG. 63. COMPUTATION OF STRESSES IN THE TRUSS OF FIG. 62 BY USING A SUBSTITUTE MEMBER.

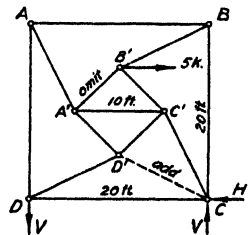
PROBLEMS

52. In Fig. 63 compute the stresses in members U_4U_5 , L_2U_5 and L_2L_3 by superposition of (a) + 3(c). Then pass a vertical section cutting these members and U_5L_3 and show that the part of the truss to the left of this section is in equilibrium.

53. Compute the stresses in members L_1L_2 , U_2L_3 , L_1U_4 , and U_2U_3 of the unmodified truss studied in Fig. 63. Then pass a vertical section cutting these four members and show that the part of the truss to the right of this section is in equilibrium.

54. Remove the member $A'B'$ and substitute $D'C$ in the planar complex truss illustrated. Then compute all stresses by the method of joints. Next determine the forces to be applied to the modified truss in line with $A'B'$ that will reduce the stress in $D'C$ to zero. From these data compute the correct stresses in the unmodified truss. Check the stress in $A'C'$ by virtual work.

Ans. Stress in $A'C' = 2.5 k$.



PROBLEM 54.

79. Survey of Elementary Stress Analysis. This chapter has been limited to a study of a few of the many applications of statics to stress analysis. Structural analysis begins with the study of the laws of statics. The reactions, the bar stresses, the distribution of load to welds, rivets, and pins, and even the stresses in the most minute fibers within these small parts are all controlled by the laws of statics. Statics may not be sufficient for the complete analysis of a complicated structure, but, nevertheless, the stress distribution obtained by the aid of some other tool must *always* satisfy the equations of statics, $\Sigma H = 0$, $\Sigma V = 0$, and $\Sigma M = 0$.

Both algebraic and graphical methods of analysis have been given for the determination of the reactions and stresses in trusses, and the reactions, shears, and moments in beams. Probably every algebraic method has a graphical counterpart, but this does not mean that both are always of equal importance. In truss analysis, for instance, the main advantage of the graphical treatment lies in the fact that forces at many angles may be handled almost as easily as horizontal and vertical forces. Moreover, it is not necessary that we even know the angles at which such forces act provided that a *scale* drawing of the structure with its loads is available. This fact is sufficient reason for choosing a graphical construction in preference to an algebraic method when such conditions exist. The common point of view is that graphical analysis is preferable for roof trusses, but that the stresses in most bridge trusses and the reactions for all structures are perhaps more easily obtained algebraically.

Virtual work or the work of imaginary displacements is a powerful tool of stress analysis. However, its main advantage over the use of statics is in those special cases where we must compute a single stress or bending moment and wish to avoid computations of unneeded reactions or stresses.

CHAPTER 3

INDUSTRIAL BUILDINGS

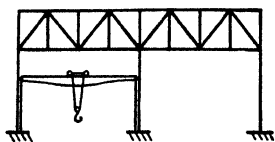
80. Types of Industrial Buildings. The term *industrial building* has come to mean any building structure used by industry where at least a part and usually most of the enclosed area is of one story height. Included are buildings for the steel mills, train sheds, locomotive repair shops, as well as automotive assembly plants and aircraft factories. These diverse structures all have the common requirement of large open floor areas frequently requiring roof trusses with adequate headroom for the use of an overhead traveling crane.

The great height to the roof, often 30 feet or more, makes it possible to arrange a balcony along one side or end of the industrial building for office space. Often the construction is extremely light and the wall enclosure is merely sheet iron or corrugated asbestos sheets. Unless insulated, such construction should be limited in use to heavy industry where the process itself produces an excess of heat or workmen are engaged in heavy physical labor. More permanent wall construction would be justified except where low first cost is essential. Glass enclosures are becoming increasingly common.

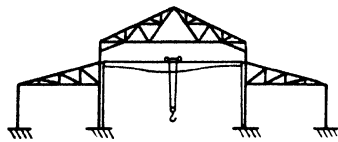
The Structural Frame. A simple type of steel-building frame is shown in Fig. 64(f). Around the main figure are small line diagrams of several types of building bents that are in common use. These are, of course, merely roof trusses that are connected rigidly to their supporting columns. The diagrams show bents with several kinds of roof trusses, with roof ventilators, with two stories, and with side sheds.

It is seen from a study of Fig. 64 and Fig. 65 that the industrial building frame consists of diagonally braced side bents or portals and several transverse bents formed from each roof truss with its supporting pair of columns. The whole structure is made rigid by the use of diagonal bracing in the planes of the top and bottom chords of the roof trusses.

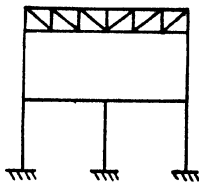
Purpose of Diagonal Bracing. Since each truss with its columns and knee braces forms a rigid bent, the function of the diagonal bracing is largely to *square* the building during erection and to prevent the building from *twisting* under a diagonal wind. Apparently bracing in one bay would serve this purpose, but it has been found that a light structure tends to vibrate and rack itself to pieces unless a considerable amount of diagonal bracing is used. It is necessary, then, to brace two bays, and it will be found advisable to *brace alternate bays* in all except temporary structures.



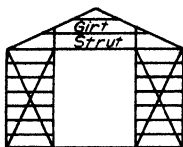
(a) Warren Type Double Bent



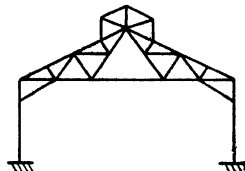
(b) Bent with Side Sheds



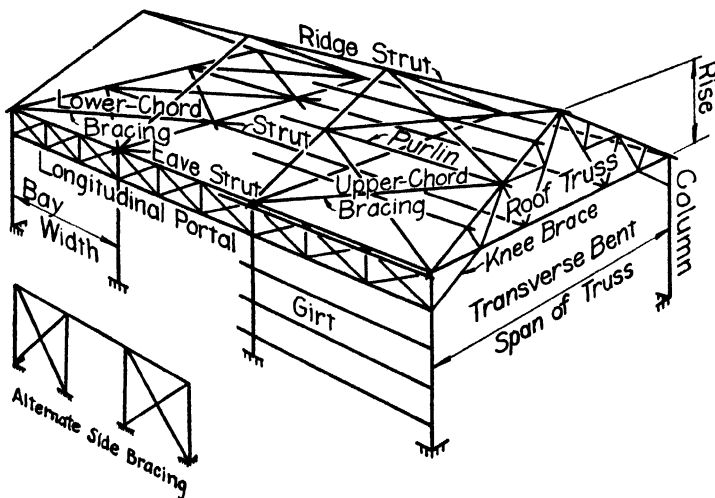
(c) Two-Story Bent



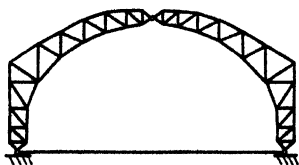
(d) End Framing



(e) Fink Type with Skylight



(f) Standard Building Frame



(g) Hinged Arch with Tie



(h) Pratt Types for Steep and Flat Roofs

FIG. 64. INDUSTRIAL BUILDING FRAMES.

In connection with the diagonal bracing, all buildings require the use of continuous longitudinal struts for the full depth of the structure. These struts reduce vibration and act as members of the systems of bracing in the

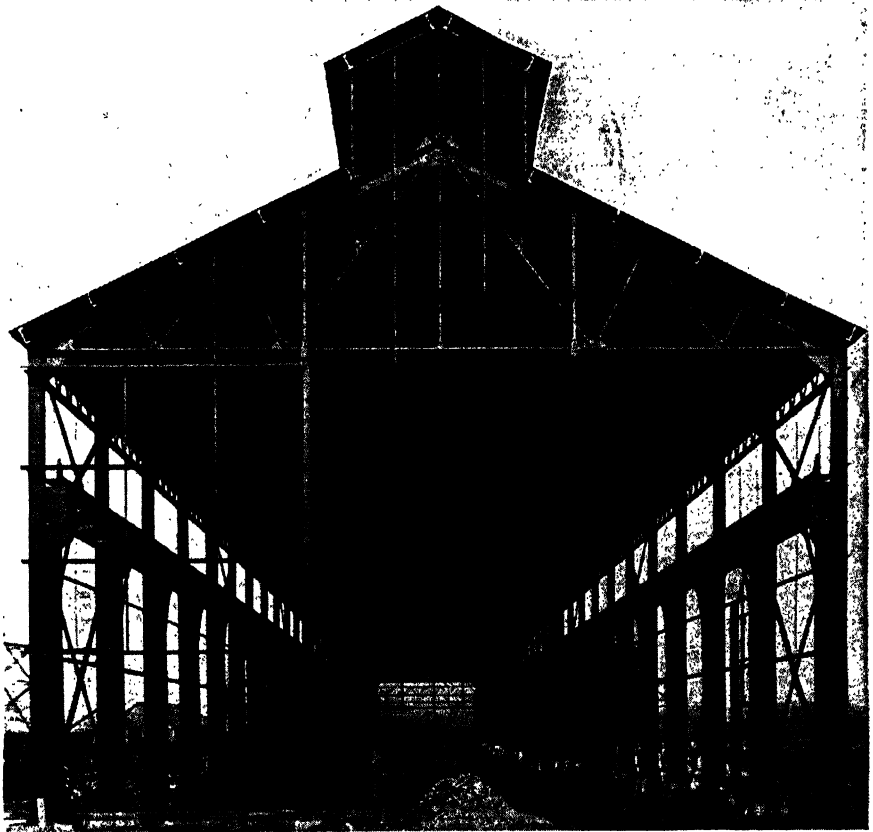


FIG. 65. INDUSTRIAL BUILDING UNDER CONSTRUCTION.

Features that are plainly visible are: the crane columns and track girder, the horizontal girts and the end framing, the diagonal bracing in each fifth panel along the sides, the longitudinal struts and the diagonal bracing in the plane of the lower chords of the trusses, and the channel purlins placed between the panel points of the upper chord.

planes of the upper and lower chords. A ridge strut, two eave struts, and at least one strut connecting the lower chords of the roof trusses are required. These may be seen in the center sketch in Fig. 64.

Flat Roof Buildings. Modern industrial architecture makes wide use of the flat roof as shown in Fig. 66 which illustrates a metallurgical research laboratory and foundry building at Illinois Institute of Technology. A flat

roof may be obtained by use of a roof truss as illustrated in Fig. 64(a) which may be extended to several bays wide. Such a building may be built with crane columns, skylights and flat roofed side sheds producing many possible variations. However, the modern trend is to eliminate the roof truss (at least for spans under 60 ft.) and to use a roof girder field welded or field

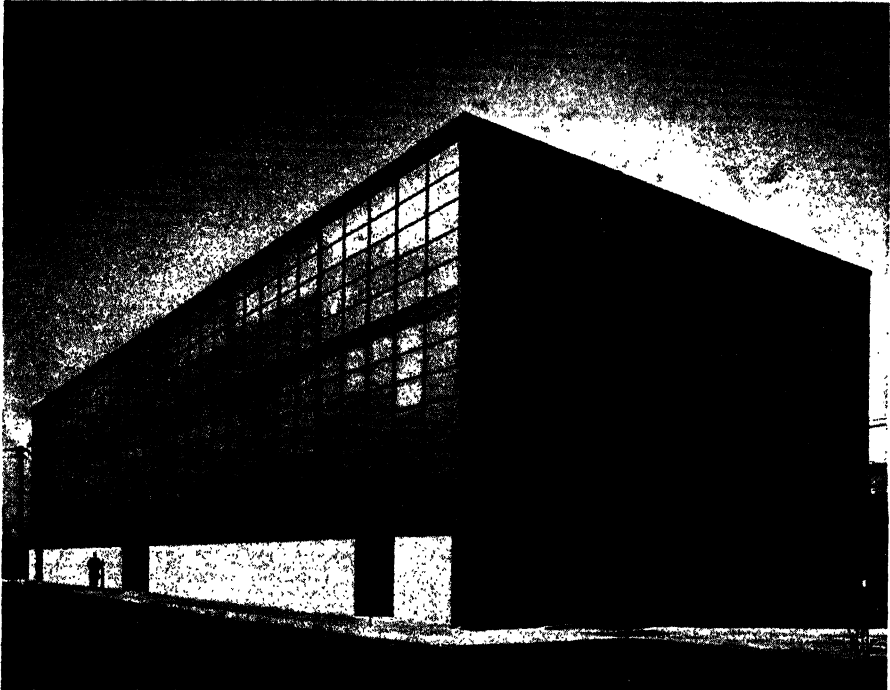


FIG. 66. METALLURGICAL RESEARCH BUILDING OF ARMOUR RESEARCH FOUNDATION AT ILLINOIS INSTITUTE OF TECHNOLOGY.

This building, designed by Mies van der Rohe shows plainly the structural framework which is exposed on the face of the building. Modern industrial buildings may consist merely of a structural frame filled in or enclosed with glass and prefabricated panels.

riveted to the columns. This is the basic structure used for the building of Fig. 66. We will later give attention to the analysis of both girder bents and trussed bents.

81. Structural Variations. Many variations of the industrial building frame shown in Fig. 64(f) are found in common use. However, the structural elements are essentially the same except for four variations.

(1) *Diagonal Side Bracing.* In some buildings where it is not necessary to have wide openings in all bays of the sides, it is possible to substitute diagonal side bracing for the longitudinal portals. A more rigid structure as well as a less expensive one can be obtained

in this way. It is necessary to place diagonal bracing in at least two bays of each side, and the use of bracing in alternate bays is desirable. A frame which has diagonal bracing in two bays of the side is shown in Fig. 67.

(2) *Continuous Roof Bracing.* Another variation of the standard type of industrial building frame occurs when knee braces are omitted in the interior bents and the *entire wind load* is transferred to the ends of the building by continuous roof bracing. This bracing may

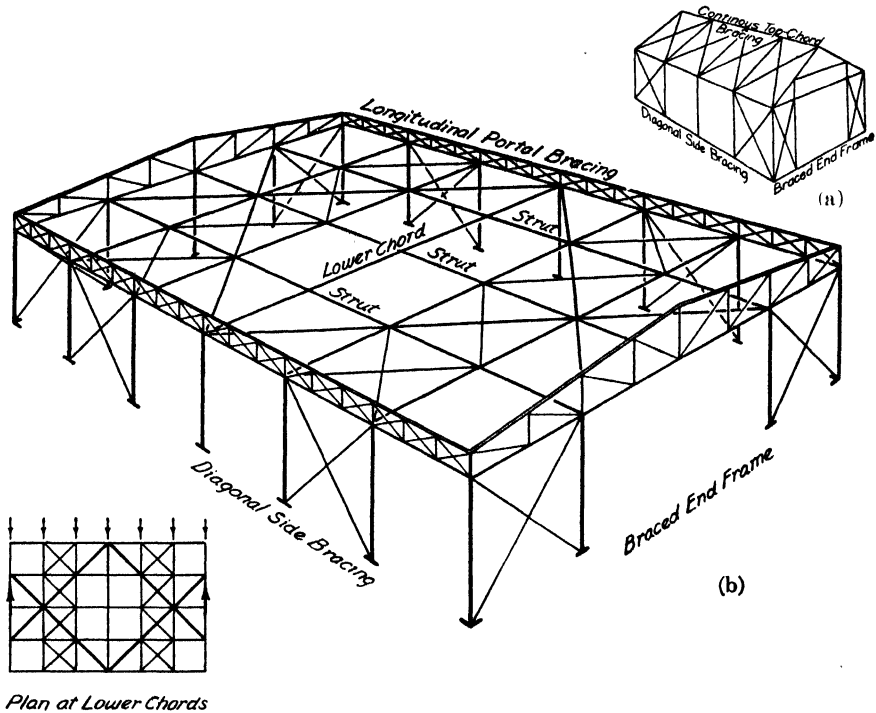


FIG. 67. USE OF LONG TRUSSES DOUBLY BRACED.

For greater rigidity long trusses may be connected to the columns to form transverse bents and longitudinal portals, and they may also have lower-chord bracing as illustrated to transfer a part of the wind load to diagonally braced end frames. In the illustration only the lower chords of interior trusses could be shown. The figure (a) is an alternate arrangement using diagonal bracing in the plane of the upper chords and in the sides and ends of the building.

be placed only in the plane of the lower chords of the roof trusses or in both planes of the chords. In either case, it is necessary to make use of solid masonry end walls or of diagonally braced end frames to resist the thrust of the wind. A knee-braced end bent would seldom be used in such a building because the heavy bending moments in the columns could not be resisted economically. Diagonally braced end frames are shown in Fig. 64 and Fig. 67.

(3) *The Hinged Arch.* A third variation of the standard industrial building is the two-hinged or three-hinged arch structure. See Fig. 64(g). If diagonal bracing is used either

continuously or in alternate bays, the arch will be braced properly. The arch building is used only where relatively great clear spans are desired, as, for instance, for an armory, a hanger, or a colosseum. A horizontal tie rod may be used to furnish the necessary horizontal reaction at the base if concrete abutments prove to be too costly. A diagram of a *tied arch* is shown in Fig. 64(g).

(4) *The Two-Story Bent.* A fourth structural variation consists in the addition of a second floor to form a two-story building. See Fig. 64(c). The structural frame is identical with that of the single-story building except for the floor system, which consists of girders framed between each pair of columns and supported at their centers by short posts. Steel floor joists run in the opposite direction at from 3-ft. to 6-ft. centers and commonly support a concrete floor slab. Where a clear first story is desired, the floor has sometimes been supported by hangers from the roof trusses in place of the posts. It is desirable that all heavy machinery should be placed on the bottom floor to reduce vibration and to cheapen the structure.

82. Roofs and Sidewalls. The roof of an industrial building must span between the roof trusses. Since the roof trusses are seldom spaced closer than 15 ft., reasonably heavy beams are required to span between the trusses and carry the roof load. These beams are called *purlins*. Their spacing will be found to vary from 3 to 7 ft. depending upon the allowable span of the roofing material or of the *sheathing*. Originally sheathing consisted of a wood deck resting on the purlins and covered by a light metal or mastic roofing. Modern construction with its flat or slightly sloping roofs often makes use of precast slabs of light-weight concrete that span from 4 to 7 feet between the purlins. Such slabs may be only $1\frac{1}{2}$ in. thick since they are machine cast with stiffening ribs along each edge. A slab with ribs, size 3 x 6 ft., will weigh about 350 lb. which permits some handling by hand. The concrete deck is covered with several coats of asphalt and roofing paper.

No sheathing is required under a corrugated metal roof or a corrugated asbestos roof, but roll roofings as well as composition shingles, slate, and tile, all require sheathing for their support. Corrugated metal roofs or corrugated asbestos roofs are therefore cheaper and are usually selected for temporary or low-cost industrial buildings. They are also non-combustible materials unless they have been dipped in asphalt. For insulation in cold climates and also to prevent moisture condensation and dripping from the roofs of boiler houses, we may pour a roof slab of gypsum or else use gypsum blocks on sheathing. Otherwise an *anti-condensation* insulative lining may be needed.

Wall Construction. The sidewall of an industrial building is commonly constructed in the same manner as the roof. *Girts* are used to span between the columns and either corrugated steel, aluminum or corrugated asbestos siding is attached to the girts. Except in the cheapest buildings this thin surface material will be backed up by insulation and an inside finish material. Much study and effort has been given to the problem of machine production

of prefabricated wall panels. Results indicate that this method of construction will eventually supersede fabrication by hand on the job. Windows take up a great deal of the sidewall area. Steel or aluminum sash and frames are used. Special girts must be placed above and below the windows for their support. Sidewall between windows may be constructed of cement plaster on expanded metal. Any of the usual stucco finishes may be specified. An uncommon type of construction is the use of *reinforced brick walls* of single thickness.* Walls of *glass bricks* have received attention.†

Details. The reader is referred to Sweet's Architectural Catalogue for details of roof and sidewall coverings, windows, doors, etc. Clips for attaching corrugated asbestos roofing to purlins and girts are shown on Fig. 68.

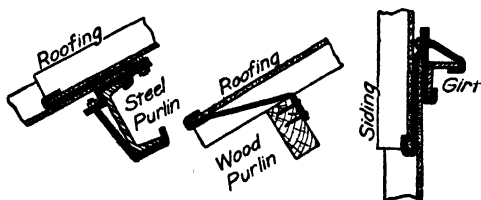


FIG. 68. CLIPS FOR ATTACHING CORRUGATED ASBESTOS.

Corrugated metal roofing is often attached to steel members with self threading screws hammer driven.

83. Permanent Construction. Industrial buildings that contain offices or drafting rooms, and also assembly halls, gymnasiums, etc., are frequently of more permanent construction than the industrial building described above. The usual practice is to make the walls of brick, usually 12 in. thick,‡ although the laboratory and foundry building of Fig. 66 has 8-in. brick walls that merely fill in between the structural steel members that are exposed on the face of the building. Corrugated asbestos supported directly upon steel purlins forms a satisfactory roof for such a structure provided that a suspended ceiling is used. A ceiling is desirable to reduce heat loss through the roof and to improve the inside appearance of the building. If the roof is flat or of low pitch, a precast concrete deck with an asphalt roofing is the common solution. Thin metal formed into continuous channels four to eight or more inches deep provides a cheap roof deck to span up to 15 feet or more. However, this "Q" deck presents the same problems of heat loss, etc. as corrugated metal roofing.

Expansion of Roof Truss. The manner in which the roof trusses are attached to the brick walls is of importance. For roof trusses of short span, the change in length of the bottom chord from temperature change will be small, and the ends of the trusses may be *bolted down* to the walls. When the trusses exceed 40 ft. in length, it is usually the practice to place one end

* H. S. Haworth, *Reinforced brick masonry for industrial use*, Civil Engineering, April, 1933, p. 204.

† Engineering News-Record, March 26, 1936, pp. 459-461.

‡ H. A. Sweet, *Resistance of brick walls to wind*, Eng. News-Record, Oct. 21, 1943, p. 121; also discussion by Jacob Feld, Nov. 18, 1943, p. 74.

of the truss upon a *sliding plate* to take care of expansion. A truss of greater span than 70 ft. should have one end supported on a *rocker* or on *rollers*. The reaction of such a support is assumed to be vertical; a truly vertical reaction would exist if the rollers were frictionless. All horizontal reaction must be furnished by the opposite support.

DEAD LOADS AND LIVE LOADS FOR BUILDINGS

84. Weight of Roof and Sidewalls. When a steel-frame building is selected because of its cheapness the entire construction is made as light as possible. For this reason a roofing material that is self supporting is desirable. Corrugated steel sheets can be obtained to span from 3 to $4\frac{1}{2}$ ft. between purlins. Corrugated asbestos roofing will span from 4 to 6 ft. between purlins. These same materials may be used to enclose the sides of the building, where they will span from 4 to 7 ft. between girts.

Corrugated Roofing. The following weights of corrugated steel, aluminum and corrugated asbestos are recommended where the load is in lb. per sq. ft. of roof surface. The use of corrugated asbestos and corrugated aluminum has increased rapidly because of neat appearance and freedom from maintenance.

CORRUGATED STEEL. Load 40 lb. per sq. ft.

- No. 22 gage (U. S. Standard) wt. 1.5 lb. per sq. ft., span 3 ft.-6 in.
- No. 20 gage (U. S. Standard) wt. 1.8 lb. per sq. ft., span 3 ft.-10 in.
- No. 18 gage (U. S. Standard) wt. 2.3 lb. per sq. ft., span 4 ft.-6 in.

CORRUGATED ALUMINUM.

0.032-in. thickness, 0.56 lb. per sq. ft., width 35 in., length 5 to 12 ft.		
Purlin Spacing	Allowable Load	
7' - 0"	25 lb. per sq. ft.	For factor of safety of 2.0 based upon ultimate flexural strength of 35,000 lb. per sq. in.
6' - 0"	35	
5' - 6"	40	
5' - 0"	50	
4' - 0"	80	

CORRUGATED ASBESTOS. Load 40 lb. per sq. ft.

- $\frac{1}{4}$ -in. thickness, wt. 3 lb. per sq. ft., span 4 ft.-0 in.
- $\frac{1}{3}$ -in. thickness, wt. 4 lb. per sq. ft., span 5 ft.-0 in.
- $\frac{3}{8}$ -in. thickness, wt. $4\frac{1}{2}$ lb. per sq. ft., span 6 ft.-0 in.

For a load of 30 lb. per sq. ft., decrease to the next lighter weight.

For a load of 50 lb. per sq. ft., increase to the next heavier weight.

Other Common Materials. Brick walls and a slate or tile roof are sometimes used for a more permanent structure. Brick walls that support roof trusses ordinarily are required to be at least 12 in. thick. The slate or tile roof is not self supporting and sheathing is necessary between purlins. Common 1-in. wood sheathing will span from $1\frac{1}{2}$ to 2 ft. between purlins and

support light loads. Thicker sheathing, 2 or 3 in., to support heavier loads for a wider spacing of purlins, is more common. Fireproof construction is obtained with a slate, tile or sheet metal roof on a thin concrete slab. By the use of a light-weight burned-clay aggregate, a 2-in. slab (precast) can be obtained which weighs but 15 lb. per sq. ft. for spans up to 6 ft. between purlins.

The following *weights of roofing materials* are suggested per square foot of roof surface. They are given as upper and lower limits since considerable variation of weight is possible.

Wood sheathing (2 in. thick)	6 to 8 lb.
Concrete roof slab (3 in. thick, depending on aggregate)	20 to 35 lb.
Slate ($\frac{1}{8}$ to $\frac{1}{4}$ in. thick lapped)	4 to 10 lb.
Tiles	8 to 20 lb.
Composition shingles	2 to 5 lb.
Composition sheet roofing	0.5 to 1.5 lb.
Tar and gravel roofing (3 to 5 ply felt)	5 to 8 lb.

85. Weights of Steel Purlins. The main structural members in the roof are the steel purlins. The weight of purlins varies with the spacing between purlins, with the amount of dead load and live load, and particularly with the spacing of roof trusses. Because of these variables, the weight of steel purlins can be stated only within wide limits. Usually this weight will be found within the limits of 2 and 4.5 lb. per sq. ft. of roof surface. The lower value is applicable where the span between roof trusses is short (under 16 ft.) and the loads are light. The upper value of 4.5 lb. per sq. ft. may be exceeded for long spans (over 20 ft.) where a heavy type of roof such as precast slab sheathing is selected.

86. Weight of Roof Truss and Roof Bracing. Roof trusses are of many types, depending upon the slope of the roof and upon the number of panel points needed for the support of the purlins. If the purlins are allowed to rest upon the top chord of the roof truss *between joints*, a heavy top-chord section will be required, which may increase the weight of the roof truss considerably. The major factors that influence the weight of a roof truss are the span, the amount of load to be carried, and the *pitch*.* A roof truss of large pitch (steep slope) will certainly weigh less per square foot of *roof surface* than one of small pitch. The rules given below for estimating the approximate weights of roof trusses are satisfactory for design purposes.

Roof Truss. The *weights of roof trusses* of from $\frac{1}{3}$ to $\frac{1}{4}$ pitch may be assumed to vary from 2.0 to 3.5 lb. per sq. ft. of roof surface where the span is 40 ft. The lower value may be used for welded construction where a corrugated metal roof or a corrugated asbestos roof is used. The gusset

* The *pitch* of a roof truss is the rise divided by the span. See Fig. 64 (f). The pitch is *not* the tangent of the angle at the eave; it is *one-half* of the tangent of that angle.

plates used with riveted construction will increase the truss weight about 15 per cent. For longer spans, add $\frac{1}{2}$ lb. for each additional 10 ft. up to 80 ft. For flat roofs, increase the above weights from $\frac{1}{2}$ lb. to 1 lb., and for very steep roofs, decrease the weights from $\frac{1}{2}$ lb. to 1 lb. The formula, $w = 0.5 + 0.05L$, may be used as a guide to the variation of weight per square foot of roof surface with the span L (in feet) for trusses of $\frac{1}{4}$ pitch.

Roof Bracing. In addition to the weight of the roof trusses, there is the *weight of diagonal bracing and longitudinal struts* to be considered. This bracing may be assumed roughly to add from $\frac{1}{2}$ to $1\frac{1}{2}$ lb. per sq. ft. to the dead load. The lower value applies to small structures where diagonal bracing is not needed in the plane of the lower chord.

From the known weight of the roof proper, which is designed *first*, and from the estimated weight of the trusses and bracing, we have at hand sufficient data for determining the dead loads acting on the roof truss.

EXAMPLE OF LOW-WEIGHT ROOF. A low-cost roof described in *Engineering News-Record*, May 27, 1948, p. 85 was used for a low warehouse with masonry walls. The stability of this so-called "umbrella truss" is partly dependent upon the walls. For one building the total weight of steel was 5.8 lb. per sq. ft. This total was divided approximately as follows: (1) Trusses, 43-ft. span 2.5, purlins, 17-ft. span, 2.0, interior pipe columns, 0.3, bracing 1.0 lb. per sq. ft. of roof surface.

87. Snow Load.* The amount of snow load that should be used in the design of an industrial building is largely a matter of opinion. Certainly, the actual snow load will be larger in the northern states than in the southern states. Probably the value of the snow load should represent the maximum weight of damp snow that can reasonably be expected to fall in any particular locality over a period of several winters. Dry snow is of little importance since it is readily swept from the roof by the accompanying wind. In general, it may be said that the weight of snow usually considered in design varies from 5 to 30 lb. per sq. ft. of roof surface. For roofs of $\frac{1}{3}$ or $\frac{1}{4}$ pitch, a snow load of from 12 to 20 lb. is commonly assumed for the central states where a 24-in. snow is exceptional and from 5 to 10 lb. for the southern states. These values may be halved for very steep roofs and they should be nearly doubled for flat roofs. They are given in pounds per square foot of *roof surface*.

88. Wind Load. The wind load depends upon wind pressure which in turn depends upon velocity of the wind. Early tests gave the relationship $p = 0.004 V^2$ as the total force in pounds on a rectangular plate 1 sq. ft. in area when exposed to a constant wind velocity of V miles per hour. A part of this force p is a pressure on the windward face and a part is accounted for by suction on the leeward face.

* A. F. Meyer, *Elements of Hydrology*, Wiley, 1928, pp. 81-89. Survey of snow and ice fall, with illustrations. Also, see *Snow loads on buildings*, Eng. News-Record, July 7, 1949, p. 76.

More recent tests have indicated that the actual pressure on the windward face of a building of reasonable depth is but one-half of the value given above or $0.002V^2$ and that a suction of about $0.0013V^2$ will occur on the leeward face. Hence, the recommendations of the A.S.C.E. Committee on Wind Bracing in Steel Buildings (Transactions A.S.C.E., 1940, p. 1715) was that the total wind force on the building be computed from the relationship

$$(1) \quad p = 0.0033V^2 \text{ (pressure plus suction, A.S.C.E., 1940)}$$

Where p is the wind force in pounds per square foot and V is the wind velocity in miles per hour.

Theoretical Considerations. The wind force against a surface that is considered to be essentially frictionless is not produced by loss of energy but by change in momentum of the air. Figure 69(b) and (c) show both the similarity

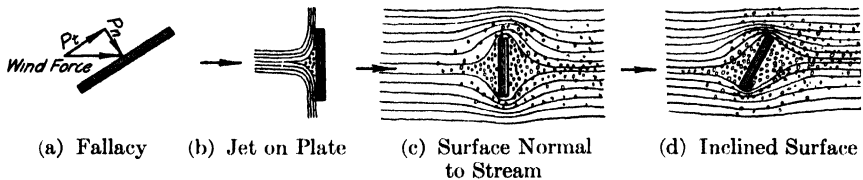


FIG. 69. WIND PRESSURE PRODUCED BY IMPACT.

and the difference between the problem of a jet of fluid striking a plate (complete loss of momentum in the direction of the jet) and the problem of a plate or building placed in a moving stream of air. Some of the momentum in (c) or (d) is conserved due to the fact that the fluid passes around the obstruction without losing all of its forward velocity.

DERIVATION. We may make use of the impulse-momentum equation for a constant force P

$$(2) \quad P \int dt = Mv_1 - Mv_2.$$

For the case of the complete obstruction of a small jet where v_2 is zero and $\int dt = 1$ sec., we have

$$(3) \quad P = Mv = \frac{W}{g} v = \left(\frac{wAv}{g} \right) v = \frac{wAv^2}{g}.$$

Here M and W represent respectively the mass and weight of fluid impinging on the obstruction per second, A is the area of the jet in square feet, v is the velocity in feet per second and w is the weight of the fluid per cubic foot.

Hence, for air of specific weight 0.0765 lb. per cu. ft. (at 15° C. and 760 mm.) the maximum total force on a square foot of area would be

$$p = \frac{0.0765}{32.2} v^2 = 0.00238v^2 \text{ (where } v \text{ is in ft. per sec.)}$$

or, by changing units

$$(4) \quad p = 0.00515V^2 \text{ (where } V \text{ is in miles per hour).}$$

This relation gives the largest possible force exerted in pounds per square foot and is to be compared with the A.S.C.E. specification of $p = 0.0033V^2$ (equation (1)). Evidently the wind velocity v_2 in equation (2) is not reduced to zero by an obstruction such as a building. Instead, the wind retains a forward velocity which effectively reduces the total force or pressure p by about one-third.

Velocity of the Wind. The value of the wind velocity at any particular location may approach the maximum probable wind velocity only a few times in the life of a structure; nevertheless, the engineer must give consideration to the maximum probable velocity in his design. It has been shown that the wind near the surface of the earth is broken up into gusts that cover relatively small areas. The importance of the maximum gust velocity is less than that of the average 5-min. velocity because a single gust would cover only a small part of the structure. This is particularly true around cities where the numerous buildings produce an extreme turbulence in the flow of air. The area covered by a single gust probably would be much greater at an elevation of 500 or 1000 ft., for there would be few obstacles to produce turbulence at such heights. It follows that the *maximum gust pressure* might be the proper force to use in the design of a tall office building or of a radio tower but that the *average pressure existing over several minutes* would be the proper force to use in the design of a low industrial building.

Gusts. Observations have been taken for a number of years in many cities of the United States. These observations have shown that an *average 5-min. velocity* exceeding 60 miles per hour may be expected through the general region of the central and eastern states. Much higher velocities are known to have occurred in tropical storms through the Florida and Gulf areas, while tornados have greatly exceeded this velocity in southern Illinois and elsewhere. Maximum gust velocities exceeding 150 miles per hour are believed to have existed in tornados. The only unquestionable record of a wind velocity exceeding 200 miles per hour was obtained on Mt. Washington in New Hampshire, April 12, 1934.*

89. Wind Pressure on Low Buildings. If we accept the A.S.C.E. formula ($p = 0.0033V^2$) we see that a velocity of 77.8 miles per hour produces a total wind force (part pressure and part suction) of 20 lb. per sq. ft. of exposed face. It is generally considered that 20 lb. per sq. ft. is a safe pressure for the design of a low building shielded by other neighboring structures where the average 5-minute velocity controls design. In an exposed or isolated location a design pressure of 30 lb. per sq. ft. corresponding to a gust velocity of 95 miles per hour is recommended. Bridges are placed in unprotected sites, which explains the fact that they must be designed for a wind pressure of from 30 to 50 lb. per sq. ft.

* Wind of 231-mile velocity recorded in New Hampshire, *Engineering News-Record*, May 10, 1934, p. 604.

Increase of Pressure with Height. Experimental evidence has established the fact that the maximum gust velocity will commonly exceed the average 5-minute velocity by fully 50 per cent. For an average 5-minute velocity of 77.8 miles per hour (20 lb. per sq. ft. pressure) the corresponding gust velocity of 117 miles per hour produces a gust pressure of 45 lb. per sq. ft. If the normal velocity over large areas at an elevation of 1000 to 1500 feet is taken to equal the gust velocity at ground level, we conclude that the pressure diagram assumed for designing a tall building or tower should increase from 20 lb. per sq. ft. at the base to 45 lb. per sq. ft. above the 1000-ft. level. The A.S.C.E. recommendation (Transactions A.S.C.E., 1940, p. 1714) is for a minimum design pressure of 20 lb. per sq. ft. up to the 300-ft. level to be increased by an additional pound of pressure for each 40 ft. of height above 300 ft. At 1500 ft. the resulting design pressure is $20 + (1500 - 300) 1/40 = 50$ lb. per sq. ft. This simple relationship seems adequate and acceptable.

Internal Wind Forces. Open windows, doors, or glass broken by a storm will cause internal wind pressure if openings may occur on the windward side. A suction or internal negative pressure will occur if openings can exist on the leeward side. The following formulas were recommended in the A.S.C.E., 1940 report. The factor n is the percentage of wall surface composed of windows, doors and other openings ranging from 0 to 30%.

$$(5) \quad p_{ip} = 4.5 + 0.25n \text{ (Internal pressure in pounds per sq. ft.)}$$

$$(6) \quad p_{is} = 4.5 + 0.15n \text{ (Internal suction in pounds per sq. ft.)}$$

It is recommended that the internal pressure or suction be used in the design of walls and for anchorage of the roof. The most unfavorable combination with external pressure or suction is to be chosen for each part of the structure. For example, on a wall parallel to the direction of the wind the internal pressure from equation (5) is to be combined with an external suction specified to be 9 lb. per sq. ft.

90. Sloping and Rounded Roof Surfaces. The normal pressure on a sloping surface can *not* be obtained by determining the component taken normal to the slope of the roof of the pressure on a vertical surface. See Fig. 69(a). Such procedure assumes that there can be a large tangential component of pressure P_t (friction) which tests have shown to be negligibly small.

As the obstruction to wind flow is turned away from the perpendicular as shown in Fig. 69(d), the loss of momentum reduces and the wind force on the body decreases. Sloping roofs and curved roofs produce such conditions. The A.S.C.E. (1940) specified wind pressures *in pounds per square foot of roof surface* are as follows:

Windward Slope, where θ is the angle with the horizontal.

(a) For θ not greater than 20° , an external suction of 12 lb. per sq. ft. When $\theta = 0$ (for a flat roof) this external suction force must not be neglected.

(b) For θ from 20° to 30° an external suction of

$$(7) \quad p_{es} = 1.20\theta - 36$$

(c) For θ from 30° to 60° an external pressure of

$$(8) \quad p_{ep} = 0.30\theta - 9$$

(d) For θ above 60° an external pressure of 9 lb. per sq. ft. along with (e) on the leeward slope.

Leeward Slope.

(e) For θ greater than 0° an external suction of 9 lb. per sq. ft. which is raised to 12 lb. per sq. ft. for a flat roof.

Rounded Roofs.

(f) For varying rise ratios and for each quarter of the arc the A.S.C.E. report (Transactions A.S.C.E., 1940, p. 1717) gives a different formula to control the design pressure or suction. Fig. 70 illustrates the need for careful study of the pressure and suction variation across a curved roof surface.

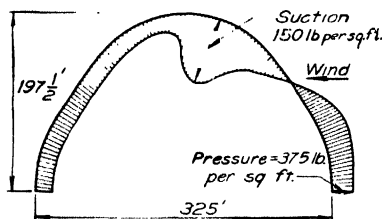


FIG. 70. WIND PRESSURE ON THE GOODYEAR ZEPPELIN DOCK.*

Empirical Formulas for Sloping Roofs.

Several formulas have long been in use for determining the total wind force (pressure plus suction) to be used in the design of a sloping roof truss. In the use of these formulas the wind is considered to be merely a pressure effect on the windward side of the roof which

tends to give conservative stresses for design in most cases. θ is the angle at the eave. The pressure p_n is normal to the roof.

$$(9) \quad p_n = p \left(\frac{2 \sin \theta}{1 + \sin^2 \theta} \right) \text{ (Duchemin formula, original tests made in 1829)}$$

$$(10) \quad p_n = p \sin \theta^{1.84 \cos \theta - 1} \text{ (Hutton formula)}$$

$$(11) \quad p_n = p \frac{\theta}{45} \text{ (Ketchum or straight-line formula)}$$

The Duchemin formula is considered to be the most reliable, but the straight-line formula has been much used because of its simplicity. The Hutton formula and the straight-line formula are in close agreement below an angle of 35 degrees (about $\frac{1}{2}$ pitch).

91. Additional Data on Wind Pressure.† *Barometric pressure* has an appreciable effect upon wind pressure and may account for a maximum variation of about 15 per cent. Curved surfaces receive less pressure from wind than flat surfaces. The pressure on a *cylinder* is approximately $\frac{3}{4}$ of the

* Civil Engineering, November, 1930, pp. 71-76.

† Valuable data on the action of wind are available in a series of articles by W. Watters Pagon, *Aerodynamics and the civil engineer*, Engineering News-Record: March 15, 1934, p. 348; July 12, 1934, p. 41; October 11, 1934, p. 456; December 27, 1934, p. 814; April 25, 1935, p. 582; May 9, 1935, p. 665; May 23, 1935, p. 742; October 31, 1935, p. 601.

pressure on a plane through its diameter; this factor may be reduced to $\frac{1}{2}$ for a *sphere*. For *closely spaced members* that are parallel, it is common to allow 50% more than the pressure on the windward member. There is reason to believe that twice the pressure on one truss should be used in bridge design where the two trusses are at least 15 to 20 ft. apart.

Anemometers. In studying wind pressure data, it is of importance to realize that the velocities recorded by standard anemometers used previous to 1927 were about 25% too high. The present anemometer (3-cup Robinson) records a velocity which is about 5% high. An excellent discussion of wind pressures is given in a book by Robins Fleming, *Wind Stresses in Buildings*. Albert Smith discusses the subject of wind pressure in an article published in the *Engineering News-Record* for June 11, 1931, page 971.

92. Load Combinations to be Used in Design. Observation shows that a heavy snow will be blown from a sloping roof before the wind velocity approaches 60 miles per hour. Therefore we do not consider it necessary to use both *full snow load and full wind load* in the design of a roof. Instead, the following load combinations are used. The *design stress* in any member is the largest stress produced by any one of these three combinations.

- (1) Dead load and snow load.
- (2) Dead load and wind pressure from either direction.
- (3) Dead load, *one-half* snow load, and wind pressure from either direction.

The term *one-half snow load* as used here means snow or rather ice over the entire roof of one-half the weight of the maximum snow load. It does not mean full snow load on one-half of the roof, which is a *very improbable loading* when combined with full wind load.

Alternate Loading. The use of a uniform vertical load over the entire area of the roof in place of the combined effect of dead load, snow load, and wind is entirely reasonable where the trusses are supported on masonry walls. A minimum vertical load of 30 lb. per sq. ft. frequently is specified.

PROBLEMS

55. The plan of an industrial building is 60×160 ft. Roof trusses are spaced 20 ft. apart. The purlin spacing is 4.8 ft. The roof is of $\frac{1}{4}$ pitch. The roofing material is corrugated asbestos. Estimate the probable dead load to be supported by one interior roof truss. Tabulate your estimate in neat form.

56. A gymnasium roof spans 80 ft. and the trusses are spaced 23 ft. apart. The rise of the roof is 3 ft.-0 in. A 3-in. concrete slab spans 5 ft. between steel purlins and carries a 5-ply tar and gravel roof. The concrete weighs 90 lb. per cu. ft. Produce a neat tabulated estimate of the total dead load acting on one truss.

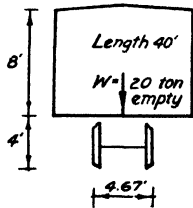
57. Plot the Duchemin, Hutton, and straight-line formulas for the special cases of $P = 20$ and $P = 30$ lb. per sq. ft. Plot all graphs on a single sheet. Add A.S.C.E. data from § 90. Values of p_n are to be plotted as ordinates and values of θ as abscissas.

58. A building is 160 ft. long, 60 ft. wide, and 25 ft. to the roof. It has a flat roof. Find the resultant pressure on the windward side of the building caused by a wind blowing

50 miles per hour at an angle of 30 degrees with the shorter side. Use A.S.C.E. formula, $p = 0.00201V^2$ for pressure on the windward side. Make use of the Duchemin formula.

Neglect skin friction.

Ans. 20,800 lb.



PROBLEM 60.

59. A solid concrete cylinder 2 ft. in diameter and 7 ft. high forming a gate post is found overturned by the effect of a hurricane. Both the post and its base are undamaged. Assume that the concrete weighs 150 lb. per cu. ft. and calculate the necessary wind velocity. Use $p = 0.0033V^2$ for the combined effects of pressure and suction but reduce coefficient to agree with § 91.

Ans. 165 m.p.h.

60. It is not uncommon for empty freight cars to be overturned in a wind storm. From data given on the size and weight of cars compute the probable velocity of the wind. Neglect wind pressure on undercarriage, but use $p = 0.004V^2$.

Ans. About 100 m.p.h.

61. Same as Problem 58 except follow all A.S.C.E. specifications from § 90 for determining pressure or suction on all four faces of this building.

ROOF-TRUSS ANALYSIS

93. **Types of Roof Trusses.** There are numerous variations of the standard types of roof trusses. However, the principles of analysis are the same for all. Truss diagrams are shown in Fig. 71 for a number of roof trusses.



(a) Fink



(b) Howe
(Wood with Steel Verticals)



(c) Pratt



(d) Pratt



(e) French



(f) Fan



(g) Saw Tooth



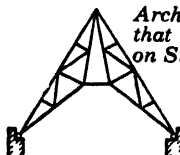
(h) Warren



(i) Quadrangular

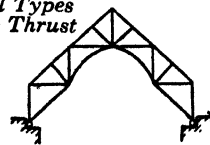


(j) Three-Hinged Arch



(k) High-Rise French

*Architectural Types
that Produce Thrust
on Sidewalls*



(l) Hammer-Beam Truss

FIG. 71. ROOF TRUSSES.

Of those shown, the *Fink* and *Pratt* types are most used for roofs of large rise ratio and the *Warren* and *Pratt* types are most used for flat roofs.

Note that the diagonals of the two Pratt trusses shown in Fig. 71(c) and (d) slope in opposite directions. The Pratt truss is constructed so that the diagonal web members will be in tension under dead load. In contrast, the *Howe* truss has its verticals in tension and its diagonals in compression under dead load. The reason for this difference is that the Howe truss is usually of wood construction with steel verticals. Wood diagonals are quite capable of resisting compression, but long steel diagonals should be sloped in the opposite direction to act in tension. The *quadrangular* truss is used for long spans and is constructed of steel. The diagonals are reversed near the center of the span where they would otherwise be acting in compression. The *French* and *Fan* trusses are less important variations of the standard Fink truss. *Saw-tooth* trusses are used to obtain adequate sky light for a wide building. The glazed surfaces should be faced north to avoid direct sunlight. The possibility of economy through use of the *three-hinged arch* should be investigated for spans exceeding 100 ft. The *hammer-beam* truss is constructed of wood. It is occasionally used in churches and auditoriums. It produces a lateral thrust comparable to the thrust of a three-hinged arch.

94. Calculations of Loads and Reactions. Panel concentrations of snow load and wind load are obtained by multiplying the area tributary to each panel point by the load per square foot. The entire dead load of roofing material, sheathing, purlins and truss is divided among the upper-chord panel points according to their tributary areas. The refinement of placing a part of the dead load of the trusses at the lower-chord panel points would be wasted precision. Ceiling loads may be caused by suspended ceilings, hanging balconies, light fixtures, shafts and suspended machinery or monorail cranes. These loads, including the effect of impact, must be taken into consideration in the design.

Reaction at a Sliding Plate. The calculations for standard cases of truss reactions caused by dead load and wind have been discussed in Chapter 2. A special case arises when a sliding plate is used at one end of the truss. The horizontal reaction at the sliding end is limited to the product of the vertical reaction there (caused by vertical loading as well as wind) and the coefficient of friction (about $\frac{1}{3}$ as a rough estimate). The value of the vertical reaction is *independent* of the coefficient of friction. Of course, if the plates corrode and fail to move (a very probable condition) both ends of the truss become fixed; hence a better approximation of the stresses may be obtained by making the horizontal reaction at each end equal to 50% of the horizontal component of the wind load.

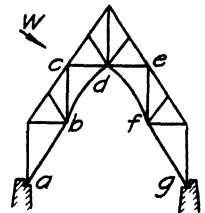


FIG. 72. HAMMER-BEAM TRUSS.

The Hammer-Beam Truss. The calculation of the reactions of the hammer-beam truss of Fig. 72 is an interesting problem. There are two sloping reactions (4 unknowns), but the reactions can be calculated by statics. Both curved members, bd and df , have zero stresses under dead load. These curved members are loose compression struts. Under the action of the wind from the left, the member bd will have a zero stress and can be removed. The result is that the structure becomes a three-hinged arch with hinges at a , c , and g . The analysis follows the procedure outlined in § 50.

95. Algebraic and Graphical Methods of Analysis. All of the standard tools of algebraic and graphical truss analysis have already been discussed. Roof trusses are commonly analyzed by means of the stress diagram. The algebraic method of joints is preferred by some. In order to obtain accurate stresses by the graphical method, we must start with a large-scale drawing of the truss drawn with extreme care. Most inaccuracies develop from our failure to obtain lines in the stress diagram that are *exactly parallel* to the corresponding truss members.

96. Ambiguous Trusses. *Algebraic Analysis.* We can start the analysis of the Fink truss of Fig. 73(a) by the method of joints. From joint L_0 , we

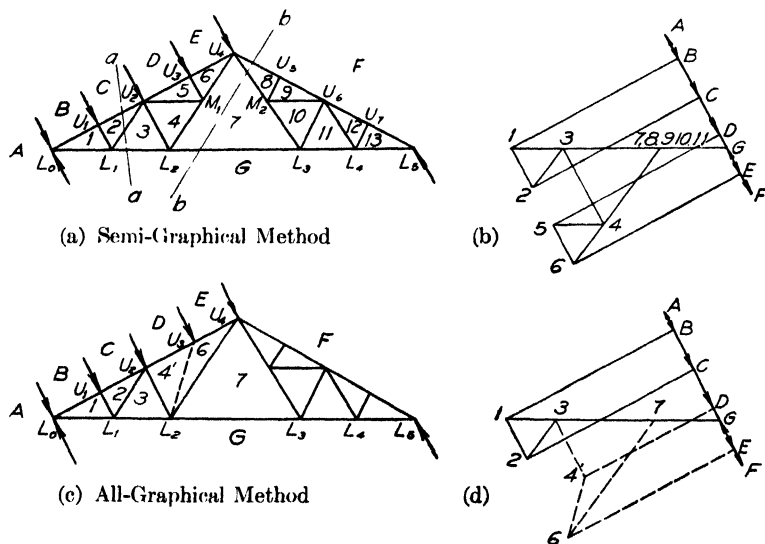


FIG. 73. GRAPHICAL ANALYSIS OF A FINK ROOF TRUSS.

continue by the usual procedure to U_1 and L_1 . Nevertheless, despite the fact that all stresses cut by the section $a-a$ are now known, there remain three unknown stresses at each of the joints U_2 and L_2 . It is possible, however, to cut the section $b-b$ and to compute the stress in L_2L_3 from an equation of

moments about U_4 . With the stress in L_2L_3 known there are but two unknown stresses at the joint L_2 . It therefore becomes possible to continue by the method of joints in the following succession: L_2 , U_2 , U_3 , M_1 , U_4 , etc.

Semi-Graphical Analysis. The same difficulty of ambiguity develops in drawing the stress diagram in (b). The points 1, 2 and 3 are located by drawing force polygons for the joints L_0 , L_1 , and U_1 . Since three unknowns exist both at U_2 and L_2 , the diagram cannot be continued. The semi-graphical method of procedure involves the algebraic determination of the stress 7- G from an equation of moments about the joint U_4 . Then the location of the point 7 on the diagram is determined by scaling the value of 7- G backward from G , as a tension stress. The stress diagram can then be continued in the order of joints mentioned above.

Graphical Analysis. An entirely graphical method of removing the ambiguity from the Fink truss is shown in Fig. 73(c) and (d). The members 4-5 and 5-6 in (a) are removed and the single member 4'-6 is substituted for these two members. The stress diagram for the substitute structure is drawn in the usual manner until the point 7 is located. Evidently the stress in the member 7- G and therefore the location of the point 7 would be the same for the original truss and for the substitute truss, since the moment about the joint U_4 is the same in both. Also the points 1, 2, 3, and 6 would not be moved. With the point 7 located as shown by the broken lines in (d), it is possible to replace the original truss members as in (a) and to complete the diagram, which will then appear as in (b).

The truss shown in Fig. 74(a) must have substitute members introduced as in (b) before the analysis can be completed graphically. In this case it

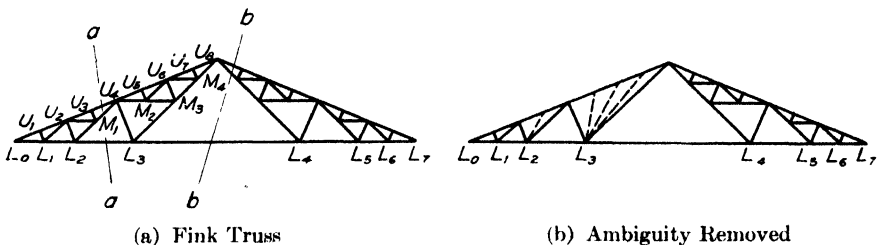


FIG. 74. SUBSTITUTE MEMBERS FOR A FINK ROOF TRUSS.

would seem simpler to make use of the combined algebraic and graphical method. The stresses L_2L_3 and L_3L_4 are computed from the sections $a-a$ and $b-b$ by the use of equations of moments about the joints U_4 and U_5 , respectively.

Simple Method of Analysis for the Fink Truss. Fink trusses are usually constructed with equal panel lengths along the top chord, so that they carry

equal panel loads. When this is true, the member U_2L_2 , Fig. 74(a), has *twice* the stress of the member U_1L_1 , and the member U_4L_3 has *four times* the stress of the member U_1L_1 . This relationship holds either for vertical or sloping loads, but it assumes that there are no loads at the lower-chord panel points. This simple device removes the ambiguity from the Fink truss since there are then but two unknown stresses at the joints L_2 and L_3 .

This study justifies the conclusion that ambiguity is caused simply by an unusual arrangement of members. It is in no way connected with statical indeterminacy. For instance, the ambiguous truss of Fig. 74(a) has 59 members and 31 joints; $2 \times 31 - 3 = 59$; the truss is statically determinate.

97. Example of the Analysis of a Flat Warren Truss. The truss shown in Fig. 75 will be analyzed and the maximum stresses determined. The truss

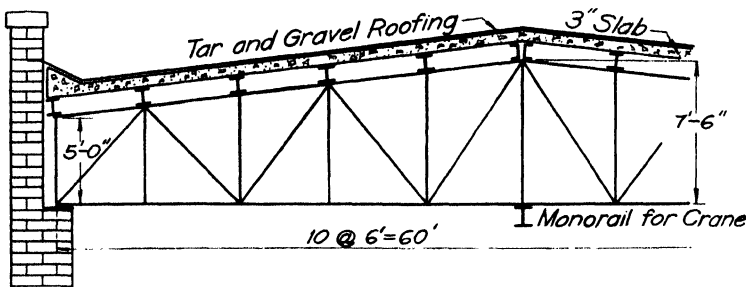


FIG. 75. ROOF OF AN INDUSTRIAL BUILDING WITH WARREN TRUSSES.

must carry the dead load of the roof, a snow load of 30 lb. per sq. ft., and a concentrated load of 20,000 lb. caused by a monorail crane. The truss is protected by a parapet wall, and therefore no positive wind load will be considered. The roof should be checked for stability under a suction of 12 lb. per sq. ft. as specified for flat slopes in § 90 and the internal wind pressure given by equation (5).

DATA.

Type of truss: Warren with verticals.

Span: 60 ft.-0 in.

Height: 7 ft.-6 in. at center, 5 ft.-0 in. at each end.

Spacing of roof trusses: 15 ft.-0 in.

Purlin spacing: 6 ft.-0 $\frac{1}{4}$ in. between panel points.

Snow load: 30 lb. per sq. ft. of roof surface.

Crane load: 20,000 lb. at center of truss (including impact).

External wind suction following § 90 = 12 lb. per sq. ft.

Internal wind pressure for 30% openings by equation (5) = $4.5 + 0.25 \times 80 = 12$ lb. per sq. ft.

Total upward wind force on roof of $12 + 12 = 24$ lb. per sq. ft.

PANEL LOADS.

Roof slab: 3-in. slab constructed with light-weight aggregate (100 lb. per cu. ft.); weight, 25 lb. per sq. ft. of roof surface. (Adequate to resist lifting force of wind.)

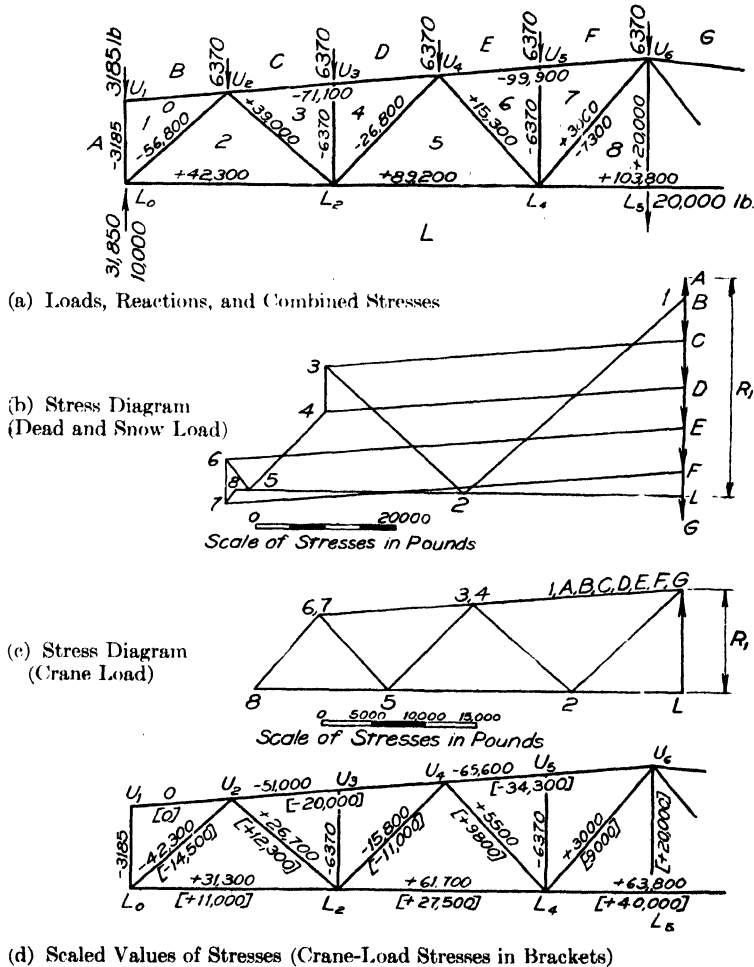


FIG. 76. ANALYSIS OF A FLAT WARREN ROOF TRUSS.

Roof covering: 5-ply tar and gravel roofing; weight, $7\frac{3}{4}$ lb. per sq. ft. of roof surface.

Purlins: 6-in. standard I-beams at 12.5 lb. per ft.; weight per sq. ft. of roof surface = $12.5 \div 6 = 2.1$ lb.

Roof truss: The loading is not particularly heavy; hence the basic weight (40-ft. span) is taken midway between the limits of 2 and 3.5 lb. per sq. ft., or 2.75 lb. per sq. ft. See § 86. This weight is increased 1 lb. per sq. ft. to account for the 60-ft. span and 1 lb. to allow for the flat pitch. The estimated weight is 4.75 lb. per sq. ft. of roof surface.

Bracing: Diagonal bracing will be used in the planes of the upper and lower chords. The estimated weight is 1.0 lb. per sq. ft. of roof surface.

Summary of panel loads:

Roof slab	= 25.0 lb. per sq. ft.
Roof covering	= 7.75
Purlins	= 2.1
Roof trusses	= 4.75
Bracing	= 1.0
Snow load	= 30.0
Total	= 70.6 lb. per sq. ft.
Panel load	= $70.6 \times 6.02 \times 15 = 6370$ lb.

STRESS CALCULATIONS.

Stress Diagram for Dead Load and Snow Load. Since there is no wind load to be considered, a combined diagram for dead load and snow load is drawn. See Fig. 76(b). The stresses are placed on the truss diagram in (d). Note that the symmetry of truss and loading makes it unnecessary for us to draw the stress diagram for the right-hand half of the truss. If we should become interested in the dead-load stresses, they may be obtained by multiplying the combined dead-load and snow-load stresses by the *constant ratio* 40.6/70.6.

Stress Diagram for the Monorail Crane Load. This diagram is shown in (c) of Fig. 76. The stresses are placed on the truss diagram in (d). These stresses are shown enclosed in brackets. The analysis presents no special difficulties.

Combined Stresses. Combined stresses for design are given on the truss diagram, Fig. 76 (a). For all members except L_4U_6 , the combined or maximum stresses may be obtained by summing the stresses as given in (d). The dead-load and snow-load stress in L_4U_6 is + 3000 lb. and the crane-load stress is - 9000 lb. The maximum possible tension is clearly 3000 lb., as recorded in (a). The maximum compression is the crane-load stress of 9000 lb. *minus* the dead-load tension of 3000(40.6/70.6), or $- 9000 + 1700 = - 7300$ lb. This value is recorded as the other combined stress for L_4U_6 in (a). The member must be capable of resisting *either of the combined stresses*.

98. Example of the Analysis of a Fink Roof Truss. The truss shown in Fig. 77 will be analyzed and the maximum stresses determined. The truss must carry the dead load of the roof, a snow load of 15 lb. per sq. ft. on the horizontal, and a wind pressure of 20 lb. per sq. ft. on the vertical.

DATA.

Type of truss: Standard Fink.
 Span: 36 ft.-0 in.
 Rise: 9 ft.-0 in.
 Spacing of roof trusses: 16 ft.-3 in.
 Snow load: 15 lb. per sq. ft. on a horizontal surface
 Wind load: 20 lb. per sq. ft. on a vertical surface.

DEAD LOAD.

Roof coverings: $\frac{3}{8}$ -in. corrugated asbestos; weight, $4\frac{1}{2}$ lb. per sq. ft.

Purlins: Steel purlins at 5-ft. centers to carry light loads for a span of 16 ft. are estimated to weigh 3 lb. per sq. ft. of roof surface.

Roof trusses: Estimated at the lower limit of 2 lb. per sq. ft. of roof surface because of the short span and light loads.

Bracing: Adequate bracing is estimated to weigh 1 lb. per sq. ft. of roof surface.

Fixtures and shafts: An allowance of $\frac{1}{2}$ lb. per sq. ft. of roof surface will care for such light loads.

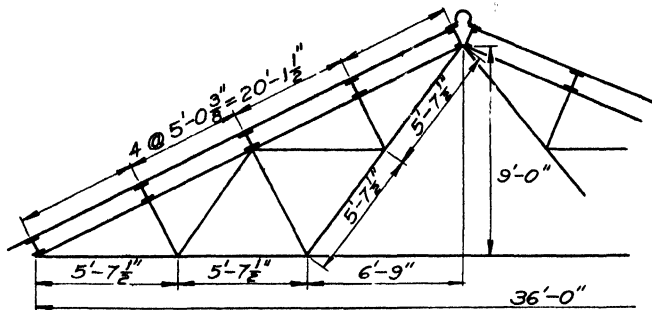


FIG. 77. ROOF WITH FINK TRUSS.

DEAD PANEL LOAD.

Roof covering = 4.5 lb. per sq. ft.

Purlins = 3.0

Roof trusses = 2.0

Bracing = 1.0

Fixtures, shafts = 0.5

Total = 11.0 lb. per sq. ft.

Panel load = $11.0 \times 5.03 \times 16.25 = 900$ lb.

SNOW LOAD. 15 lb. per sq. ft. on the horizontal corresponds to about 10 lb. per sq. ft. on a roof of $\frac{1}{4}$ pitch.

Panel load = $10 \times 5.03 \times 16.25 = 820$ lb.

WIND LOAD. The wind pressure on a roof of $\frac{1}{4}$ pitch ($\theta = 26^{\circ}:40'$) is $p_n = 20 \times 26.67 \div 45 = 11.8$ lb. per sq. ft. (straight-line formula).

Panel load = $11.8 \times 5.03 \times 16.25 = 960$ lb.

STRESS CALCULATIONS.

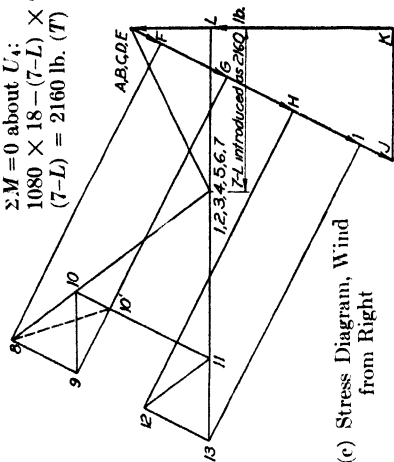
Stress Diagram for Dead Load. This diagram, shown in Fig. 78, presents no difficulty other than the need for overcoming the ambiguity of the Fink truss. The use of the substitute member U_3L_2 and the introduction of the calculated value of the stress 7-L are both illustrated on the diagram.

Stress Diagrams for Wind. Two wind-stress diagrams are shown on Fig. 78. Two diagrams are required because the entire horizontal reaction occurs at the right-hand end of the truss, and, therefore, the stresses produced by wind from the left are not simply reversed from those produced by wind from the right.

$$\Sigma M = 0 \text{ about } U_4;$$

$$1080 \times 18 - (7-L) \times 9 = 0$$

$$(7-L) = 2160 \text{ lb. (T)}$$

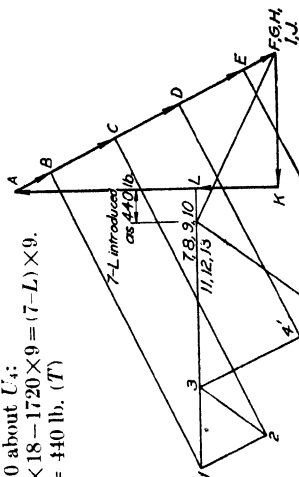


(c) Stress Diagram, Wind from Right

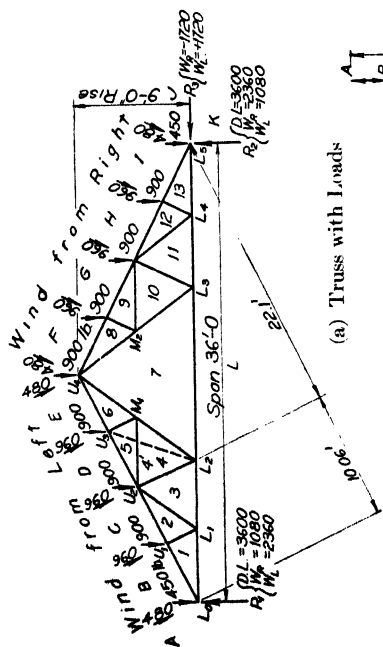
$$\Sigma M = 0 \text{ about } U_4;$$

$$1080 \times 18 - 1720 \times 9 = (7-L) \times 9.$$

$$7-L = 440 \text{ lb. (T)}$$



(d) Stress Diagram, Wind from Left



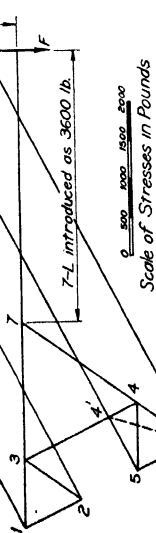
(a) Truss with Loads

$$\Sigma M = 0 \text{ about } U_4;$$

$$3600 \times 18 - 450 \times 18$$

$$-2700 \times 9 = (7-L) \times 9$$

$$(7-L) = 3600 \text{ lb. (T)}$$



(b) Stress Diagram for DL

FIG. 78. STRESS DIAGRAMS FOR A FINK ROOF TRUSS.

Reversal of Wind Reactions. In Fig. 78 it is possible to make one wind stress diagram do double duty by considering the effect of moving the horizontal reaction from L_5 to L_6 without reversing the direction of the wind. Assume that diagram (d) of Fig. 78 has been drawn for wind from the left. If the horizontal reaction is merely moved to L_6 , the pattern of bar stresses becomes the mirror image (left-to-right reversal) of the bar stress pattern obtained from Fig. 78(c). Hence, Fig. 78(d) with a revised force-reaction polygon to agree with the new position of the reactions and with the points renumbered can take the place of the diagram (c).

Combined Stresses. Table 1 shows how the stresses may be combined for maximum. In filling out this table, we obtain column (2) for snow load from the dead-load stresses of column (1) by direct proportion using the ratio of the corresponding panel loads, that is, $820/900 = 0.91$. Maximum combined stresses are given in column (7). There are no members that undergo a reversal of stress. All corresponding stresses for the two halves of the truss are equal except for the members of the *lower chord*. For every member, the combination of $DL + \frac{1}{2} \text{ Snow} + \text{Wind}$ gives a larger stress than $DL + \text{Snow}$. This result is to be expected where the snow load is relatively small.

Alternate Loading. Column (8) of Table 1 gives the stresses for an alternate vertical loading of 25 lb. per sq. ft. of roof surface. The chord stresses range from 98 to 118% of their computed maximum values. The web stresses range from 86 to 90% of their maximum stresses. A large amount of roof-truss design is based upon an alternate vertical loading.

Anchorage. For a roof slope of 26° - $40'$ the specified external suction is 9 lb. per sq. ft. on the leeward slope and $1.20\theta - 36 = (1.20 \times 26.7) - 36 = -4$ lb. per sq. ft. on the windward slope (equation (7), § 90). By equation (5) § 89 for 30% openings, there may be an internal pressure of $4.5 + 0.25 \times 30 = 12$ lb. per sq. ft. of roof surface. The maximum lifting force on one half the roof is $(9 + 12) \times 16.25 \times 18.0 = 6150$ lb. This lift is resisted by four panels of DL or 3600 lb. The remainder ($6150 - 3600 = 2550$ lb.) must be resisted by the anchor bolts at each end. Unanchored roofs are frequently lifted from the walls in wind storms. In fact, it has been observed by many engineers that the collapse of roofs in hurricanes or earthquakes is almost always due to failure of some small detail such as an anchor bolt or a seat angle.

99. Stress Coefficients and Nomographic Chart. The bar stresses in a given roof truss of *fixed pitch* vary directly with the panel load and are *independent* of the span. If the panel load is taken as 1 lb., the corresponding bar stresses are called stress coefficients. Tables of stress coefficients are available for most standard trusses in the structural handbooks. Figure 79 is a nomographic chart, prepared by Professor J. R. Griffith, from which either the dead-load stresses or the wind stresses can be obtained for a standard eight-panel Fink truss for any given panel loading. A new chart is required for each change in pitch and for each new arrangement of members. Any span may be assigned since the stresses are dependent upon the pitch rather than the span for fixed panel loads. The wind stresses in Fig. 79 are based upon the assumption of parallel reactions. Draw a straight line from the value of the panel load through the number of the member on the vertical center line and extend to the scale of the total stress. Try this method of checking the dead-load stresses in Table 1 for the truss of Fig. 78.

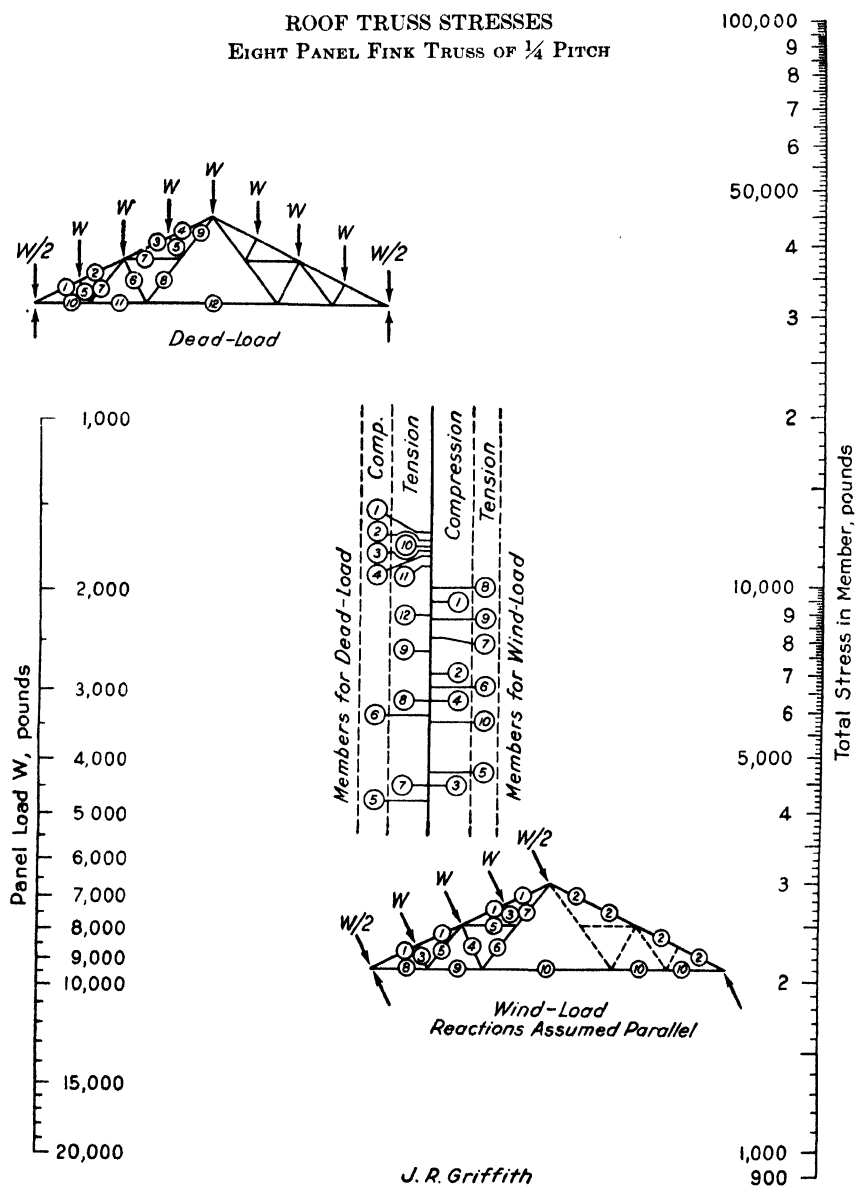


FIG. 79. NOMOGRAPH OF STRESSES FOR A STANDARD FINK ROOF TRUSS.

Draw a straight line from the value of the panel load through the number of the member on the vertical center line and extend to the scale of the total stress at the right.

TABLE 1
STRESSES IN THE FINK TRUSS OF FIG. 78

STRESS	(1) DEAD LOAD	(2) FULL SNOW	(3) HALF SNOW	(4) WIND FROM LEFT	(5) WIND FROM RIGHT	(6) COLUMNS TO BE COM- BINED FOR MAXIMUM	(7) MAXIMUM STRESS*	(8) STRESS CAUSED BY A VERTICAL LOAD OF 25 LB. PER SQ. FT. AS AN ALTERNATE LOADING	MEM- BER
1-B	-7050	-6420	-3210	-4320	-2440	1, 3, 4	-14,580	-16,100	U_1L_0
2-C	-6650	-6050	-3030	-4320	-2440	1, 3, 4	-14,000	-15,100	U_1U_2
5-D	-6250	-5700	-2850	-4320	-2440	1, 3, 4	-13,420	-14,200	U_2U_3
6-E	-5850	-5320	-2660	-4320	-2440	1, 3, 4	-12,830	-13,300	U_3U_4
13-I	-7050	-6420	-3210	-2440	-4320	1, 3, 5	-14,580	-16,100	U_7L_5
12-H	-6650	-6050	-3030	-2440	-4320	1, 3, 5	-14,000	-15,100	U_7U_8
9-G	-6250	-5700	-2850	-2440	-4320	1, 3, 5	-13,420	-14,200	U_6U_5
8-F	-5850	-5320	-2660	-2440	-4320	1, 3, 5	-12,830	-13,300	U_6U_4
1-L	+6300	+5730	+2860	+3650	+2160	1, 3, 4	+12,810	+14,300	L_0L_1
3-L	+5400	+4920	+2460	+2600	+2160	1, 3, 4	+10,460	+12,300	L_1L_2
7-L	+3600	+3280	+1640	+ 440	+2160	1, 3, 5	+ 7,400	+ 8,200	L_2L_3
11-L	+5400	+4920	+2460	+ 440	+4300	1, 3, 5	+12,160	+12,300	L_3L_4
13-L	+6300	+5730	+2860	+ 440	+5400	1, 3, 5	+14,560	+14,300	L_4L_5
1-2	- 810	- 740	- 370	- 960	0	1, 3, 4	- 2140	- 1840	U_1L_1
2-3	+ 900	+ 820	+ 410	+1070	0	1, 3, 4	+ 2380	+ 2050	U_2L_1
3-4	-1620	-1480	- 740	-1920	0	1, 3, 4	- 4280	- 3680	U_2L_2
4-5	+ 900	+ 820	+ 410	+1070	0	1, 3, 4	+ 2380	+ 2050	U_2M_1
5-6	- 810	- 740	- 370	- 960	0	1, 3, 4	- 2140	- 1840	U_3M_1
6-7	+2700	+2460	+1230	+3200	0	1, 3, 4	+ 7130	+ 6130	U_4M_1
4-7	+1800	+1640	+ 820	+2130	0	1, 3, 4	+ 4750	+ 4090	M_1L_2
13-12	- 810	- 740	- 370	0	- 960	1, 3, 5	- 2140	- 1840	U_7L_4
12-11	+ 900	+ 820	+ 410	0	+1070	1, 3, 5	+ 2380	+ 2050	U_6L_4
11-10	-1620	-1480	- 740	0	-1920	1, 3, 5	- 4280	- 3680	U_6L_3
10-9	+ 900	+ 820	+ 410	0	+1070	1, 3, 5	+ 2380	+ 2050	U_6M_2
9-8	- 810	- 740	- 370	0	- 960	1, 3, 5	- 2140	- 1840	U_6M_3
8-7	+2700	+2460	+1230	0	+3200	1, 3, 5	+ 7130	+ 6130	U_4M_2
10-7	+1800	+1640	+ 820	0	+2130	1, 3, 5	+ 4750	+ 4090	M_2L_3

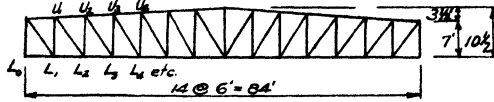
PROBLEMS

62. The Pratt truss shown supports a concrete roof slab $2\frac{1}{2}$ in. thick (weight 150 lb. per cu. ft.). The trusses are spaced 20 ft. apart and the purlins are 8-in., 11.5-lb. channels placed at each upper joint. The weight of roof truss and bracing may be taken at 7.5 lb. per sq. ft. of roof surface. The roof covering is 5-ply tar and gravel weighing 6.5 lb. per sq. ft. Analyze the truss for dead load and for a snow load of 25 lb. per sq. ft. of roof surface. Determine the stresses produced by a crane load of 10,000 lb. placed at the first panel point to the left of the center and combine for maximum stresses.

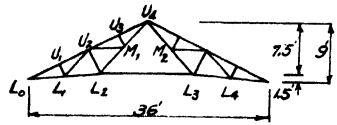
Ans. $L_6L_7 = +145$; $U_5U_6 = -146$; $U_6U_7 = -139$; $L_6U_6 = +4.3$ or -2.6 ;
 $U_6L_6 = +7.8$; $U_6L_7 = -13.3$ kips.

63. Analyze the French truss for exactly the same loads that were used in the example of § 98. Assume that the wind reactions are parallel to the wind loads.

Ans. $L_2L_3 = + 8.2$; $U_3U_4 = - 19.2$ kips.



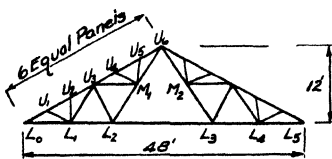
PROBLEM 62.



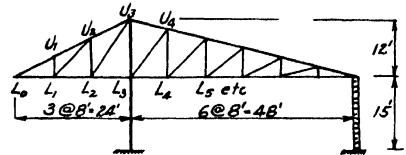
PROBLEM 63.

64. Lay out a Warren truss to replace the Pratt truss of Problem 62. Use the same loads and determine the maximum stresses.

65. The Fan truss shown carries a dead load of 14 lb. per sq. ft. of roof surface. Trusses are spaced 16 ft. apart. The snow load is 12 lb. per sq. ft. of roof surface and the wind pressure is 15 lb. per sq. ft. on a vertical surface. Calculate normal wind pressure by the Ketchum formula ($p_n = p\theta/45$). The left end of the truss rests on a sliding plate having a friction coefficient of 0.3. Allow for a friction reaction due to $DL + Wind$. If the horizontal component of the wind load is less than twice the maximum possible friction reaction at the left, divide this H -component equally between the two reactions. For a larger H -component of wind load all excess above the maximum value of the friction reaction at the left will be resisted at the right. Compute the wind stresses and the design stresses.



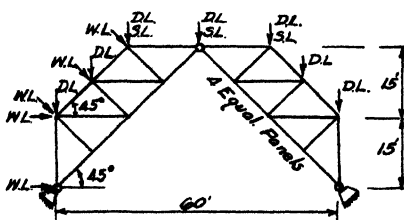
PROBLEM 65.



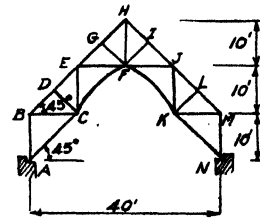
PROBLEM 66.

66. The cantilever truss covers a grandstand. Panel lengths along the lower chord are all equal. Dead load is estimated at 15 lb. per sq. ft. of roof surface. Trusses are 20 ft. apart. No snow is to be considered, but the wind may blow from either direction with a pressure of 25 lb. per sq. ft. on the vertical. Make use of the Duchemin formula. Assume that all horizontal reaction is furnished by the rear wall. Determine maximum stresses. If the diagonals cannot take compression, which panels will require counters (diagonals sloping in the opposite direction)?

Ans. $L_3L_4 = + 0.7, - 14.4$; $L_4L_5 = + 9.6, - 9.4$; $L_5L_6 = + 18.3, - 4.5$ kips.



PROBLEM 67.



PROBLEM 68.

67. Determine the maximum stresses for the three-hinged arch. The dead load is 20 lb. per sq. ft. of horizontal projection. The wind pressure is 20 lb. per sq. ft. of vertical surface. It may be transformed into normal pressure by the straight-line formula ($p_n = p\theta/45$). The snow load is 20 lb. per sq. ft. on a horizontal surface and zero on a surface sloping at 45 degrees. In this case combine dead load with full snow load on the horizontal surface and full wind pressure from either direction. Trusses are 25 ft. apart. Check stresses given in table and complete the analysis for the remaining members.

Answer to Problem 67

MEMBER	DEAD LOAD	SNOW	WIND (LEFT)	WIND (RIGHT)	MAX. STRESS	REVERSAL
U_0U_1	- 7950	-2650	-10,600	0	-21,200	
U_1U_2	- 6630	-3975	- 6630	0	-17,200	
U_2U_3	- 3750	-1875	- 9380	0	-15,000	
L_1L_2	-10,600	-7960	+ 9300	-6630	-25,200	
L_3U_3	- 5300	-5300	+ 6630	-6630	-17,200	+1300
U_0L_2	+ 5625	+1875	+ 1880	0	+ 9400	
U_1L_2	- 3975	-1325	- 1330	0	- 6600	

68. Determine the maximum combined stresses for the hammer-beam truss. Take the dead load at 23 lb. per sq. ft. of roof surface and the wind pressure at 30 lb. per sq. ft. on a vertical surface or on a surface at 45 degrees. There is no snow load. Trusses are spaced 15 ft. apart. Assume that the curved members can carry compression only, and that each has a zero dead-load stress. Horizontal reactions occur at both walls.

Ans. $AB = -10.3$; $BC = +4.9$; $AC = -17.5$; $CF = -14.3$; $EF = -9.3$;
 $CE = +2.4$; $HF = +2.4$ or -5.5 kips.

DIAGONALLY BRACED BUILDING FRAMES

100. **Bearing Walls vs. Steel Frame Construction.** The examples of roof truss analysis in §§ 97 and 98 are for roof trusses supported upon masonry walls or upon well-braced columns. Today, the high labor costs of brick masonry, the architect's desire for continuous window areas and the costly foundations required to support heavy walls have led to the wide use of light steel construction with wall enclosures of glass and of prefabricated panels. The result is to place the entire burden of resisting wind forces upon the steel frame. In the following chapter we will consider the analysis of stresses in partially braced buildings wherein lateral resistance to wind forces is provided in part by the flexural resistance of the steel columns. In this chapter we will restrict ourselves to consideration of building frames that are adequately braced with diagonal bracing in the sides, ends and the planes of the upper or lower chords of the roof trusses.

101. Diagonal Side and End Bracing. Full diagonal bracing is to be preferred to partial portal bracing in the side and end of the building because it removes the bending from the columns and produces a more rigid structure.* See Fig. 64(f) and (d). It is usually cheaper to resist loads by direct stress in diagonal bracing than by column flexure. The use of diagonal bracing is objectionable in those bays of the sides and ends of the building where doors, windows and unobstructed openings for the movement of materials are desired. Wherever diagonal bracing is permissible, it is best to use angles and to consider only the tension angle to be in operation. The minimum angle that will meet the specification for maximum slenderness ratio (L/r value) frequently will be more than able to carry the stress.

The use of diagonal bracing in only one bay of each side of the building is sufficient theoretically to maintain the stability of the structure. However, it is unwise to be niggardly in the use of bracing because the useful life of the structure may be considerably reduced thereby. Moreover, its cost is not great. The diagonal bracing might represent from 10 to 15% of the weight of structural steel and perhaps from 1 to 2% of the cost of the finished building. It is recommended that alternate bays be braced except in temporary structures, for which two bays may be braced in each side of the building.

Expansion Stresses in Diagonals. The use of diagonals in the end bays of the building, as shown in Fig. 80 has been common. However, it is bad design for a long structure because of temperature changes. When a long building expands the column bases do not move. Hence the columns are forced outward at the top particularly toward the ends of the building. The diagonal bracing tries to resist the motion and may be overstressed. To avoid this overstress place the diagonal bracing in several panels near the middle of the length of the building and leave the end panels without diagonals.

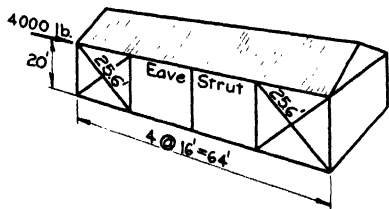


FIG. 80. DIAGONAL SIDE BRACING.

End Bracing. Usually, large doors must be placed in the ends of an industrial building, but even then two panels of diagonal bracing may still be used as shown in Fig. 64(d). A large central opening could be introduced in the end frame of Fig. 81. The diagonal bracing may pass across windows since this is not always considered objectionable by modern architects.

ACTION OF DOUBLE DIAGONALS. It is desirable to detail all diagonal-bracing angles about 0.02 in. short for each 10 ft. of length so that they will

* A. A. Weitsman, *Bracing of industrial buildings*, Engineering News-Record, Nov. 16, 1944, p. 91; also discussion by N. L. Ashton, Dec. 14, 1944, p. 77.

have to be *drifted** into place and will carry about 5000 lb. per sq. in. of *initial tension*. $((30,000,000 \times 0.02)/(10 \times 12) = 5000)$ The advantage of this is that the structure is rigidly held together by these prestressed diagonals so that it does not wear loose at the joints. The action of the wind increases the tension in one angle and reduces it in the other instead of buckling one of the angles by the introduction of a compressive stress. Both diagonals will be in action under wind load when they are erected with sufficient initial tension. It would seem reasonable, then, to assume this condition as a basis for stress analysis except for the fact that *inaccuracies of fabrication* may result in certain angles being long enough to have only a slight amount of initial tension or even none at all. Hence it is usual to assume that only one diagonal is in action in each panel or bay. Would the resultant calculated stress be different for the two cases? Would the design of the member be affected? Prove that the answer is *no* to each question.

EXAMPLE. When diagonal bracing is used in several bays, all diagonals sloping in one direction may be taken as resisting the wind shear. For instance, the side bracing shown in Fig. 80 must resist a total horizontal shear of 4000 lb. Assuming that this shear is resisted by the two tension diagonals, we find the stress per diagonal to be $(4000/2)(25.6/16) = 3200$ lb. The maximum stress in the cave strut is equal to the load, or 4000 lb. compression.

102. Diagonal Chord Bracing. Each roof truss connected rigidly to its columns will form a transverse bent. For this type of construction the chord bracing serves merely to *square* the building during erection and to prevent collapse from the *twisting* action of a diagonal wind. Bracing must be used in at least two bays in the plane of one chord, but the bracing of alternate bays in the planes of both upper and lower chords is probably justified.

Horizontal Truss for Lateral Resistance. If the only lateral resistance in the building is furnished by end walls or braced end frames, the diagonal bracing in the plane of either the upper or the lower chord must form a horizontal or inclined truss which requires that the chord bracing be continuous from *end to end* of the building. This chord bracing forms a horizontal or inclined truss whose reactions are furnished by the end frames and which carries horizontal wind loads at its panel points. A panel concentration is all of the horizontal component of the wind pressure acting on one bay of the roof and one-half of the wind pressure on one bay of the sidewall. This panel concentration may be considered to be divided equally between the upper-chord and the lower-chord trusses if both are continuous from end to end of the building.

Longitudinal Struts. Whenever diagonal bracing is used, the panel points where the diagonals join the chord members must be separated by struts. A longitudinal strut is always used along the tops of the columns, where it also

* The members are stretched into place by the use of tapered pins called *drift bolts*.

serves as a member of the side bracing, and perhaps of both upper-chord and lower-chord bracing. This member is called the *eave strut*. The purlins and the *ridge strut* also serve as members of the upper-chord bracing. For buildings of ordinary widths, the lower-chord bracing usually consists of three pairs of diagonals per bay which makes necessary the use of two *longitudinal interior struts*. See Fig. 81(a). These struts are made continuous for the full length of the building to prevent vibration of the roof trusses, even though only a few panels have diagonal bracing as may be seen in Fig. 64(f) where two longitudinal struts are shown.

The design of a strut is not always controlled by stress. Ordinarily the strut either transfers directly a determinable amount of wind force or else it receives a calculated stress as a member of a system of bracing. More commonly, however, the specification of limiting slenderness ratio or L/r value will determine its size. Struts are unchanged in section from one end of the building to the other.

Diagonal Bracing Between Struts of Upper and Lower Chords. When roof trusses are being erected, some diagonal bracing in a vertical or nearly vertical plane is required. Hence, along the center line of the building or in two or three longitudinal vertical planes we find diagonal bracing either continuously or in alternate bays. The use of diagonal bracing in alternate bays permits erection of the roof trusses in pairs. Since there is no vertical shear between roof trusses such diagonals have no determinable stresses.

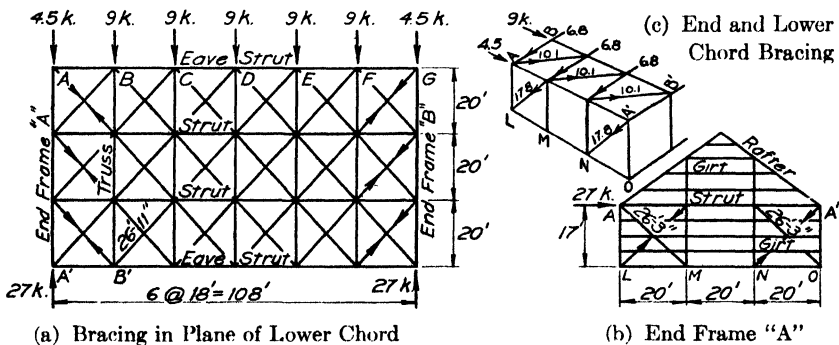


FIG. 81. ROOF AND END BRACING.

EXAMPLE. Stresses in a Diagonally Braced Building. A typical layout of continuous lower-chord bracing is shown in Fig. 81(a). The maximum panel shear is $27 - 4.5 = 22.5$ kips. This shear is resisted by the three tension diagonals in the end panel. The stress per diagonal is $(22,500/3)(26.9/20) = 10,100$ lb. The maximum stress in a strut is $10,100 \times (18/26.9) = 6800$ lb. The wind stress in the diagonals of panels nearer to the center of the building will be less, but even a center panel will not have a zero shear, because the wind force must be considered movable. Accordingly, for maximum shear in panel

BC , the loads at A and B are removed, and for maximum shear in panel CD , the load at C is also removed. Counter diagonals have the same stresses as the main diagonals when the wind reverses its direction.

The End Frame. The diagonal bracing of the *end frame* is shown in Fig. 81(b). The end frame is loaded with the reaction of 27 kips from the horizontal truss shown in (a). Again, it is assumed that the horizontal shear is resisted by the two tension diagonals. The stress per diagonal is $(27,000/2)(26.3/20) = 17,800$ lb. The maximum stress in the strut placed at the level of the eaves in the end frame is $27,000/2 = 13,500$ lb. This will be evident when we observe in the perspective sketch that the reaction of 27 kips shown in Fig. 81(a) actually is produced by two separate diagonal forces of 17.8 kips in the vertical plane of the end frame whose horizontal components of 13.5 kips provide the 27-kip reaction. Hence the single load of 27 kips shown in Fig. 81(b) is really two loads of 13.5 kips each. The strut AA' is subjected to the components in the direction AA' of the stresses in the five diagonals shown in the perspective sketch in addition to the load of 4.5 kips at A .

PROBLEMS

69. Draw up neat sketches of the complete structural frame of a standard industrial building that has the following dimensions: width, 70 ft.; depth, 160 ft.; height to lower chord of roof truss, 20 ft.; truss to be of $\frac{1}{4}$ pitch. All wind load is carried to the end frames. The wind pressure is 25 lb. per sq. ft. on the vertical. Determine the maximum wind stresses in all diagonals and struts. Many reasonable solutions are possible.

70. Repeat Problem 69 for a two-story building — 30 ft. to the flat roof level.

71. Choose any one of the forms of roof trusses shown in Fig. 64 or Fig. 71 and use it in the layout of an appropriate building. Establish all dimensions. Place diagonal bracing as needed to provide stability without reliance upon masonry walls or the flexural resistance of columns. Show doors and windows with dimensions. Compute the stresses in all diagonals and struts (not in roof truss members) for a wind pressure of 20 lb. per sq. ft. on a vertical surface. Use specifications of § 90 as needed.

103. Standards of Accuracy in Building Analysis and Design. A common mistake made by young engineers is that of placing too much emphasis upon *mathematical* analysis and too little upon the other important features of analysis. The choice of basic assumptions and the estimate of the loads are *fully* as important as the mathematical calculation of stresses. Such features of the analysis involve the use of engineering judgment.

It should be evident that a preliminary estimate of the dead loads involves approximations while the probable snow load or wind load is a matter of opinion rather than of fact. In view of the uncertainties involved in the selection of the loads, it seems rather foolish to attempt great precision in the calculation of stresses. A sure indication of lack of experience is observed in the engineer who confidently states that the dead-load stress in a certain member is 29,223 lb. The first four figures express the value of the stress more accurately than the dead load could possibly be estimated, and even more accurately than it is possible to read an ordinary slide rule. A recorded stress of 29,200 lb. would be proper. A stress ten times as large would be

recorded as 292,000 lb. and not as 292,200 lb., unless it was obtained by direct addition of several smaller stresses. Even then, the fourth significant figure is of no real importance.

Naturally the accuracy of an analysis is dependent upon the methods employed. If algebraic methods are used, an error of 1% may develop from slight inaccuracies in setting a slide rule. About the same standard of accuracy can be maintained in graphical analysis. If his stress computations involve no errors exceeding 1%, the student rightly may feel that his work is "exact" within the usual standards of engineering accuracy. Errors of 4 or 5% might not be objectionable because of their effect upon the design of the structure, but inaccuracies of this magnitude usually point to careless work; hence we could hardly feel certain that an error of 20 or 25% might not exist in *some* of our calculations. These remarks are intended to apply to simple structures. Different standards of accuracy must be applied to continuous structures, such as tall building frames.

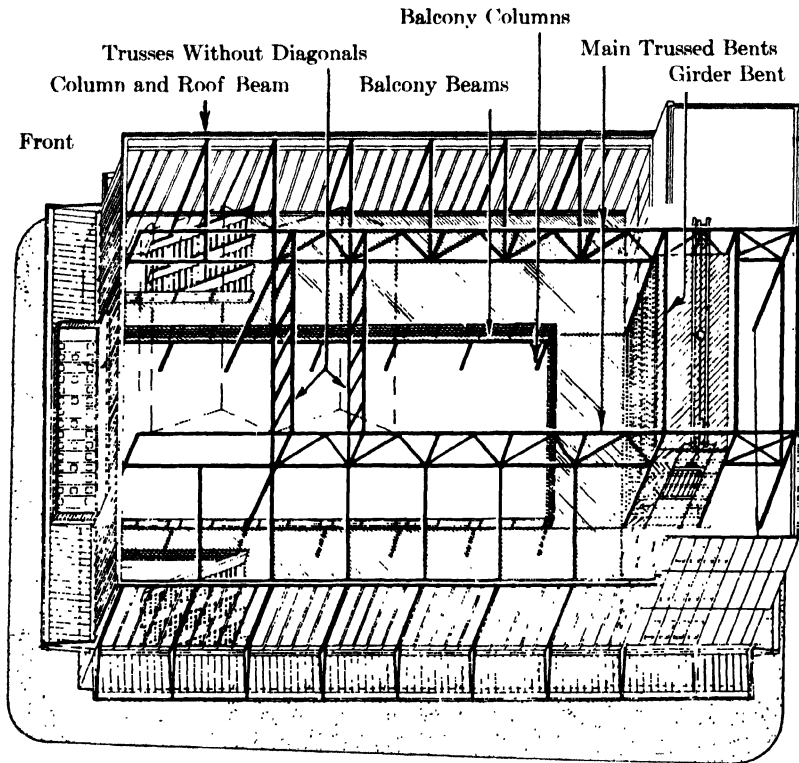
CHAPTER 4

INDUSTRIAL BUILDING BENTS

104. Building Bents or Portals. A study of the steel building frame of Fig. 64(f) will show that the essential structural elements are the transverse bent and the longitudinal bent or portal. These two structures serve similar purposes in making the frame resistant to horizontal wind forces, but the transverse bent serves the additional purpose of supporting the roof loads. Diagonal bracing in the planes of the upper and lower chords of the roof trusses serve to square the building during erection and to resist torsional wind effects when the walls and floors do not serve this purpose. In Fig. 82 we have a more complex building also braced exclusively with longitudinal trussed bents and transverse girder bents or portals. Even this structure is relatively clean and simple in its concept. Many buildings are structurally much more complex. The analysis of the continuous longitudinal bent or portal of the industrial building will be found to depend upon the same principles as does the analysis of the single-bay bent or portal.

105. Girder Portal with Pin-End Columns. The simplest type of girder bent or portal has pin-end columns, as shown in Fig. 83(a). A free-body diagram of the structure is shown in (b). Since the moments at the base must be zero (pin-end columns) there are but four reactions, and the structure is statically indeterminate to the *first* degree. In order to determine the reactions by statics, it is necessary to make an assumption which will eliminate *one* unknown. The usual assumption is that $H_1 = H_2$. Then, from the equation of statics $\Sigma H = 0$, it is seen that $H_1 = H_2 = P/2$; from $\Sigma V = 0$, it is seen that $V_1 = -V_2$; and from $\Sigma M_B = 0$, it is found that numerically $V_1 = V_2 = Ph/a$. The directions of the reactions are as shown in (b) of Fig. 83.

Shear and Moment Diagrams. The shears and moments may be calculated at any point in the structure from the values of the reactions. The shear at any point in the column is equal to $H = P/2$, and the column moment is $(P/2)y$. The shear at any point in the girder is $V = Ph/a$, and its moment at any point becomes $(P/2)h - (Ph/a)(x)$. The moment diagram for the portal is shown in (d) where the moment diagram is plotted on the tension sides of the members.



Courtesy Progressive Architecture

FIG. 82. BENTS PERPENDICULAR TO EACH OTHER PROVIDE A STABLE STRUCTURE.

This illustration depicts the structure of a French market building. With a movable, motor driven roof and second floor, this building can be changed from an open air market to a closed social hall or theater in less than one hour. The taller office block is at the rear. Notice the longitudinal trussed bents, on two widely spaced columns in the front, joined at the roof level to a narrow trussed bent in the rear. The three cross bents at the rear are of the simple girder type on twin columns. The first cross bent in the front is on four columns with a central truss of the Vierendeel or rigid-frame type without diagonals. The outside columns and beams would be so weak in comparison to the central columns and their Vierendeel trusses that they might be neglected in the analysis of wind stresses. However, since this structure is welded it acts as a unit. Much may be learned about structural arrangement by a careful study of this illustration. For example, the illustration clarifies the fact that all structures are in reality space frames. When we treat a complex framework as a series of planar frames and trusses we are introducing a simplifying assumption that is far from universally true. Every structure should be studied carefully to determine whether its analysis as a series of simplified planar frames is acceptable.

Approximation Justified. A justification for the fundamental assumption that $H_1 = H_2$ is easily found. Since the girder is made of the same material as the columns, that is, steel, its small direct stress produced by the load P will not shorten it appreciably. The deflection curves of the two columns as

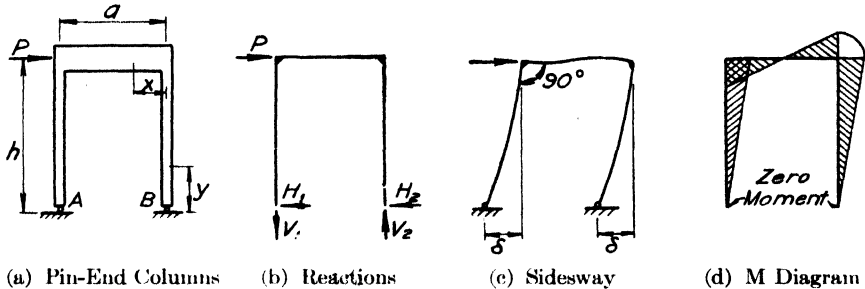


FIG. 83. GIRDER BENT OR PORTAL WITH PIN-END COLUMNS.

shown in Fig. 83(c) are therefore practically identical. The columns may be looked upon as identical cantilever beams (extending downward from the girder) loaded with the forces H_1 and H_2 . In order to have equal deflections, these identical cantilever beams must resist the same shears and moments, which leads us to conclude that H_1 and H_2 are equal.

106. Girder Portal with Fixed-End Columns. This structure, shown in Fig. 84(a), has six components of reaction. It follows that the fixed-end portal is three times statically indeterminate. The deflected structure (b) shows plainly the points of contraflexure near the mid-heights of the columns.

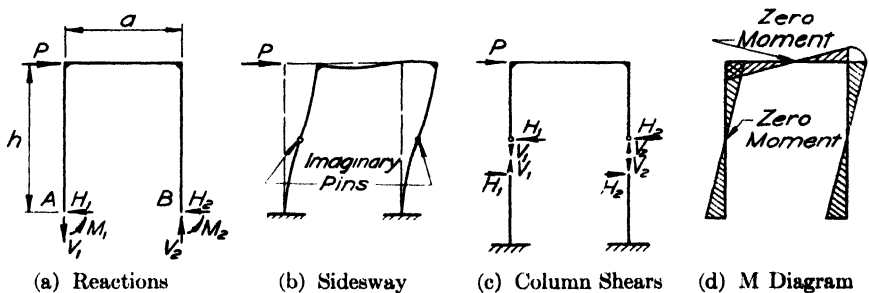


FIG. 84. GIRDER BENT OR PORTAL WITH FIXED-END COLUMNS.

Since points of contraflexure are points of zero moment, that part of the structure above these points may be isolated, as in (c), and treated as a portal with shortened columns and pin-end bases. The calculations for the reactions, shears, and moments follow as for the pin-end portal. A final moment diagram is shown in Fig. 84(d). Keep in mind that the analysis of this

indeterminate structure is made possible by statics only when *three reaction unknowns* are eliminated by *two basic assumptions*: (1) there are points of contraflexure in the columns at the mid-height of the bent, (2) the total horizontal shear or horizontal reaction is divided equally between the two legs.

107. Column Restraint. The designer must necessarily be very careful when considering the effect of column fixation. If the point of contraflexure is assumed to be located at the mid-height of the column, the maximum moment is only one-half of the corresponding maximum moment obtained when the point of zero moment is at *either* the top or bottom of the column.

It is important to visualize the action of the portal as it deflects. If the fixed base is released gradually and allowed to rotate, the point of contraflexure or of zero moment travels downward and is located at the base for the limiting case of a pin end. The restraint at the top is furnished by the cross-girder. When the base is fixed, the point of contraflexure moves upward as the girder is made more flexible, that is, as its moment of inertia is decreased. As a limiting case the bent would become merely two flagpoles with a light cross-bar, in which case the points of zero moment would be at the tops of the poles. It follows that the point of contraflexure is located by the relative restraints at the top and bottom of the columns, which places its location at the mid-height for equal end restraints. Otherwise the point of contraflexure is found *nearer the less restrained end*.

Common Assumptions. It is common practice to assume that the points of contraflexure are at the mid-heights of the columns. This assumption is dependent upon the further assumption that there will be only sufficient distortion or *give* in the bottom anchorage to balance the slight flexibility of the top girder. Where reasonably favorable conditions for bottom fixation do not exist, the designer should locate the points of contraflexure much nearer the column bases.* The analysis is made for pin ends if no attempt is made to fix the column bases.

108. Column Anchorage. A column that carries a heavy direct stress may be fixed against the rotating effect of a small moment without the aid of anchor bolts. True fixation exists as long as the moment does not *reverse* the bearing pressure under one edge of the base plate, i.e., $P/A > Mc/I$. In case tension is found there, the tension must be carried by the anchor bolt, and some rotation will occur because of the stretch of the anchor bolt. In order to reduce this rotation, it is wise to reduce the working stress for the design of the anchor bolts. It is also desirable to remove all flexibility as far as possible from the angles and plates of the base. A welded base is ideal for this

* For convenience, examples and problems in this text will be worked out on the assumption that points of contraflexure are at the mid-heights of the columns. It is not intended to imply thereby that this is the correct assumption in all cases, even though columns are fixed at their bases.

purpose. In addition, the masonry pier of the footing must be designed so that it will not rotate appreciably on its earth base. Initial tension in the anchor bolts also aids in the reduction of base rotation.

EXAMPLE OF INITIAL TENSION IN ANCHOR BOLTS. Another method of reducing the rotation of the base is by producing initial tension in the anchor bolts by tightening down on the nuts. A bolt with initial tension

will not stretch appreciably until the applied tensile load on the bolt exceeds the initial stress. In Fig. 85(a), the initial tension in the bolt produces the forces C in the block. When the external force T equals the value of the initial stress, the forces C will disappear (the block being assumed inelastic), and the bolt stress will be T . The bolt will *not* have changed in length.

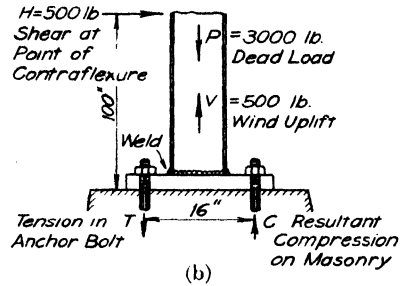


FIG. 85. STRESSES IN ANCHOR BOLTS.

The calculation of stress in an anchor bolt after the initial tension has been *considerably* exceeded is

illustrated by Fig. 85(b). It is *assumed* that the resultant pressure under the base is at the right-hand anchor bolt. By taking moments of all forces about this point, we obtain the value of the bolt tension T in the form

$$T = \frac{500 \times 100 - (3000 - 500) \times 8}{16} = 1870 \text{ lb.}$$

For a practical case where all dimensions are known, we could probably improve upon the assumption above as to the location of C , the resultant pressure on the base. Actually, in a proper design, the initial tension in the anchor bolt would never be exceeded and compression would exist over the entire base.

109. Points of Contraflexure for the Framed Portal. The girder bent which has been discussed is a common type of welded steel structure. How-

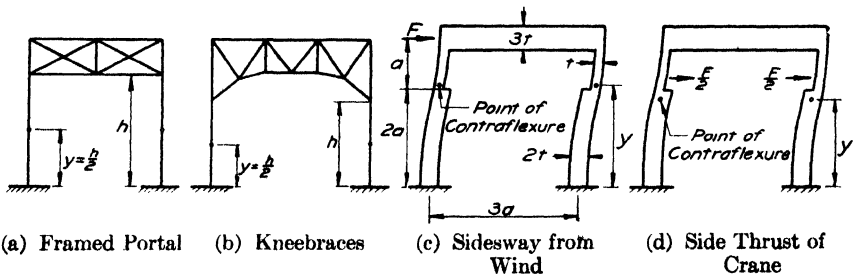


FIG. 86. ASSUMED LOCATIONS OF THE POINTS OF CONTRAFLEXURE.

ever, both in buildings and in bridges we also find use for the framed bent or portal. Again, it is necessary for the designer to use his judgment as to the location of the points of contraflexure in the columns. For a bent with fixed

bases the points of contraflexure are usually taken at one-half of the distance up from the base to the foot of the knee brace or to the bottom of the truss. See Fig. 86. It is common practice to assume pin-end conditions where the column anchorage offers less than full resistance to the moment at the column base. The assumption of equal column shears, which corresponds to equal horizontal reactions at the column bases, does not introduce an appreciable error for the simple portal with identical columns.

Points of Contraflexure in Crane Columns. The moment of inertia of the cross-section of a crane column is increased greatly below the seat for the crane girder. This produces a change in the position of the point of contraflexure. When the base of the crane column is fixed, the point of contraflexure occurs near the point where the column changes section, which is a common assumption made in design. Figure 86(d) illustrates the effect of the lateral forces produced by side thrust of the crane. For this loading the points of contraflexure were found by a *model test* to be slightly below the point of change of section. The points of contraflexure were slightly above the point where the section changed when the lateral force was placed at the top of the bent, as shown in Fig. 86(c).

110. Example of the Analysis of a Framed Portal. The portal shown in Fig. 87 will be analyzed for the stresses caused by a horizontal force of 1000 lb. acting at the top. The columns are assumed to be fixed at their lower ends.

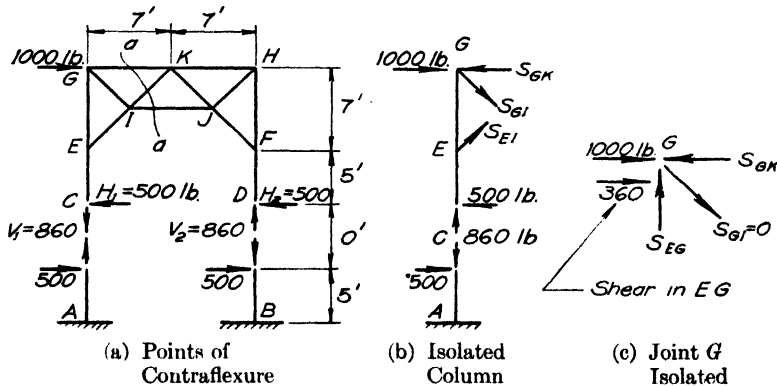


FIG. 87. ANALYSIS OF A SIMPLE PORTAL.

The shear in each column is taken as one-half of the load, or 500 lb. The points of contraflexure are located at the mid-height from the base to the foot of the knee brace or 5 ft. above the base.

Signs of Stresses. In the calculations that follow, each force is assumed to be acting in the direction shown in Fig. 87. A negative value for a stress simply means that the assumed direction should be reversed.

VERTICAL REACTIONS. Use the isolated section above the points of contraflexure, Fig. 87(a).

$$\Sigma M_C = 0; \quad 1000 \times 12 - V_2 \times 14 = 0; \quad V_2 = 860 \text{ lb.} \quad (C).$$

$$\Sigma V = 0; \quad V_1 = -V_2 = 860 \text{ lb.} \quad (T).$$

MOMENT AT COLUMN BASE.

$$M_A = M_B = 500 \times 5 = 2500 \text{ ft-lb.}$$

STRESS IN EI . Isolate the left-hand column above the point of contraflexure as in (b) of Fig. 87.

$$\Sigma M_G = 0; \quad 500 \times 12 - H\text{-comp.}S_{EI} \times 7 = 0; \quad H\text{-comp.}S_{EI} = +860 \text{ lb.}$$

$$S_{EI} = 1.41 \times 860 = 1210 \text{ lb.} \quad (T).$$

STRESS IN GI . Use the same section as for the member EI .

$$\Sigma V = 0; \quad -860 + V\text{-comp.}S_{EI} - V\text{-comp.}S_{GI} = 0; \quad S_{GI} = 0$$

STRESS IN GK . Use the same section as for the member EI .

$$\Sigma M_C = 0; \quad 1000 \times 12 + H\text{-comp.}S_{EI} \times 5 - S_{GK} \times 12 = 0; \quad S_{GK} = 1360 \text{ lb.} \quad (C).$$

Moment and Shear Diagrams. These diagrams may now be drawn for the left-hand column. They are shown in Fig. 88(b) and (c). The remainder of the stresses may be obtained by any desired combination of the methods of joints, moments, and sections. For instance, the use of the section $a-a$ of Fig. 87 and the equation $\Sigma V = 0$ is a convenient way to determine the stress in the member IK . Evidently the stress is $1.41 \times 860 = 1210$ lb. (T). A casual inspection of the joint G might mislead us into thinking that the stress in the member GK is 1000 lb. (C) ($\Sigma H = 0$). This conclusion is erroneous because the equation

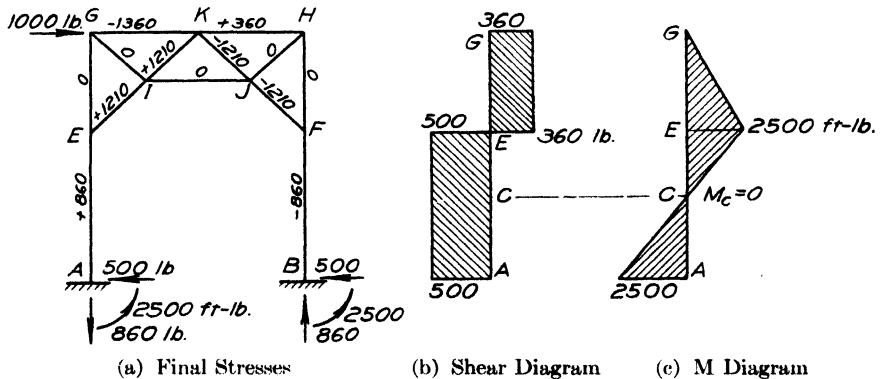


FIG. 88. STRESSES IN THE SIMPLE PORTAL OF FIG. 87.

$\Sigma H = 0$ at G [Fig. 87(c)] must include the shear at G in the post EG . The stress in the member GK then becomes $1000 + 360 = 1360$ lb. (C), which agrees with the value determined above.

Final Stresses. The computed stresses are given in Fig. 88(a). Considering the stresses in the members KH , JH , and JF as forces on the right-hand column, we may determine its shear and moment diagrams, which should be identical with those for the left-hand column. The equilibrium of the right-hand column should be checked under the action of the reactions and of the stresses in the connecting members.

111. Graphical Analysis of a Portal. In order to make an analysis by means of the stress diagram, it is necessary to *remove all bending from the columns*. This is accomplished by the addition of the substitute members (broken lines) shown in Fig. 89. The reactions of this structure will be the

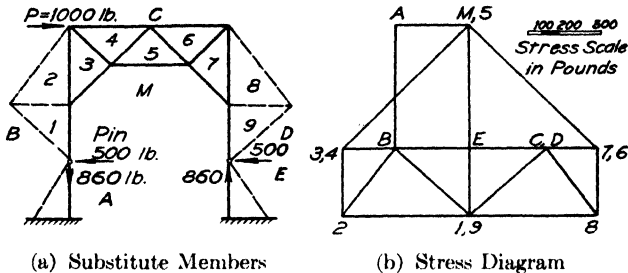


FIG. 89. GRAPHICAL ANALYSIS OF A PORTAL.

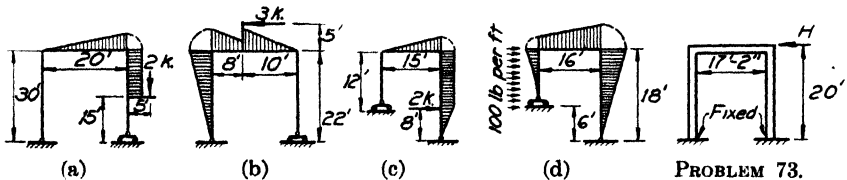
same at the pins as those in the original bent at the points of contraflexure and they are determined in the same way. The part of the structure above the pins is then isolated and a stress diagram is drawn in the usual manner. The stresses obtained are correct for all members except the columns. The direct stresses in the columns and their shear and moment diagrams can be obtained algebraically, as has been explained in the preceding section.

PROBLEMS

72. The stiff frames (a), (b), (c), and (d) have one end on rollers. These rollers and pins can resist uplift. Compute the values of critical moments and draw the shear diagrams.

73. The welded bent shown is formed of 12-in. beams with their webs in the plane of the bent. Find H to produce a fiber stress of 20,000 lb. per sq. in. in the column at the bottom of the girder. The section modulus is 100 in.³ and the area is 18.9 in.² for all members. Repeat with pin ends.

Ans. $H = 16,700$ lb. (pin ends).



PROBLEM 72.

74. This base plate is welded to the end of a 12-in., 35-lb. I-beam column. The direct load is 30,000 lb. Four $1\frac{1}{4}$ -in. anchor bolts are prestressed to 5000 lb. per sq. in. on their gross areas. (a) Compute the bending moment in each direction to produce a zero edge pressure. (b) Determine the approximate moment to stress bolts b and c to 15,000 lb. per sq. in. on the gross section. What is the corresponding column stress if $A = 10.2$ in.² and $S = 37.8$ in.³

Ans. to (b). 830,000 in.-lb. and 24,900 lb. per sq. in.

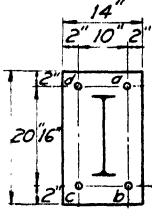
75 to 77. Analyze for the stresses in these portals produced by the loads shown on the sketches. Draw shear and moment diagrams for the columns.

Ans. The maximum stress for each portal is successively:

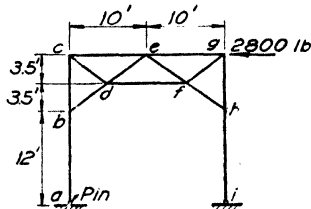
$$S_{cg} = -5200; S_{gt} = +4130; S_{ce}, S_{ch} = \pm 3460 \text{ lb.}$$

78. Analyze for the stresses in this portal and draw shear and moment diagrams for the columns. Divide the vertical shear equally between the stiff diagonals.

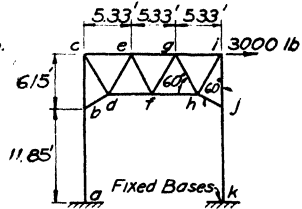
$$\text{Ans. } S_{be} = -2520 \text{ lb.}; S_{ee} = +4170 \text{ lb.}$$



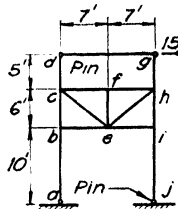
PROBLEM 74.



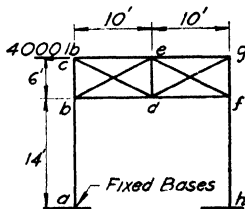
PROBLEM 75.



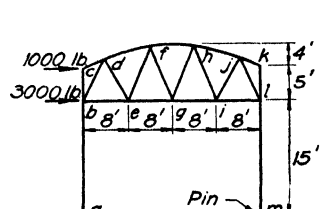
PROBLEM 76.



PROBLEM 77.



PROBLEM 78.



PROBLEM 79.

79. Analyze this portal by the graphical method and then draw shear and moment diagrams for the legs. The upper panel points lie on a parabola whose maximum rise is 4 ft. above point *k*.

$$\text{Ans. } S_{eg} = -790 \text{ lb.}; S_{gt} = -5240 \text{ lb.}$$

112. Multiple Portal with Identical Columns. The multiple portal with fixed bases has three reactions at the base of each column or twelve reactions for a structure with three bays. See Fig. 92. Since but *three* reactions can be determined by statics, the *other nine* must be eliminated by assumptions as to the action of the structure. When the columns are alike, it is usually assumed that their shears are equal. Hence four of the reactions are determined from the single equation $\Sigma H = 0$. The second assumption is that a point of contraflexure exists at the mid-height of each column from the base to the foot of the kneebrace. The same rule governs the locations of the points of contraflexure in single and multiple portals. This assumption makes possible the calculation of the moment reactions at the bases of the columns.

The Vertical Reactions. The two equations of statics, $\Sigma M = 0$ and $\Sigma V = 0$, are available for the calculation of the four vertical reactions. The usual assumption to make the calculation of the vertical reactions possible is that

the unit direct column stress varies in proportion to the distance of the column out from the centroid of the column areas. This is the same basic assumption made in the derivation of the flexure formula, $f = My/I$. This formula can be adapted to the present case by noting that M is the overturning moment about a horizontal line drawn *through* the points of contraflexure, y is the horizontal distance out from the centroid of the column areas to any column, and I is ΣAy^2 , where A is the cross-sectional area of a column.* Of course, f is the unit direct stress in the column under consideration. The vertical reaction at this column will be $f \times A$, the product of the unit stress and the column area.

113. Unlike Columns. The method outlined above for the calculation of the vertical reactions can also be applied to a portal with unlike columns.

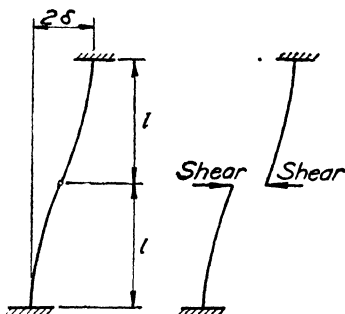


FIG. 90. COLUMN DEFORMATION.

The division of shear to columns of different cross-sections needs consideration. The typical deflection curve of a column is the reversed curve shown in Fig. 90. This is clearly identical with the combined curve for two cantilevers of the same length, one extending upward from the base and the other extending downward from the top. It is convenient to consider these cantilevers as loaded at their ends by concentrated loads equal to the column shears, which actually gives the correct moment diagram for the column. The end deflection of such a cantilever is $\delta = P^3/(3EI)$,

or the shear P is $3EI\delta/l^3$.† But the values of E , δ , and l are the same for all the columns. Hence the total shear is to be divided between the columns in proportion to the moments of inertia of their cross-sectional areas.

Evaluation of Assumptions. We should pause to reconsider all of the assumptions that are tied together into this approximate method of analysis of the multiple portal.

- (1) Points of contraflexure are located by judgment.
- (2) Shear division is based upon the assumption of identical deflection curves for all columns.
- (3) Vertical reactions are computed on the assumption that the upper truss is stiff enough to produce a planer distribution of vertical stress.

A precise mathematical analysis based upon assumptions gives results that are *no more accurate than the assumptions themselves*. If we multiply a number

* We may prefer to equate *overturning* and *resisting* moments. $M = Ph = 2(f_1A_1y_1 + f_2A_2y_2)$; also, $f_1/y_1 = f_2/y_2$. Solve simultaneously for f_1 and f_2 ; then $V_1 = f_1A_1$, $V_2 = f_2A_2$. (Fig. 92.)

† I/l commonly defines *stiffness*, but I/l^3 defines *resistance to side lurch*.

which is 10% in error by another number which is correct to five decimal places, the result is still 10% in error. Hence, if we wish to improve upon the accuracy of our analysis, we should study the structure to determine how its physical makeup is going to affect the fundamental assumptions of analysis, and, if possible, we should improve upon these assumptions. For instance, in a three-bay portal the two outside columns might have fixed bases and the two inside columns might be almost entirely unrestrained. The reactions and stresses would be changed greatly because of the *unlike deflection curves* of the columns. As will be shown in the following section the column deflection curves (controlled by their moment diagrams for fixed bases) are not necessarily identical even for a symmetrical bent with equal bays and identical columns.

114. Exact and Approximate Reactions for a Continuous Bent. The moment diagram of Fig. 91 was determined by a method of analysis of statically indeterminate structures. Column bases are fixed and moments of inertia and lengths of all members are identical. Note that the shear division is not 20% to each column but varies from 17.5% to 21.9%. Points of

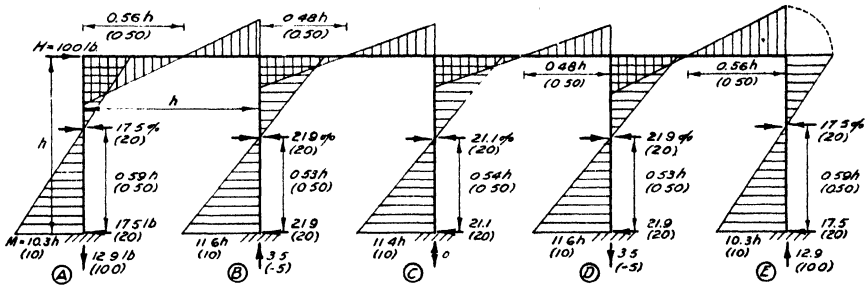


FIG. 91. COMPARISON OF REACTIONS OBTAINED BY ASSUMPTIONS AND BY ELASTIC ANALYSIS.

Reaction values in parentheses are for the usual assumptions of equal division of H to the columns and for a straight-line variation of V . Note reversal of signs for these approximate V -reactions at B and D from the true directions which are shown by the arrows.

contraflexure are not at the midpoints of the members, but their positions vary from 52% to 59% of the lengths. Vertical reactions do not agree with the assumption of planar distribution which would give us a vertical reaction of $100 \times 0.5h \times (2h)/10h^2 = 10$ lb. for the outside columns instead of 12.9 lb. Note that even the sign of the small reaction for the first interior column does not agree with planar distribution but is actually reversed in direction. Fortunately, this is not a very important stress. Nevertheless, an assumption in this case that only the exterior columns carry vertical stress would be more nearly correct since it would give us an exterior reaction of $100 (0.5h)/4h = 12.5$ lb. It is evident, however, that no fixed set of assumptions can be

expected to give exactly correct results for different numbers of bays, variations in the widths of bays, and variations in the cross-sections of the members.

Percentage of Error. When assumptions are adopted it should be with an understanding of the maximum probable percentage error in controlling stresses that may be produced thereby. Note, however, that the elastic analysis also involves an assumption—that of perfect fixation of the column bases. With some distortion of a base anchorage the point of contraflexure in the corresponding column will move downward. Hence, analysis can never give us a perfect picture of the action of the structure in the field. Nevertheless, as a crude measure of the variation between analysis based upon reasonable assumptions and elastic analysis, we observe that the greatest variation of a critical bending moment in a member of Fig. 91 is 16 per cent. Although the designer would like to know the stresses in the structure with less variation than 15–20%, he recognizes that such knowledge is often unobtainable.

Further Studies. It is not possible to go further into the effects of these assumptions here, but when the analysis of the stresses in tall building frames is investigated, in §§ 288 to 294, the errors introduced by the common assumptions will be evaluated with greater care. Then we will understand why the assumption of linearity of column stresses, for example, might be reasonable for three or four bays but not for six or eight bays (§ 293).

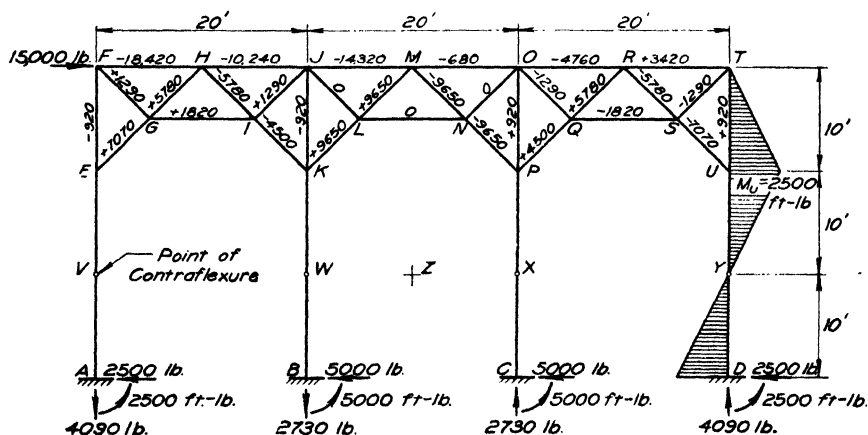


FIG. 92. STRESSES IN A MULTIPLE PORTAL.

115. Example of the Analysis of a Multiple Portal. The multiple portal shown in Fig. 92 will be analyzed for the stresses caused by the horizontal force of 15,000 lb. at the top.

ASSUMED CONDITIONS.

- (1) Bases are anchored.
- (2) Points of contraflexure in all columns are at the mid-height from base to foot of knee brace.
- (3) Cross-sectional areas and moments of inertia of the interior columns are twice these values for the exterior columns.
- (4) For convenience the area of an exterior column will be taken as unity.
- (5) The upper truss will be designed stiff enough to produce a straight-line variation of direct column stress.

COLUMN SHEARS. The interior columns are assumed to resist twice as much shear as the exterior columns because of their relative moments of inertia.

$$\text{Shear in one exterior column} = 15,000 \div 6 = 2500 \text{ lb.}$$

$$\text{Shear in one interior column} = 2500 \times 2 = 5000 \text{ lb.}$$

DIRECT COLUMN STRESSES. The formula, $f = My/I$, is applied to the structure about a center of moments at Z .

$$M_Z = 15,000 \times 20 = 300,000 \text{ ft-lb.}$$

$$I = \Sigma Ay^2 = 1(30^2 + 30^2) + 2(10^2 + 10^2) = 2200.$$

$$\text{Stress in the column } A = \frac{300,000 \times 30}{2200} = 4090 \text{ lb. per unit of area.}$$

$$\text{Vertical reaction at } A = 4090 \times 1 = 4090 \text{ lb.}$$

$$\text{Stress in the column } B = \frac{300,000 \times 10}{2200} = 1365 \text{ lb. per unit of area.}$$

$$\text{Vertical reaction at } B = 1365 \times 2 = 2730 \text{ lb.}$$

$$\text{Vertical reactions at } C \text{ and } D \text{ are } 2730 \text{ and } 4090 \text{ lb. respectively.}$$

$$\begin{aligned} \text{Check on vertical reactions: } \Sigma M_Z &= 15,000 \times 20 - (4090 \times 30)2 \\ &\quad - (2730 \times 10)2 = 0; \quad 0 = 0. \end{aligned}$$

Column Moments. The moment diagram is shown for the column D . The diagram for the column A is identical with that for the column D , while the columns B and C must resist moments twice as large. Equal moments in each column are found at the base and at the foot of the knee brace. Each maximum moment is equal to the column shear times the lever arm of 10 ft.

Stresses in Truss Members. First, the top half of the column A is isolated, and the stresses in EG , FG , and FH are calculated from the equations $\Sigma M_F = 0$, $\Sigma V = 0$, and $\Sigma M_V = 0$. All other stresses in the first bay can then be found by the methods of joints and sections, as in a simple portal. Then the top half of the column B is isolated. The known stresses in IK , IJ , and HJ may be looked upon as external forces. The stresses in KL , JL , and JM are obtained from the equations $\Sigma M_J = 0$, $\Sigma V = 0$, and $\Sigma M_W = 0$. This process is repeated from bay to bay until the entire structure has been analyzed. A check may be obtained by isolating the top half of the column D . Take the calculated stresses in RT , ST , and SU as the external loads which must be in equilibrium with the column shear and the column direct stress at the point of contraflexure.

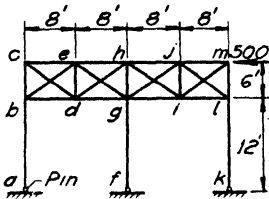
PROBLEMS

80. Analyze for the stresses and draw the shear and moment diagrams for this double portal. Divide the vertical shear equally between the stiff diagonals. The three columns are identical.

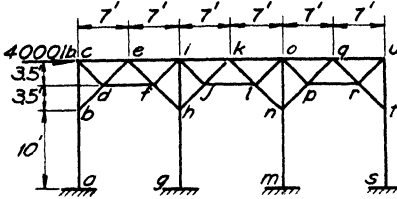
$$\text{Ans. } S_{bc} = -2340; \quad S_{ed} = 0; \quad S_{jm} = -6460 \text{ lb.}$$

81. Analyze for the stresses in this multiple portal. The columns are all identical. The bases are fixed.

Ans. $S_{np} = +490$; $S_{tl} = 0$; $S_{ce} = -5400$ lb.



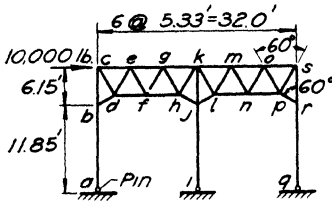
PROBLEM 80.



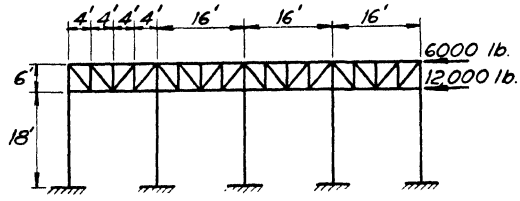
PROBLEM 81.

82. Compute the stresses and draw the shear and moment diagrams for this double portal. An exterior column has three times the moment of inertia but the same area as the interior column.

Ans. $S_{ce} = -19.2$; $S_{np} = -10.2$; $S_{fu} = +6.5$ kips.



PROBLEM 82.



PROBLEM 83

83. Compute the stresses for this multiple portal. Bases are fixed. The area and moment of inertia of an exterior column are three-quarters of the corresponding values for an interior column.

Ans. Maximum stress = $-11,300$ lb.

STRESS ANALYSIS OF TRANSVERSE BENTS

116. **The Transverse Bent.** The transverse bent presents a different problem from the portal in two particulars. First, the transverse bent carries the vertical roof loads, and, second, the wind load is not concentrated at a single point, but is transferred to the panel points of the truss by the purlins and to the windward column by the girts, as shown in Fig. 93(a). In some roof trusses it is necessary to place purlins between the panel points of the truss. This arrangement produces bending in the upper-chord member of the roof truss.

Action of Dead Load and Snow Load. It is common practice to assume that the stresses in the truss from dead load and snow load may be calculated just as though the truss were *simply supported* on masonry walls. In other words, the stress in the knee brace is taken as zero, and the column is considered to have direct stress only. This method will be followed here.

Action of Wind Load. The combined action of the truss and columns is to be considered in the calculation of the wind-load stresses. The wind loads actually come to the transverse bent through the purlins and girts, as shown in Fig. 93(a). Temporarily, however, in order to make the method of analysis clear, the wind load on the side of the building will be considered to be concentrated at the top and bottom of the column, as shown in Fig. 93(b). Concentrations come from *tributary areas*. See explanation below Fig. 93.

117. Reactions of the Transverse Bent. Exactly as in the analysis of portals, the locations of the points of contraflexure in the columns will be determined from a consideration of top and bottom restraints. The point of

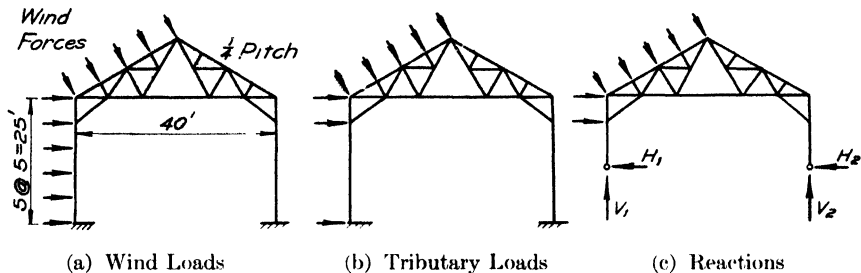


FIG. 93. REACTIONS FOR THE TRANSVERSE BENT.

Area tributary to column base is $10d$ where d is the distance between bents and the half height from the base to the foot of the knee brace is 10 ft. Areas tributary to the top of the column and to the foot of the knee brace are $2.5d$ and $12.5d$ respectively.

contraflexure may be taken as one-half of the distance from the base up to the foot of the knee brace when the column anchorage is designed to resist the greatest possible moment at the base of the column. Pin-end conditions are assumed if the degree of restraint is unknown.

Column Shears. The total horizontal component of the resultant of the wind forces that are concentrated above the points of contraflexure, as seen in Fig. 93(b) and (c), will be assumed to produce *equal shears at the points of contraflexure*. This is not the assumption that has been most used in the past, but it will be found more satisfactory.* The usual assumption has been that the horizontal reactions at the bases of the columns are equal, but this assumption is seriously in error.

Vertical Reactions. The structure above the assumed points of contraflexure will be isolated, as in (c). The vertical reaction V_1 can be found from the equation $\Sigma M = 0$ referred to a center of moments at the point of contra-

* A comparison of the reactions for the bent of Fig. 93 as obtained by the author's method and from a model test by Professor J. R. Griffith is significant. The wind pressure is taken as 30 lb. per sq. ft. and the spacing of bents is 15 ft. By model analysis, $H_1 = 9733$ lb., $H_2 = 5015$ lb. By the method of §117, $H_1 = 9630$ lb., $H_2 = 5120$ lb.

flexure in the right-hand leg, and V_2 can then be found from the equation $\Sigma V=0$ applied to the isolated part of the structure above the points of contraflexure.

118. Stresses in the Transverse Bent. From here on, the procedure may be made identical with the analysis of the single-bay portal. The left-hand column above the point of contraflexure is isolated, as shown in Fig. 94. The stress S_1 can be obtained from the equation $\Sigma M_B=0$. Then the stress S_3

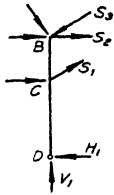


FIG. 94.
ISOLATED
COLUMN.

follows from $\Sigma V=0$. Finally, the stress S_2 can be found from the equation $\Sigma M_D=0$. Note that the moment about the point C of all the forces above C is *not* zero, but is the *bending moment* in the column at the point C. The same statement is true in regard to the moment of the forces below the point C. In order to set the moment about the point C equal to zero, we must consider all forces shown in Fig. 94 in the equation of moments. This equation is not needed.

Use of the Stress Diagram. It has been most convenient up to this point to use algebraic analysis. Since the truss members are found to have nearly as many different slopes as there are members, it may save time to complete the analysis by a graphical construction. A stress diagram can readily be drawn for the remainder of the truss by starting with the known stresses S_1 , S_2 , and S_3 as external forces.

CHECK. The stresses for those members that frame into the right-hand column are obtained from the completed stress diagram. At the same time, it is wise to isolate the column above the point of contraflexure and to check the forces acting on this part of the column for equilibrium. This is a complete check upon the graphical construction.

119. Shear and Moment Diagrams. The shear diagram and the moment diagram for the leeward column are identical in appearance with those for a simple portal. Equal maximum moments occur at the base and at the foot of the knee brace. The value of the maximum moment is equal to the shear in the leeward column times one-half the height from the base to the foot of the knee brace. Fixed bases are assumed.

Windward Column. In drawing the shear diagram and the moment diagram for the windward column, we must remember that the true loading is as shown in Fig. 93(a) and not as was assumed in Fig. 93(b). This change in loading is assumed to have no effect upon the stress in any member except the windward column. The determination of the shear diagram and the moment diagram for the windward column from the true loading and the stresses in the adjoining members completes the analysis. This procedure is illustrated by Fig. 95. The horizontal reaction H_1 can be found from the equation $\Sigma H=0$ for the forces of (a). The shear diagram is shown in (b). The moment M_1 can be found by taking moments of all forces about the base; the moment

diagram as shown in (c) can be completed by calculating the moment about each load point of the known forces above that point.

Location of Point of Contraflexure. It will be found that the point of zero moment in the windward column is considerably below its assumed location.

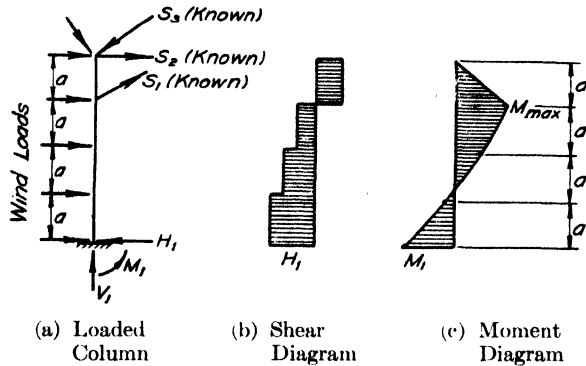


FIG. 95. SHEAR AND MOMENT DIAGRAMS FOR THE WINDWARD COLUMN OF A TRANSVERSE BENT.

which was at the mid-height. This fact will not affect the design of the structure, however, because the maximum base moment as originally calculated still exists at the base of the leeward column. The maximum moment in the windward column, found at the foot of the knee brace, has not increased appreciably from its first assumed value. All necessary information regarding column moments can therefore be obtained without actually drawing the moment diagrams. Nevertheless, it is excellent training for the student to draw the shear diagram and the moment diagram to scale.

120. Pin-End Columns. Transverse bents with pin-end columns are analyzed just as was explained above for the part of the anchored bent above the points of contraflexure. Again, the windward reaction (horizontal) should be greater than the leeward reaction by the amount of the force F_1 (Fig. 96) which is obtained from the area tributary to the base of the windward column. Figure 96 also shows how substitute members may be introduced (broken lines) just as in Fig. 89, to obtain a complete graphical analysis.

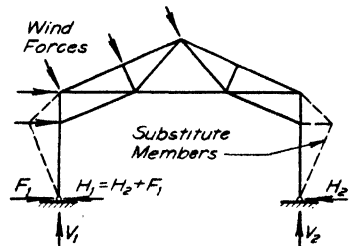
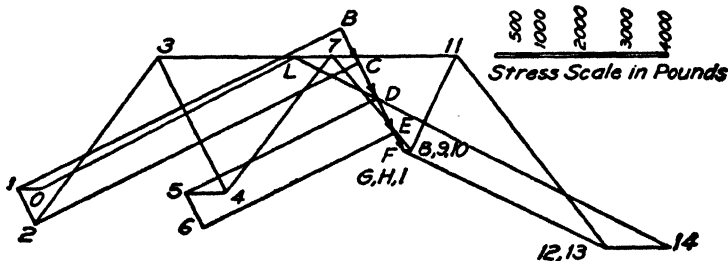
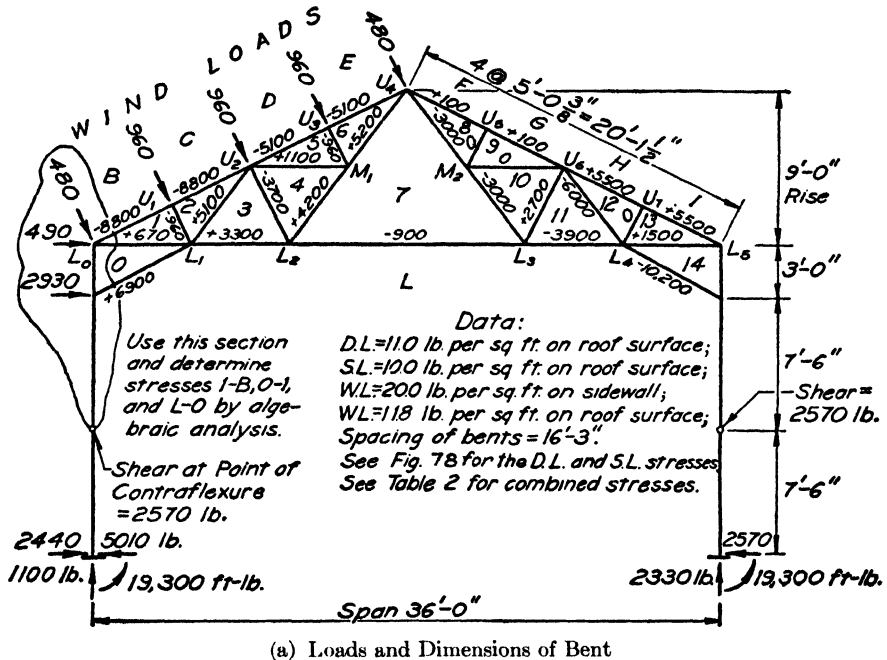


FIG. 96. REACTIONS FOR A PIN-END BENT.

121. Combined Stresses. The same procedure that was followed for the ordinary roof truss is suggested for obtaining the design stress for any member

of a transverse bent. That is, design each member for the maximum and minimum stresses caused by any of three load combinations: (1) dead load plus full wind load from either direction, (2) dead load plus full snow load,



(b) Stress Diagram for Wind
(Starting with L-0, 0-1, and 1-B Known)

FIG. 97. WIND-STRESS ANALYSIS OF A TRANSVERSE BENT.

(3) dead load plus one-half snow load plus full wind load from either direction. The proper combinations and the final design stresses are given in Table 2 for the bent analyzed in Fig. 97.

TABLE 2

STRESSES IN THE MEMBERS OF THE TRANSVERSE BENT OF FIG. 97

STRESS	DEAD LOAD	FULL SNOW	HALF SNOW	WIND FROM LEFT	WIND FROM RIGHT	COLUMNS TO BE COM- BINED FOR MAXIMUM		MAXIMUM STRESSES		STRESS CAUSED BY AN ALTER- NATE VERTICAL LOAD OF 25 LB. PER SQ. FT.	MEM- BER
						Tens.	Comp.	Tens.	Comp.		
1-B	-7050	-8420	-3210	-8800	+5500		1, 3, 4		-19,060	-16,100	U_1L_0
2-C	-6650	-6050	-3030	-8800	+5500		1, 3, 4		-18,480	-15,100	U_1U_2
5-D	-6250	-5700	-2850	-5100	+ 100		1, 3, 4		-14,200	-14,200	U_2U_3
6-E	-5850	-5320	-2660	-5100	+ 100		1, 3, 4		-13,610	-13,300	U_3U_4
13-I	-7050	-6420	-3210	+5500	-8800		1, 3, 5		-19,060	-16,100	U_1L_5
12-H	-6650	-6050	-3030	+5500	-8800		1, 3, 5		-18,480	-15,100	U_1U_5
9-G	-6250	-5700	-2850	+ 100	-5100		1, 3, 5		-14,200	-14,200	U_4U_5
8-F	-5850	-5320	-2660	+ 100	-5100		1, 3, 5		-13,610	-13,300	U_5U_4
1-O	+6300	+5730	+2860	+ 670	+1500	1 & 2		+12,030		+14,300	L_0L_1
3-L	+5400	+4920	+2460	+3300	-3900	1, 3, 4		+11,160		+12,300	L_1L_2
7-M	+3600	+3280	+1640	- 900	- 900	1 & 2		+ 6,880		+ 8,200	L_1L_3
11-L	+5400	+4920	+2460	-3900	+3300	1, 3, 5		+11,160		+12,300	L_3L_4
13-14	+6300	+5730	+2860	+1500	+ 670	1 & 2		+12,030		+14,300	L_4L_5
1-2	- 810	- 740	- 370	- 960	0		1, 3, 4		- 2140	- 1840	U_1L_1
2-3	+ 900	+ 820	+ 410	+5100	-6000	1, 3, 4	1 & 5	+ 6410	- 5100	+ 2050	U_2L_1
3-4	-1620	-1480	- 740	-3700	+2700	1 & 5	1, 3, 4	+ 1080	- 6060	- 3680	U_2L_3
4-5	+ 900	+ 820	+ 410	+1100	0	1, 3, 4		+ 2410		+ 2050	U_2M_1
5-6	- 810	- 740	- 370	- 960	0		1, 3, 4		- 2140	- 1840	U_3M_1
6-7	+2700	+2460	+1230	+5200	-3000	1, 3, 4	1 & 5	+ 9130	- 300	+ 6130	U_4M_1
4-7	+1800	+1640	+ 820	+4200	-3000	1, 3, 4	1 & 5	+ 6820	- 1200	+ 4090	M_1L_2
13-12	- 810	- 740	- 370	0	- 960		1, 3, 5		- 2140	- 1840	U_1L_4
12-11	+ 900	+ 820	+ 410	-6000	+5100	1, 3, 5	1 & 4	+ 6410	- 5100	+ 2050	U_4L_4
11-10	-1620	-1480	- 740	+2700	-3700	1 & 4	1, 3, 5	+ 1080	- 6060	- 3680	U_4L_3
10-9	+ 900	+ 820	+ 410	0	+1100	1, 3, 5		+ 2410		+ 2050	U_4M_2
9-8	- 810	- 740	- 370	0	- 960		1, 3, 5		- 2140	- 1840	U_5M_2
8-7	+2700	+2460	+1230	-3000	+5200	1, 3, 5	1 & 4	+ 9130	- 300	+ 6130	U_5M_1
10-7	+1800	+1640	+ 820	-3000	+4200	1, 3, 5	1 & 4	+ 6820	- 1200	+ 4090	M_2L_2

Equivalent Vertical Loading. The method of the equivalent vertical load is satisfactory for the analysis of *wall-bearing trusses*, but it seems entirely misused in the analysis of a transverse bent because it does not account for the important stresses introduced by the knee brace.

EXAMPLE OF DEAD-LOAD STRESS IN THE KNEE BRACE. It would be of interest to know to what extent the dead-load and the snow-load stresses for this bent would be changed if the resistance of the knee braces to vertical deflection of the truss could be determined. Since this analysis would involve a study of deflections, it can not be attempted as yet. In one bent which the author analyzed by an exact method, the horizontal shear in each column (bases fixed) was found to be nearly 3% of the vertical dead-load and snow-load reaction. A column shear of 1% existed with pin-end columns. The corresponding stress in the knee brace (bases fixed) amounted to 25% of its wind stress. The possibility of a heavy dead-load and snow-load stress in the knee brace always exists when the columns are reasonably stiff.

122. Monitor Bent. The bent with monitor or ventilator is shown in Fig. 98(a). The simplest method of analysis is to remove the monitor and to analyze it separately. A free-body diagram of the monitor is shown in (b). If it is assumed that only the tension diagonal is in operation, then the entire

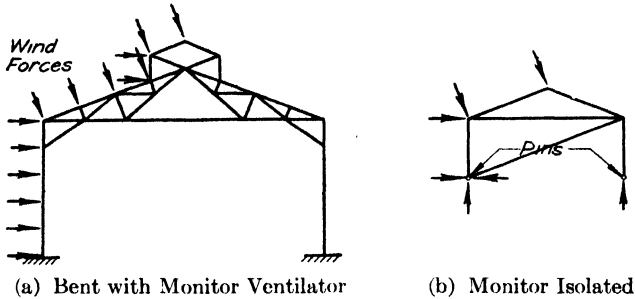


FIG. 98. SPECIAL INDUSTRIAL BUILDING BENTS.

horizontal reaction occurs at the left-hand pin. Whenever stiff diagonals are used it will be more reasonable to assume that they are stressed equally (one in tension and the other in compression), and that the horizontal component of each is one-half of the horizontal wind shear above the pins in (b).

Effect on the Roof Truss. The reactions of the monitor are reversed in direction and considered as loads upon the main truss. The analysis of the bent then proceeds in the usual manner. The reversed reactions of the monitor simply replace the wind loads on the monitor in their effect upon the main truss. Finally, we must realize that the upper-chord members in the center panels receive stress both as members of the monitor and as members of the truss proper. The final stress in such a member is found by combining the partial stresses.

123. Bent with Side Sheds. Transverse bents with side sheds are shown in Fig. 99. The bent (a) is a highly indeterminate structure that should be

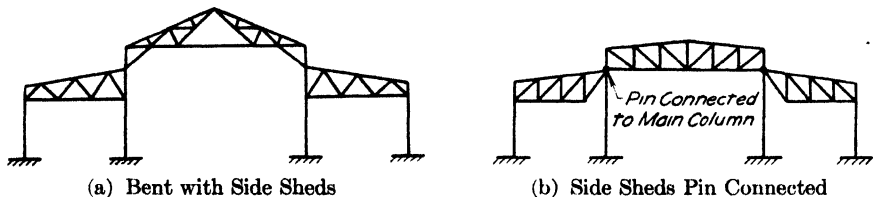
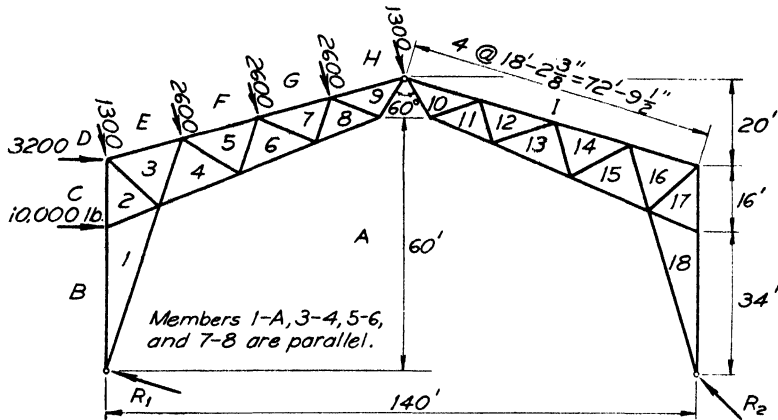


FIG. 99. COMPLEX BENTS WITH SIDE SHEDS.

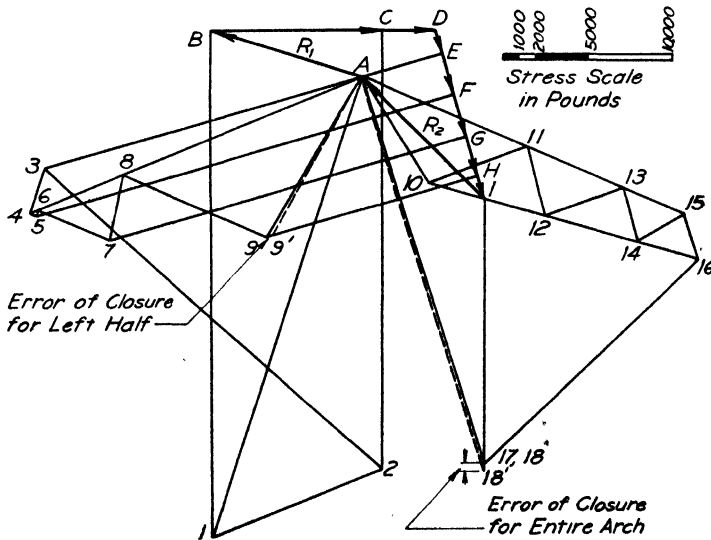
analyzed by a method of deflections. A simplified structure is shown in (b) where the side sheds are pin-connected to the center bay, the columns being continuous past the pins. In this case the stability of the structure is furnished almost entirely by the center bay acting as a framed bent. The

approximate methods discussed can be applied to (b) with reasonable confidence, but their use for the complex structure in Fig. 99(a) should be limited to a rough preliminary design.

124. Three-Hinged Bent. This structure, as shown in Fig. 100, is simply a three-hinged arch built in the form of a building bent. It is cheaper to fabricate than the polygonal arch and it is also easier to roof. Like all



(a) Loads and Dimensions of Arch



(b) Stress Diagram for Wind

FIG. 100. GRAPHICAL ANALYSIS OF WIND STRESSES FOR A THREE-HINGED BENT OF AN INDUSTRIAL BUILDING.

87. Analyze for the wind stresses in the monitor bent. Wind forces are shown on the sketch. The columns are fixed at their bases.

Ans. Stresses in the center members of the lower chord are + 1160 lb., - 2100 lb.

88. Analyze for the wind stresses in this double bent of sawtooth construction. The columns are pin-connected to their bases. All columns are alike. State your assumptions clearly.

89. Analyze for the wind stresses in this mill bent with side shed. Main columns are of the same cross-section and are pin-connected to their bases. The left-hand column has $A/3$ and $I/3$ of a main column. The wind forces are shown. Assume shear division in proportion to I/h^3 (where h is height to knee brace) and planer distribution of column unit stresses from a C.G. of the column areas.

90. Determine the maximum and minimum stresses for the three-hinged mill bent of Fig. 100 for the following data. Wind pressure, 25 lb. per sq. ft. on the vertical. Wind pressure normal to the roof to be found from A.S.C.E. specifications § 90°. Note that for this flat roof simultaneous suctions are specified for windward and leeward slopes. Use these suctions with pressure on the vertical face. Trusses spaced 24 ft. apart. Dead load, 22 lb. per sq. ft. of roof surface. Snow load, 14 lb. per sq. ft. of roof surface.

125. Modern Trends in Industrial Building Design. Flat roofed structures are preferred by modern industrial architects. With the use of light weight precast concrete slabs for sheathing, a flat roof becomes reasonably economical. For spans between columns of 50 ft. or less the girder bent is chosen. A trussed bent with flat Warren or Pratt roof truss may be used for greater spans. Welding provides economical rigid connections between roof girders and columns, thus avoiding the use of diagonal bracing where it conflicts with free passageways.

Two-Story Bents. The simple two-story bent of Fig. 101 is sometimes constructed so that the floor system is joined rigidly to the columns. The result is to produce two points of contraflexure in each column with a large reduction of wind moments. More often, however, the floor girders are merely bolted to the columns with no provision for moment resistance of the connections. The result is to make the restraining influence of the floor system upon the rest of the frame entirely negligible, and the center column becomes incapable of resisting wind shear. For such construction the wind stress analysis is made just as if the floor system did not exist. The beam reactions due to the floor loads must be considered when we calculate the stresses in the columns due to dead and live loading. These reactions fall between the limiting cases shown in Fig. 102(a) and (b). If the end connections are lightly bolted, the reactions will be those of case (a). The moment diagrams will be discussed below.

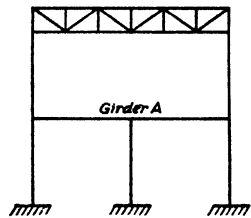


FIG. 101. TWO-STORY BENT.

Balcony Construction. The drawing (Fig. 103) of the framing of a modern industrial building shows construction of a rather high type. Again, as for

the two-story bent, the balcony may properly be neglected when wind stresses are calculated since its connections to the main columns are usually not designed to be moment resistant. Hence the wind-stress analysis of the

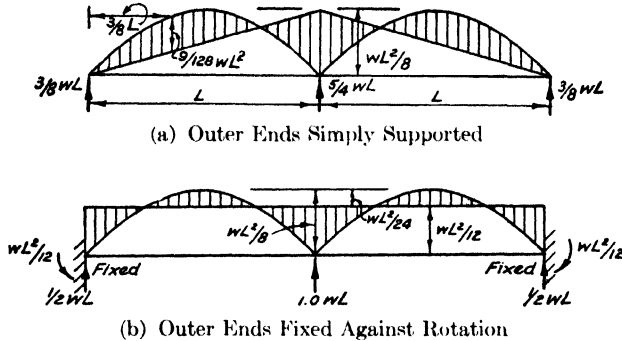


FIG. 102. M-DIAGRAMS FOR LIMITING END CONDITIONS—UNIFORM LOADING.

symmetrical two-bay bent follows from the usual assumptions whose accuracy was considered in Fig. 91.

In addition we need to consider the effect of vertical loading upon the girder bent of Fig. 103. Since concentrated loads are likely to be relatively small, if they exist at all, we will here consider only the moments produced

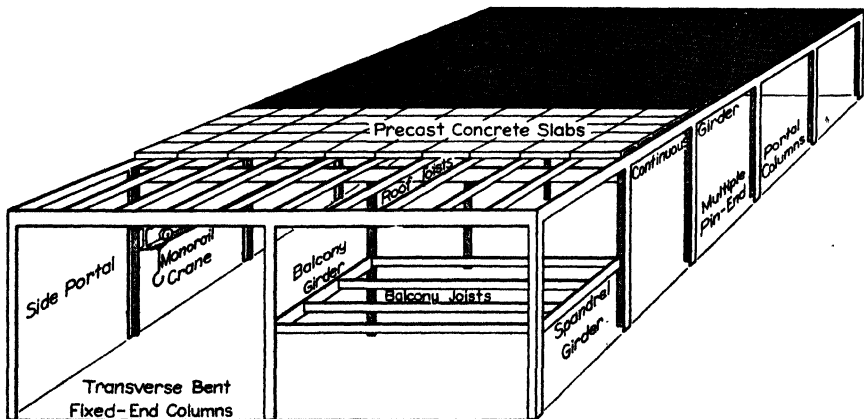


FIG. 103. HIGHEST TYPE OF MODERN INDUSTRIAL BUILDING.

by a uniform dead and snow loading. Not being as yet prepared to analyze the bent as an elastic indeterminate structure, we will merely consider extreme conditions of end restraint of the girder upon its moment diagram. (1) We

may assume that the normal end rotations of the girder are negligibly restrained by the columns. Then the moment diagram is that of Fig. 102(a). Or (2) we may assume that a light top girder is fully restrained or fixed at its outer ends by the rotational resistances of the columns. The resulting moment diagram would be Fig. 102(b). Since these are the extreme possibilities the actual moment diagram for any girder bent of two equal spans due to a full uniform loading will fall between these two diagrams. Hence the girder of such a bent can have no moment greater than $wL^2/8$ as in (a), and its maximum moment may be reduced nearly to $wL^2/12$ as in (b). A common assumption is that the maximum moment in the girder will be $wL^2/10$. The methods of indeterminate structures demonstrate that the controlling factor which we have not considered is the *ratio* of the I/L value for the girder to the comparable stiffness factor of the column. The effect of such *physical properties* of the members upon moments is a part of the theory of elasticity or of the study of statically indeterminate structures.

CHAPTER 5

TRUSS BRIDGES — DEAD LOADING

126. Parts of the Bridge. All bridges are composed of two major parts, the *superstructure* or span, and the *substructure* or foundation. This chapter will deal mainly with the steel superstructure of highway truss bridges. In most bridges, the foundation problem is the more serious from the viewpoint of the designer. In regard to costs, it is usually stated that the cost of the piers and abutments is about equal to the cost of the superstructure. However, the variation is great, and the foundation may cost from one-fourth to three-fourths of the total cost of the bridge.

The Superstructure. The four major parts of the superstructure perform separate functions. (1) The *floor system* provides a flat surface over which the traffic moves. Its structural parts must be designed to transfer its own weight

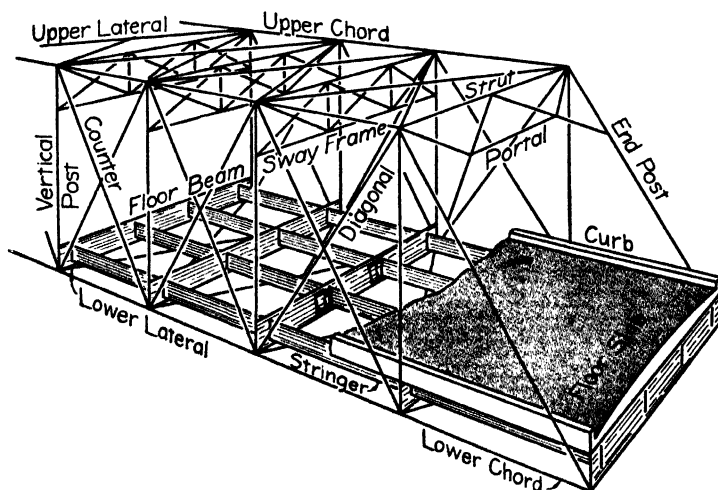


FIG. 104. THROUGH-TRUSS HIGHWAY BRIDGE.

and the wheel reactions of the moving vehicles to the panel points of the main trusses. (2) The *main trusses* support the floor system and transfer all loads to the abutments. (3) An *end shoe* spreads the reaction of one truss over the required area of masonry and provides for the longitudinal movement caused by temperature change. (4) The *lateral bracing* stiffens the structure, reduces

vibration, and holds the trusses upright, preventing their collapse under the lateral force of a side wind. The parts of a standard through-truss highway bridge are shown in the sketch, Fig. 104.

127. Low-Truss Bridges. The pony-truss or low-truss bridge is used for highway spans under 140 ft. This structure has shallow trusses that are not braced laterally in the plane of the upper chords. The trusses are stiffened laterally by connecting the verticals rigidly to the floor beams. See Fig. 106 and Fig. 107. Accordingly, verticals or posts must be placed at all panel points where floor beams occur.

Standard Types. The most common type of low-truss bridge is built with Warren trusses as shown in Fig. 108(a) and (b). The Pratt type shown in (c) is unusual in modern construction. The Warren truss of (a) and the Pratt truss of (c) are built with floor beams placed at the panel points and spaced from 12 to 16 ft. apart. Since a concrete floor slab to span 12 ft. would have to be from 10 to 14 in. thick, it is necessary to use stringers be-



FIG. 105. EARLY PIN-CONNECTED PONY TRUSS (1879).

Notice the very light diagonals and vertical posts. The pins along the upper chord are plainly visible.

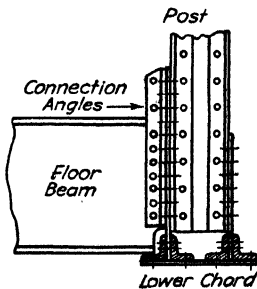


FIG. 106. FLOOR-BEAM CONNECTION.

The connection angles are shown extending above the top of the floor beam. This connection is useful for increasing the lateral stiffness of low-truss bridges. The connection could be welded.

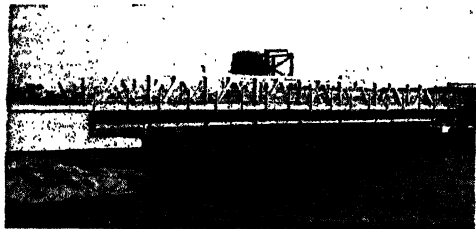


FIG. 107. WELDED LOW-TRUSS BRIDGE.

This was the first welded swing bridge built in America (1935). It is also one of our first all-welded highway bridges. Welding had progressed much more rapidly in Europe before 1940. The first all-welded railway truss bridge was constructed at Chicopee Falls, Mass., in 1928. A few cracked welds in this truss delayed the development of welded bridges.

tween the floor beams. The stringers are spaced from 2 to 5 ft. apart and a concrete slab from 5 to 7 in. thick is required. The Warren truss of Fig. 108 (b) has additional verticals for the purpose of subdividing the panels. Hence the shortened panels will be from 6 to 9 ft. long, so that a concrete slab from 7 to 10 in.

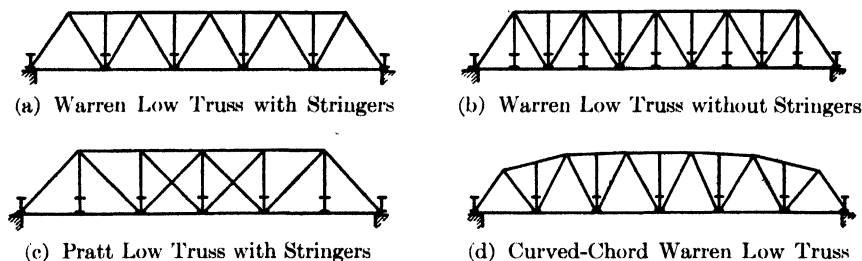


FIG. 108. LOW TRUSSES FOR HIGHWAY BRIDGES.

deep can be used without stringers. The costs of bridges with and without stringers are not greatly different for a given span. At equal costs, there is reason to prefer the stringerless bridge because *the thick floor slab adds to the rigidity and stability of the structure.*

The Warren low truss with curved chord, shown in Fig. 108(d), is used for spans of 90 ft. or more. The truss with curved upper chord has a better appearance for the longer spans. It may be found to be slightly more economical than the truss with parallel chords. Note that the "curved chord" is really a polygon.

128. High-Truss Highway Bridges. The economical depth of the truss exceeds 12 ft. for spans above 120 ft. A bridge of this height requires lateral

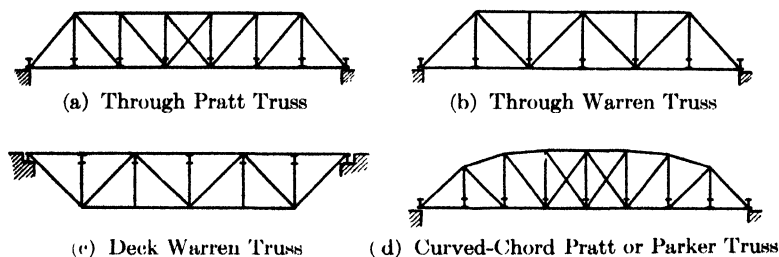
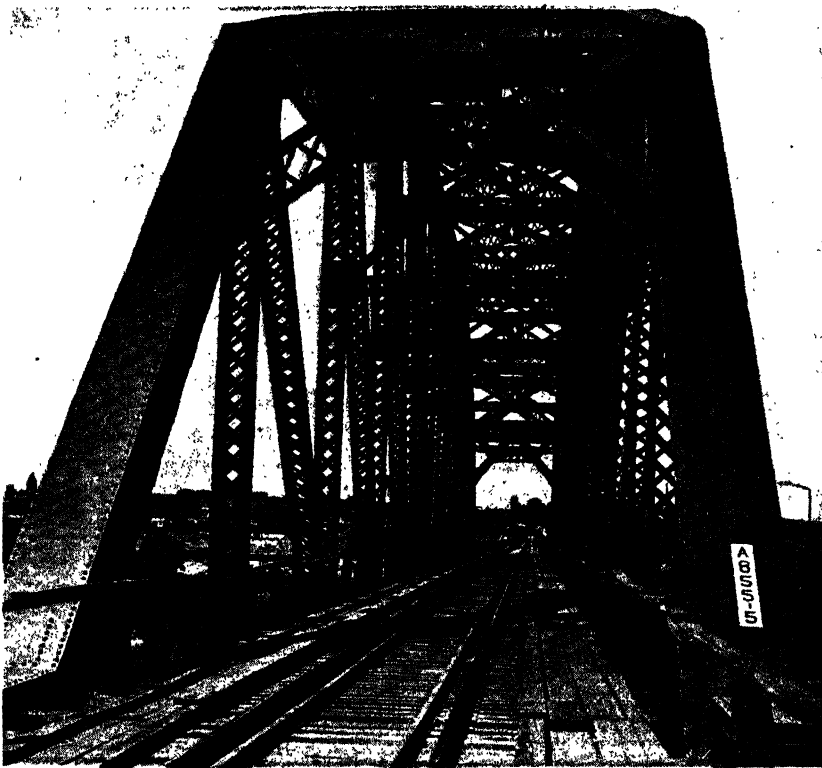


FIG. 109. DEEP TRUSSES FOR HIGHWAY BRIDGES.

bracing in the plane of the upper chords, with sway bracing or sway frames between opposite pairs of posts. Since the required clearance over the roadway is about 14 ft., the depth of a high-truss bridge with floor carried at the joints of the lower chord cannot be much under 20 ft. This estimate allows

3 ft. for the depth of the sway bracing over head and 3 ft. for the total depth of the floor beam and road slab. Stringers are framed between the floor beams so that they do not add to the depth of the floor. Occasionally the floor beams have been supported below the bottom chord, which reduces the required height of the trusses, but this construction is now seldom used.

Standard Types. The parallel-chord Pratt truss shown in Fig. 109(a) is by far the most common type of high bridge truss for through spans of moderate length. Its economical span ranges from 100 to 150 ft., although it is commonly used up to spans of 160 and even 170 ft. The Warren high truss



Courtesy Illinois Central R.R.

FIG. 110. PORTAL AND SWAY BRACING OF A RAILWAY TRUSS BRIDGE.

shown in (b) is seldom used, but this type is much used as a deck structure, Fig. 109(c). The curved-chord Pratt truss of (d) is ordinarily found economical for spans above 160 ft. This truss is also known as the Parker or Camel-back truss. The truss of Fig. 110 is of curved-chord Pratt type.

RAILWAY TRUSS BRIDGES

129. Structural Types. Steel railway bridges may be constructed as plate girders or trusses of either through or deck design. The analysis of the plate girder will be discussed separately in Chapter 9. Most of the parts of a railway bridge are essentially the same as the corresponding parts of a highway bridge. Trusses, lateral systems, and end supports may be identical in appearance in the two structures. A distinction exists in the use of pin-connected trusses, which are more satisfactory for a heavy railway bridge than for the lighter highway bridge although they are no longer in common use. Pony-truss bridges, which are used as highway structures, do not have sufficient lateral rigidity for railway service; plate girders are used instead.

The Pratt truss with horizontal chords is much used for railway-bridge spans up to 160 ft. The Parker or curved-chord Pratt truss (Fig. 110) is economical for spans from 160 to 250 ft. or even more. Above 300 ft., either a K-bar truss (Fig. 119) or a Pettit truss with subdivided panels (Fig. 153(b)) will be found desirable. The Baltimore truss (subdivided panels and horizontal chords, Fig. 153(a)) may be looked upon as intermediate between the Parker and the Pettit trusses. It is not widely used for there is an obvious economy in sloping the upper chord for long-span trusses. The excess cost of a Pratt truss with parallel chords over a curved chord truss for a span of say 200 ft. could not be very great; hence the limits given above are merely guideposts.

130. The Floor System. The greatest difference between the railway bridge and the highway bridge is found in the floor system. For single-track through bridges, there are ordinarily only two stringers, which are supported by floor beams at the panel points. The stringers are spaced about $6\frac{1}{2}$ ft. apart. They support 10-ft. ties which in turn carry the rails. Double stringers under each rail, or four in all, may be used to maintain a shallow depth. Deck girders may be placed 7 ft. apart for the support of 10-ft. ties. Deck trusses are spaced 10 ft. apart to support 14-ft. ties directly upon their top chords. Sections illustrating such open-floor construction are shown in Fig. 111.

Ballasted Floors. Floors with ballast are used for important railway structures. Either a wood deck, a steel-plate floor, or a concrete floor slab may be used to retain the ballast. Wood-deck construction is shown in Fig. 111(d). For through girder bridges, small I-beams may be placed close together as floor beams and either filled with concrete or covered by a steel-plate floor. A riveted trough floor has also been used. Sections through such floors are shown in Fig. 112. Through or deck truss bridges may have several stringers which support a concrete slab or a steel-plate floor. The stringers must be supported at panel points by transverse floor beams.

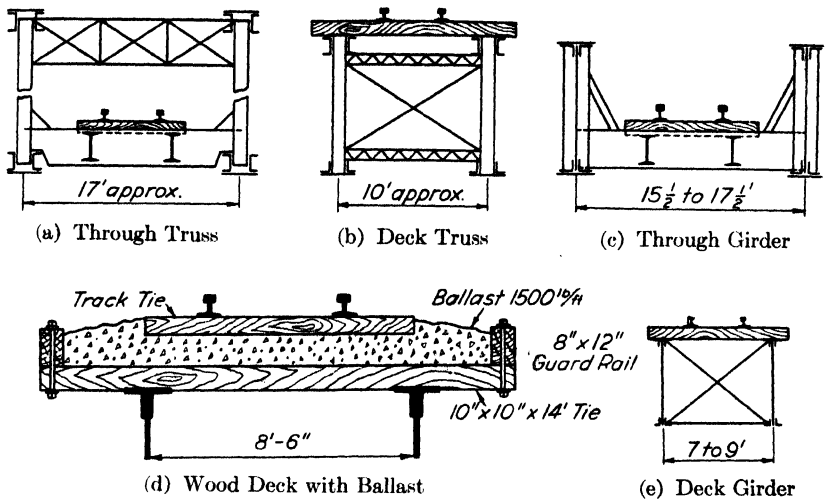


FIG. 111. CROSS-SECTIONS THROUGH RAILWAY BRIDGES.

131. Details of Bridge Trusses. The members of modern bridge trusses in most cases are connected by *riveted joints*. Only long-span structures can have *pin-connected joints*, as shown in Fig. 113(a), because a heavy dead load is needed to prevent undue vibration of a structure with pin joints. Actually, only exceptional structures will be pin connected in the future because rolled eye-bars are no longer readily obtainable in stock sizes. Welding has not as yet been widely adopted for the fabrication of bridge trusses. Only a few bridge trusses have been built with *welded joints*. Further information

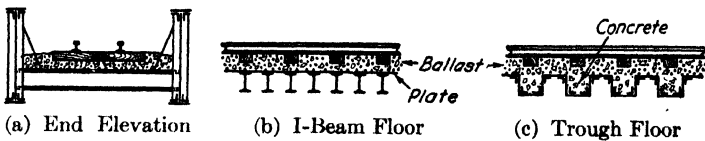


FIG. 112. BALLAST FLOORS FOR THROUGH GIRDERS.

regarding the stress distribution in welded joints will have to be made available before welded fabrication of bridge trusses can become common practice. A welded truss joint is shown in (b) of Fig. 113.

End Supports. The ordinary bridge truss has a pin-connected shoe at each end of the truss over the abutment. A *pin* is required here because the end of the truss rotates through a slight angle when the structure deflects downward under the effect of the loads. If the *shoe* or supporting block is riveted to the end of the truss, the entire weight will ride upon the inside edge of the shoe. See Fig. 113(c). This would be doubly objectionable, for

it *might* produce crushing of the masonry and certainly it would prevent the reaction from being concurrent with the stresses in the bottom chord and end

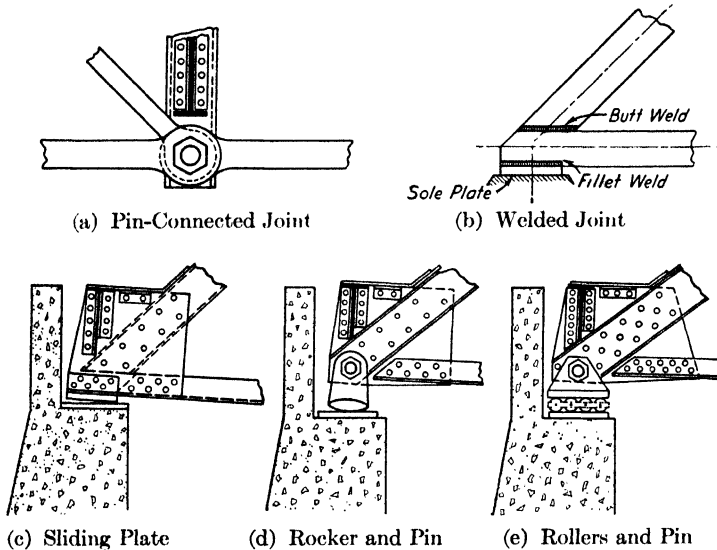


FIG. 113. TRUSS DETAILS.

post. The result would be to introduce an objectionable bending moment into the truss members at the end joint. The ordinary construction where a pin is used in connection with a *rocker* to take care of expansion is illustrated by Fig.

113(d). A *roller nest* as shown in (e) is required for heavy trusses. A pin, supported by a *fixed pedestal*, is used at the other end of the truss.

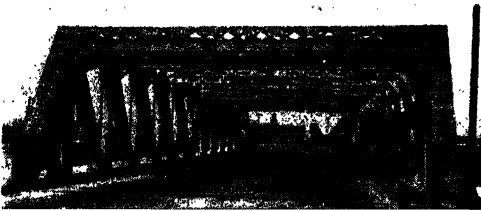


FIG. 114. PRATT TRUSSES WITH COLLISION STRUTS.

The collision strut can be seen in the first panel of the left-hand truss. It is attached at the mid-point of the end post. Its action was found to overstress the end post in flexure. It is no longer used. Notice the shallow end portal.

lution of snow would practically stop traffic. The dead load must be estimated. It will include the weight of the floor system including the floor slab, the floor

THE DEAD LOADING OF BRIDGE TRUSSES

132. Loads on Bridges. The main loads to be carried by the structure consist of dead load, live load, impact, and wind forces. Snow load is not considered important because a heavy accumulation of snow would practically stop traffic. The dead load must be estimated.

beams and the stringers; the weight of the main trusses with the laterals and the sway bracing; and a small allowance for a handrail. The live load may be specified as a line of wheel loads representing several trucks or a locomotive, or it may be specified as an *equivalent uniform load*. Impact is made dependent upon the loaded length of span and is usually fixed by specification. The wind force is determined from the side area of the structure as seen in elevation, but a minimum lateral force per lineal foot is fixed by specification to assure the design of a *reasonably stiff structure*.

133. Dead Load of the Floor of a Highway Bridge. A major part of the dead weight of a standard highway bridge comes from the reinforced concrete floor slab. The thickness of the slab can be estimated as approximately one inch for each foot that it spans between stringers or floor beams. However, a slab thinner than 5 in. should not be used, and a 6-in. overall depth is a more common minimum. The thicker slabs seldom are covered, but slabs of minimum depth ordinarily carry a bituminous wearing surface from $\frac{1}{2}$ in. to 3 in. in thickness. A thin bituminous covering tends to peel off and is not considered desirable. A minimum thickness of 1 in. usually is recommended.

Weight of Steel in the Floor System.

When the stringer spacing has been determined by the methods that will be discussed the design of the stringers and floor beams can be completed and their exact weights added to the weight of the floor slab. This information makes it possible to determine the actual loads brought to the truss joints by the dead weight of the floor system.* The weight of stringers, or of floor beams if stringers are not used, will be from 10 to 12 lb. per sq. ft. of floor surface for *medium loads*. The use of stringers reduces the total weight of floor beams about 40 per cent. Hence the corresponding weight of steel in the floor system of a bridge with stringers would be from 16 to 20 lb. per sq. ft. of roadway.

134. Dead Weight of Highway Bridge Trusses. The weights of steel trusses vary considerably, according to the specifications under which the design is made. The following formulas are for modern structures of medium

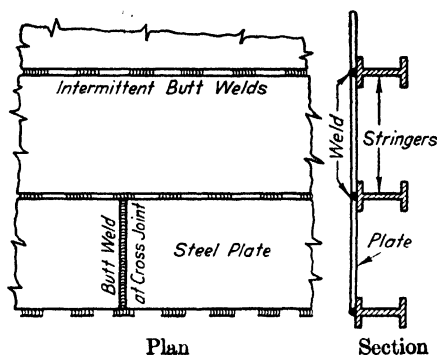


FIG. 115. BATTLEDECK FLOOR CONSTRUCTION.

This welded steel-plate floor covered with 2 in. of bituminous topping has been used as a bridge floor. Plates are $\frac{3}{8}$ in. or $\frac{1}{2}$ in. and stringers are 4 to 6 ft. apart. The stringer and plate acts as a T-section.

* See *Economics of highway bridge floorings of various unit weights*, Transactions, A.S.C.E., Vol. 103, pp. 515-550.

capacity designed to carry two 15-ton trucks (*H*-15 loading). The weights should be increased 20% for bridges designed for 20-ton trucks (*H*-20 loading). See § 146 for an explanation of the *H*-loadings.

STEEL LOW-TRUSS HIGHWAY SPANS WITH STRINGERS. For a 20-ft. roadway, the approximate weight (*w*) of total structural steel per foot of bridge, not including steel handrail or stringers, is

$$(1) \qquad w = 250 + 4.5 L,$$

where the length of truss in feet is *L*, which is limited to 100 ft. (*H*-15 loading; see Fig. 126).

STEEL LOW-TRUSS HIGHWAY SPANS WITHOUT STRINGERS. For a 20-ft. roadway, the approximate weight of total structural steel per foot of bridge, not including steel handrail or floor beams, is

$$(2) \qquad w = 250 + 4.5 L,$$

where *L* is limited to 100 ft. (*H*-15 loading; see Fig. 126 and §146).

STEEL HIGH-TRUSS HIGHWAY SPANS WITH STRINGERS. For a 20-ft. roadway, the approximate weight of total structural steel per foot of bridge, not including steel handrail or stringers, is

$$(3) \qquad w = 250 + 4.0 L,$$

where *L* is limited to the range from 100 to 160 ft. (*H*-15 loading; see Fig. 126 and §146).

Influence of Roadway. For other widths of roadway, multiply the value of *w* by the width and divide by 20 ft. After 1944, A.A.S.H.O. specifications set a minimum width of 26 ft. for the clearance between curbs of a two-lane highway bridge. These formulas will be satisfactory when the roadway surface consists of a concrete slab weighing not less than 75 lb. per sq. ft. For a wood-strip floor or a steel-plate floor of half this dead load, these weights may be cut 15 per cent.

Rule for Checking the Estimated Weight of Steel.* Another rule of weight estimate which is sometimes useful is as follows. The total weight of structural steel in pounds per foot of span in two trusses and bracing is approximately 35 times the greatest net area in square inches of the lower chord of one truss. The weight of steel per foot in the floor system must be added. Also, from 25 to 40 lb. per ft. should be added for the weight of the two steel handrails.

135. Weights of Railway Bridges. The weight of the structure will be divided into the weight of total structural steel and the weight of track or floor. In estimating the weight of a single track, the weight of rails, inside guard rails and fastenings may be taken at 200 lb. per lineal ft. per track. Concrete is estimated at 150 lb. per cu. ft. and ballast at 100 lb. per cu. ft. Rail sections are common up to 150 lb. per yd. although heavier sections have been rolled. The weight of ties and outside guard rails may be computed from an estimated weight of $4\frac{1}{2}$ lb. per ft., board measure. The total weight of an open floor including rails will be found to be about 500 lb. per

* See *Weights of metal in steel trusses*, Transactions, A.S.C.E., Vol. 101, pp. 1-34.

lineal ft. when 10-in. $\times$ 10-in. $\times$ 10-ft. ties are used at 14-in. centers. The maximum space allowed between ties is 6 in. *in the clear* although 4 in. is common. The guard rail may be a 6-in. $\times$ 8-in. timber, continuous for the full length of the bridge.

Formulas for Weight of Steel in Railway Trusses. The total weight of structural steel per foot of span may be estimated from the following formulas, each of which is designed to apply to a specific type of structure and for *E-60* loading. (The significance of the Cooper *E-loadings* will be discussed in Chapter 7. See Fig. 161 for an illustration of the *E-60* locomotive loading.) These weights may be reduced 15% for the *E-50* loading and increased 15% for the *E-72* loading.

DECK PLATE GIRDERS. (Single track *E-60* locomotive loading.)

- (4) $w = 12L + 400$ (Wood deck with ballast. Spans 40 to 100 ft. Reduce 100 lb. per ft. for open floor.)

THROUGH PLATE GIRDERS. (Single track *E-60* locomotive loading.)

- (5) $w = 12L + 1000$ (Two or four stringers and floor beams. Spans 40 to 100 ft.)

THROUGH TRUSSES. (Single track *E-60* locomotive loading.)

- (6) $w = 10L + 1000$ (Riveted or pin-connected trusses. Spans 125 to 350 ft. Ballast adds 10 to 15% to the weight of steel.)

DECK TRUSSES. (Single track *E-60* locomotive loading.)

- (7) $w = 10L + 1000$ (Riveted structures. Spans 100 to 175 ft. Open-floor construction.)

Other Variations. Naturally, the weights of railway bridges vary considerably, depending upon the allowance made for future increase in live load and upon the type of specifications which control the design, as well as upon the attitude of the designer. The estimated weights determined from the formulas above should be compared with the weights as given by other formulas listed in the footnote below,* as well as with the plotted weights of actual spans given in Ketchum's *Structural Engineer's Handbook*, Waddell's *Bridge Engineering*, and Spofford's *Theory of Structures*. Attention is again called to the fact that these formulas give the total weight of structural steel in pounds per foot of bridge for single-track structures (*E-60* loading). These weights must be increased at least 85% for a double-track bridge. They may be decreased 15% for *E-50* loading, and increased 15% for *E-72* loading. (See Fig. 161 for an illustration of the *E-60* locomotive loading.)

* The following formulas are for the *total weight* of structural steel per foot in single-track through trusses with open floors designed for *E-60* loading: Urquhart and O'Rourke, $w = 4(2L + 300)$; Constant, $w = 1.12(9L + 700)$; Swain, $w = (L + 45) + 0.089$; Morris, $w = 10L + 900$.

PROBLEMS

91. Explain why a formula for the dead weight of steel per foot of bridge should contain a constant term and one that varies with the span. Consider the floor system, the trusses and the bracing.

92. Produce a neat tabulated estimate of the dead weight of a 100-ft. low-truss highway bridge designed for *H-20* loading. The floor beams are placed at panel points 8 ft. apart. They weigh 125 lb. per ft. Trusses are spaced 29 ft.-6 in. apart to accommodate a 26-ft. clear roadway and two curbs 12 in. wide by 12 in. high. The concrete slab is 9 in. thick. Each handrail is composed of four $3 \times 3 \times \frac{5}{16}$ -in. angles.

93. Produce a neat tabulated estimate of the dead weight of a 180-ft. through highway bridge designed for *H-15* loading. The floor beams are spaced 20 ft. apart and weigh 135 lb. per ft. Trusses are spaced 27 ft.-8 in. apart to accommodate a 24-ft. clear roadway and 12 in. curbs. Seven lines of stringers (12-in., 32-lb. beams) support a 6-in. concrete slab topped with 2 in. of bituminous material weighing 120 lb. per cu. ft. Each handrail weighs 20 lb. per ft.

94. Plot on a single sheet all of the formulas for the dead weight of steel in highway and railway bridges as given in §§ 134 and 135, including those mentioned in the footnote. Plot lengths as abscissas and *total* weights of steel in the bridge as ordinates.

95. Produce neat tabulated estimates of the total weight of the following open-floor bridges designed for single-track *E-72* loading except as noted. Ties are spaced 4 in. in the clear.

- (a) Deck truss, 150-ft. span. Use 12-in. $\times$ 10-in. $\times$ 14-ft. ties.
- (b) Through truss, 220-ft. span. Use 10-in. $\times$ 8-in. $\times$ 10-ft. ties.
- (c) Double-track through bridge, 300-ft. span. Use 10-in. $\times$ 10-in. $\times$ 10-ft. ties.
- (d) Deck plate girder, 90-ft. span. Use 10-in. $\times$ 10-in. $\times$ 12-ft. ties.

96. Estimate the total weight of a 90-ft. through plate-girder bridge designed to carry *E-60* loading (single track with ballast floor). The depth of ballast is from 6 in. below the 8-in. $\times$ 8-in. $\times$ 10-ft. ties up to level with the top of the ties. The floor is constructed of 16-in., 50-lb. transverse beams 15 in. apart filled with concrete to a depth of 20 in. Girders are spaced 15 ft. apart.

DEAD-LOAD STRESSES IN BRIDGE TRUSSES

136. Joint Loads. The floor system of a highway or railway bridge concentrates its own weight at the panel points of the loaded chords, which are the lower chords in through bridges. The weight of the lateral bracing in the plane of the lower chord naturally comes to the lower-chord panel points, while the bracing in the plane of the upper chord acts at the panel points of the upper chord. The weight of a truss member is assumed to be divided equally between the joints at its two ends. Joint loads produced by dead load therefore exist at every panel point.

The division of the dead load as indicated above is not possible before the members are designed. Moreover, we shall find that the concrete slab, or ballast-floor, will concentrate a major part of the dead weight of through bridges at the lower panel points. For this reason, all of the dead load of through highway bridges with heavy concrete floors or of ballasted railway

bridges usually is considered to act at the panel points of the lower chords. A more accurate method would be to place all of the dead weight of the floor system at lower-chord joints and to divide the weight of structural steel in the ratio of two-thirds to the lower chord and one-third to the upper chord.

EXAMPLE. The following table of shop weights of steel for a standard 150-ft., 8-panel highway bridge designed for medium loads shows that this division is acceptable.

SUBDIVISIONS	TOTAL WEIGHT	WEIGHT TO UPPER CHORD	WEIGHT TO LOWER CHORD
Two Main Trusses	80,801 lb.	37,900 lb.	42,901 lb.
Lateral Bracing	5,022	2,000	3,022
Sway Bracing	9,548	9,548	0
Floor System	59,250	0	59,250
Handrails	5,094	0	5,094
End Supports	2,299	0	0
	162,014 lb.	49,448 lb.	110,267 lb.

The actual joint concentrations including the weight of a 6-in. floor slab applied entirely to the bottom chord are shown in Fig. 116(a). The concentrations when the total weight of steel is divided in the ratio of $\frac{1}{3}:\frac{2}{3}$ between chords are as shown in (b). Note that the loads are left off over the supports in (b) and that only the net reactions are given. Since the load over the support causes no stresses in the truss, it is omitted in the load diagram.

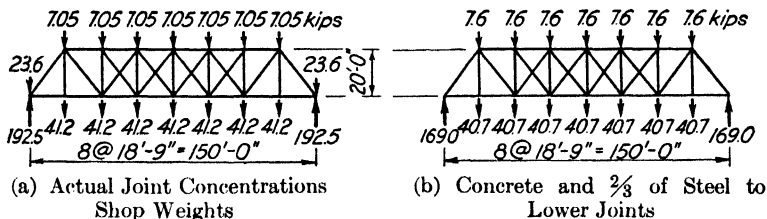


FIG. 116. COMPARISON OF ACTUAL AND CONVENTIONAL DEAD LOADING.

The gross reaction is used to control the design of the end support. The loads given in Fig. 116 are dead loads for the entire bridge; they must be divided by two to obtain the dead loads for one truss.

PROBLEMS

97. Obtain a design sheet or the detail drawings for a through truss highway bridge and take off the dead weights. Determine the actual joint concentrations and compare the division of load between chords with the division shown in Fig. 116.

98. If one knows all of the stresses in a truss due to vertical loads applied entirely to lower chord panel points, prove by cutting vertical and sloping sections that only the stresses in the vertical members are changed by moving all or any part of the loads to the upper chord panel points. Explain then a practical method of taking into account the loads at upper chord joints in Fig. 116.

137. Analysis of Trusses with Parallel Chords. *Method of Sections.* The two basic methods of truss analysis, the method of joints and the method of sections, are equally useful in solving this problem. After a little experience with either method, the reader will be able to calculate the dead-load stresses by the aid of a slide rule without even the need for scratch paper. The method of sections will be discussed first.

Web Members. The stresses in the web members are found conveniently by the method of sections. Mentally we picture vertical sections cut through the truss with the parts isolated, as shown in Fig. 117. Clearly, the vertical component of the unknown stress for each of the sections 1-1, 3-3, 4-4, and 5-5 must act downward and it must equal the summation of the vertical forces to the left of the section, or the *shear on the section* which acts upward. Hence we arrive at the common rule for the calculation of the stresses in the web members of trusses with horizontal chords:

$$(8) \quad \text{Stress in a web member} = \frac{\text{Panel shear} \times \text{Length of member}}{\text{Height of truss}}.$$

It is clear from the section 2-2, or from the joint *b*, that the member *Bb* serves simply as a hanger for the load at *b*, while the section 6-6 shows that the compression stress in the post *Dd* is equal to the load at the joint *D* of the upper chord. The stresses in Fig. 117, as elsewhere in this book, are marked with a plus sign for tension and a minus sign for compression.

Chord Members. The stresses in the chord members can be found from moment equations. The stress in each member of the upper chord is calculated from the moment about the lower-chord joint at which the diagonal or vertical cut by the section intersects the lower chord. The member *CD* of Fig. 117 will serve as an illustration. The moment of the forces to the left of the section 5-5, taken about *d*, is

$$M_d = 30(3 \times 15) - 12(2 \times 15) - 12(1 \times 15) = 15[(30 \times 3) - (12 \times 3)].$$

The stress in the chord member *CD* is this moment divided by the height of the truss, or

$$S_{CD} = \frac{15}{20} [(30 \times 3) - (12 \times 3)] = 40.5 \text{ kips compression.}$$

These calculations explain the common rule for horizontal chord stresses:

$$(9) \quad \text{Stress in a chord member} = \frac{\text{Bending moment at center of moments}}{\text{Height of truss}}.$$

The simple device used above of calculating all moments by use of *lever arms in number of panel lengths* rather than in feet will be found to save considerable time.

138. Method of Joints. Web Members. We can start at the joint *D* in Fig. 117, where the stress in *Dd* is seen to be -3 kips. At the joint *d*, the diagonal must pull upward with a vertical component equal to one-half of the total downward thrust of $9 + 3 = 12$ kips. Hence the stress in *Cd* is $(25/20) \times 6 = +7.5$ kips. Passing to the joint *C*, we notice that the equation $\Sigma V = 0$ shows that the member *Cc* has a stress equal to the vertical component of the stress in *Cd* plus the 3-kip load at *C*, or -9 kips. Hence

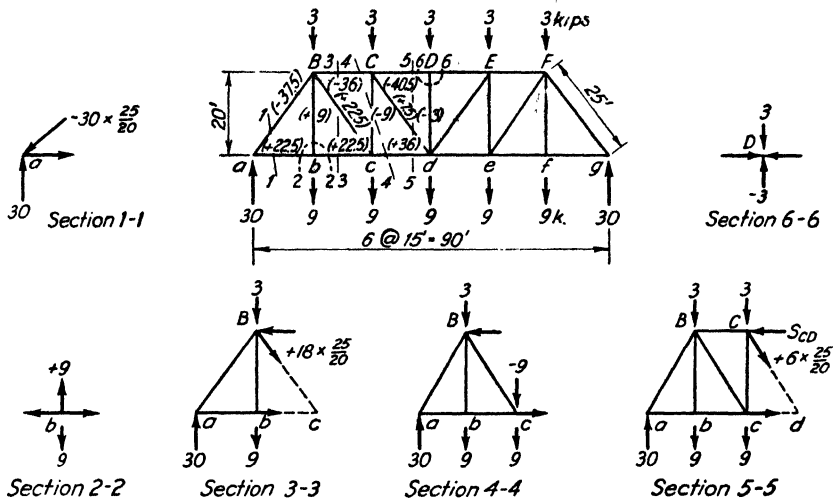


FIG. 117. DEAD-LOAD STRESSES BY THE METHOD OF SECTIONS.

the procedure for web members is simply to start at the end or center of the truss and to pass from joint to joint, calculating the web stresses by repeated use of the equation $\Sigma V = 0$.

Chord Members. We then repeat the circuit by starting at the end of the truss and apply the equation $\Sigma H = 0$ to each joint to determine the stresses in the chord members. The method of joints, when applied to a truss with parallel chords, is sometimes called *the method of chord increments* because one starts at the left end where the chord stress is low and continues to add in horizontal components of the stresses in the web members while passing from joint to joint toward the right.

139. Index Values or Stress Coefficients. An *index stress* is a vertical component of a stress rather than a stress itself. The use of index stresses is a variation of the method of joints. As explained above, we can proceed from joint to joint and calculate the vertical components of all web stresses. Only the vertical components are needed and these are written on a truss diagram as index stresses for the web members. The index stress for a chord

member is the sum of the *vertical* components of the stresses in all diagonals whose *horizontal* components add up to the stress in the chord member. If we begin at the left end of the truss in Fig. 117, the stress in the member ab may be looked upon as the horizontal component of the stress in the member aB whose vertical component is $30k$. Hence the index stress for ab is $+30$. The index stress for BC is the V component of the stress in aB plus the V component of the stress in Bc , $-30 + 18 = -48$. Index stresses for the truss of Fig. 117 are written on the truss diagram, Fig. 118(a).

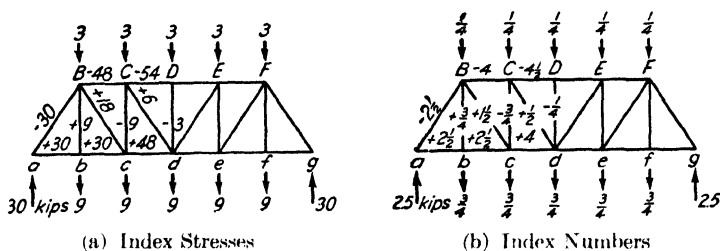


FIG. 118. INDEX VALUES FOR A PRATT TRUSS.

Automatic Procedure. The actual stress in a web member is the index stress times the length of the member divided by the height of the truss. The actual stress in a chord member is the index stress times a panel length divided by the height of the truss. These statements may be combined into the following rule:

$$(10) \quad \text{Stress in a member} = \frac{\text{Index stress} \times \text{True length of member}}{\text{Height of truss}}.$$

Index Numbers. If the index stresses are computed for total panel loads of unity, they are called index numbers, or *stress coefficients*. The only advantage gained is ease of use where panel loads may be changed to fit different widths of roadway, etc. Index numbers are handled in exactly the same way as index stresses. Index numbers for the truss of Fig. 117 are given on the truss diagram, Fig. 118(b).

PROBLEMS

99. Use the method of sections to calculate the stresses in the Warren truss caused by a dead load of 1100 lb. per ft. of truss acting at the bottom chord.

$$\text{Ans. } S_{\max.} = +94,400 \text{ lb.}$$

100. Use the method of joints to calculate the stresses in the Warren deck truss. Check three stresses near the middle by the method of sections.

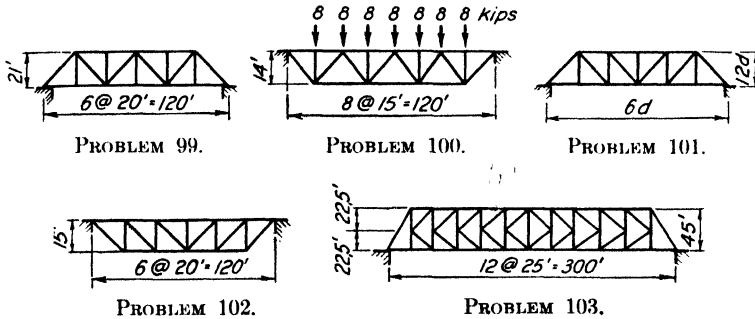
$$\text{Ans. } S_{\max.} = +68,600 \text{ lb}$$

101. Write out the index numbers for this Pratt truss for unit loads at lower-chord points. Then use these index numbers to obtain the dead-load stresses for the end

post and the center member of the bottom chord for spans of 100, 125, and 150 ft. and for loads of 1100, 1250, and 1400 lb. per ft. for each length of span. Arrange results in a stress table.

102. Write out the index stresses for this truss for a panel load of 16 kips. Check three stresses near the center of the truss by the method of sections.

Ans. $S_{\max.} = -96,000$ lb.



103. Calculate the dead-load stresses in this K-truss by any method. The dead load is 1800 lb. per ft. of truss.

SUGGESTION. Study the application of the equation $\Sigma H = 0$ to the joints at the mid-heights of the verticals.

Ans. $S_{\max.} = 438$ kips.

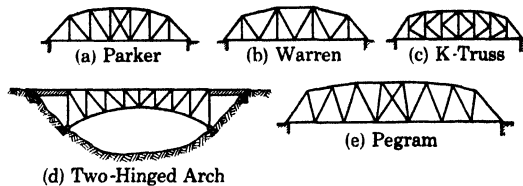


FIG. 119. CURVED-CHORD TRUSSES.

140. **Dead-Load Stresses in Simple Curved-Chord Trusses.** The most common bridge truss with curved chords is the Parker truss which is actually a curved-chord Pratt truss. See Fig. 120. Other types less frequently used are the Warren truss, the K-truss, the arched truss, and the Pegram truss. All five types are illustrated in Fig. 119. The arched truss, a deck structure, would be uneconomical as a simple span because of the shallow depth at the center, but it is widely used as a two-hinged arch. It is statically indeterminate to the first degree. A three-hinged arch is statically determinate.

The method of joints can be used to compute the dead-load stresses for a truss with curved chords, but the method of sections or the method of moments is perhaps more satisfactory. The application of the method of moments to the analysis of the truss of Fig. 121(a) will be given in detail below.

141. Method of Moments for Chord Members. All chord stresses for the truss of Fig. 121(a), except the stress in the member BC , are calculated just as for a truss with parallel chords. The center of moments for the member cd is at C . For the member CD , the center of moments is at d , etc. These chord stresses are found by dividing the moment by the height of the truss at the center of moments.

The upper-chord member BC is cut by the section 2-2. The intersection of the other two members, which is always the proper center of moments, lies

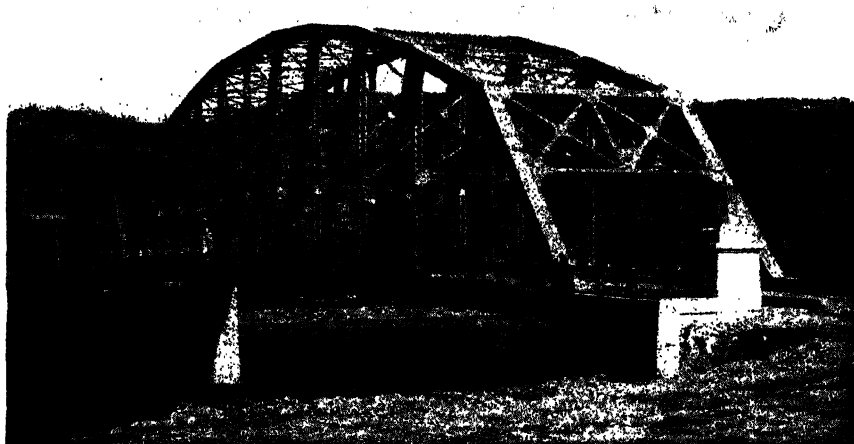


FIG. 120. ALLEGHANY RIVER BRIDGE AT KITTANNING, PA.

at the joint c . The moment about c is computed for the forces to the left of the section 2-2, as shown in Fig. 121(b). The stress in BC is found by dividing this moment by the lever arm r_1 . This is an application of the method of moments as described in § 63.

Use of Components. A more convenient analysis is possible by the use of *stress components*. The stress in BC is broken up into components at C , so that its V component will pass through the center of moments at c . The H component of the stress in the member BC is equal to the moment about c divided by the height h_2 . The stress in BC is found by multiplying its H component by its true length and then by dividing the result by the panel length d which is the horizontal component of the length BC .

142. Method of Modified Shears for Web Members. The web member Cd receives the same stress as the web member of a truss with horizontal chords, that is, its vertical component is equal to the shear in the panel. The stress in the diagonal Bc can be found in a similar manner by the method of modified shears. The total shear on the section 2-2, which is the shear in

the panel bc , must be resisted by the sum of the vertical components of the stresses in the members BC and Bc , as shown in Fig. 121(b). In other words, the diagonal Bc must have a vertical component equal to the vertical shear minus the V component of the stress in BC , which is termed the *modified panel shear*. The reason for analyzing the members of the upper chord first now becomes apparent. The total panel shear can then be reduced by the known V component of the upper-chord stress to obtain the modified panel shear. Hence we have the relation:

$$(11) \text{ Stress in a web member} = \frac{\text{Modified panel shear} \times \text{True length of web member}}{V \text{ component of length of web member}}.$$

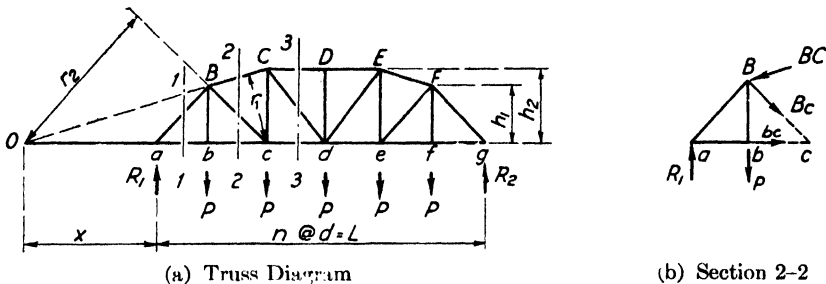


FIG. 121. METHOD OF MOMENTS FOR CURVED-CHORD TRUSSES.

If the web member is vertical, its true length and its V component of length are equal. Therefore, the stress in a vertical member is the modified panel shear. Cases will be encountered, although they are not common, where the modified shear is *greater* than the panel shear. This simply means that the V component of the chord stress acts *with* the panel shear rather than in opposition to it. The stress in the web member is correspondingly increased.

143. Method of Moments for Web Members. Web members of a curved-chord truss can be analyzed without any knowledge of the chord stresses by a direct application of the method of moments. From the section 2-2 of Fig. 121(a), it is noticed that the upper-chord member BC and the lower-chord member bc intersect at O , a distance x to the left of the reaction R_1 . The stress in the diagonal Bc can be found by dividing the moment about the point O of all forces either to the left or right of the section 2-2 by the lever arm r_2 . Considering the free body to the left of the section 2-2 as shown in Fig. 121(b), the resultant moment of R_1 and P about O is counterclockwise. Hence the member Bc must be in tension, and its stress acts downward, as shown in (b), to produce a clockwise resisting moment. Such an investigation of signs is necessary since a diagonal in a panel near the center of the truss

$$\begin{aligned}
 M_b &= 2500 \times 25 = \dots\dots\dots + 62,500 \text{ ft.-lb.} && \text{(Forces to left of } b\text{)} \\
 M_c &= 2500 \times 50 - 1000 \times 25 = \dots\dots\dots + 100,000 && \text{(Forces to left of } c\text{)} \\
 M_d &= 2500 \times 75 - 1000 \times 75 = \dots\dots\dots + 112,500 && \text{(Forces to left of } d\text{)} \\
 M_n &= -2500 \times 39.3 + 1000 \times 64.3 = \dots\dots\dots - 34,000 && \text{(Forces to left of Section 1-1)} \\
 M_o &= -2500 \times 39.3 + 1000 \times 153.6 = \dots\dots\dots + 55,300 && \text{(Forces to left of Section 2-2)} \\
 M_p &= -2500 \times 75 + 1000 \times 225 = \dots\dots\dots + 37,500 && \text{(Forces to left of Section 3-3)}
 \end{aligned}$$
$$\begin{aligned}
 S_{aB} &= - \left(2500 \times \frac{30.8}{18.0} \right) = -4280 \text{ lb} \dots \dots (\Sigma V = 0 \text{ at joint a}) \\
 S_{ab} &= - \left(-2500 \times \frac{25}{18} \right) = +3470 \dots \dots (\Sigma H = 0 \text{ at joint a}) \\
 S_{bc} &= +3470 \dots \dots (\Sigma H = 0 \text{ at joint b}) \\
 S_{cd} &= +100,000 \div 25 = +4000 \dots \dots (\Sigma M_c = 0, \text{ Section 2-2}) \\
 S_{cD} &= -(112,500 \div 29.4) = -3820 \dots \dots (\Sigma M_d = 0, \text{ Section 3-3}) \\
 S_{BC} &= -(100,000 \div 25) \frac{26}{25} = -4160 \dots \dots (\Sigma M_e = 0, \text{ Section 2-2, Components}) \\
 S_{Bb} &= +1000 \dots \dots (\Sigma V = 0 \text{ at joint b}) \\
 S_{Be} &= -(-34,000 + 52.1) = +650 \dots \dots (\Sigma M_n = 0, \text{ Section 1-1}) \\
 S_{Ce} &= +55,300 + 89.3 = +620 \dots \dots (\Sigma M_n = 0, \text{ Section 2-2}) \\
 &= - \left(650 \times \frac{18}{30.8} - 1000 \right) = +620 \dots \dots (\Sigma V = 0 \text{ at joint c}) \\
 S_{Cd} &= -(137,500 \div 150) \frac{35.3}{25.0} = -350 \dots \dots (\Sigma M_o = 0, \text{ Section 3-3, Components}) \\
 S_{Dd} &= 2 \left(3820 \times \frac{5}{25.5} \right) = +1500 \dots \dots (\Sigma V = 0 \text{ at joint D})
 \end{aligned}$$
$$\begin{aligned} S_{Be} &= \left(2500 - 1000 - 4160 \times \frac{7}{26} \right) \frac{30.8}{18} = +650 \dots\dots\dots (\text{Section 1-1}) \\ S_{Cd} &= \left(2500 - 2000 - 3820 \times \frac{5}{25.5} \right) \frac{35.3}{25} = -350 \dots\dots\dots (\text{Section 3-3}) \\ S_{Ce} &= - \left(2500 - 2000 - 4160 \times \frac{7}{26} \right) = +620 \dots\dots\dots (\text{Section 2-2}) \end{aligned}$$

Fig. 122.

where the top chord *slopes sharply upward* may be in compression, that is, the modified shear may be *negative*.

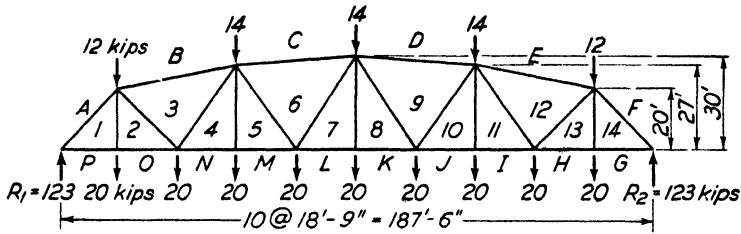
Use of Components. The use of components of stress simplifies the analysis. The stress Bc , Fig. 121(a), may be divided into components at c so that its H component will pass through the moment center O . The V component of the stress in the member Bc may be found by dividing the moment of R_1 and P about the point O by the horizontal arm from O to c , or, in this case, by the quantity $(x + 2d)$. The stress in Bc is equal to its V component times the true length of the member divided by h_1 , the vertical component of its length. The use of diagonal lever arms is convenient when they can be scaled from a large drawing of the truss; otherwise, the use of stress components will save time.

EXAMPLE. A complete analysis of the dead-load stresses in a curved-chord Pratt truss or a Parker truss is given in Fig. 122. Two or three members are analyzed by each of the methods suggested above. This is not the way in which the analysis would be made in a design office, but it is offered for illustration. *Signs* of terms are determined by the assumption that each member acts in tension. The equations that are given in Fig. 122 are obtained when all terms except the unknowns are transferred to their right hand sides.

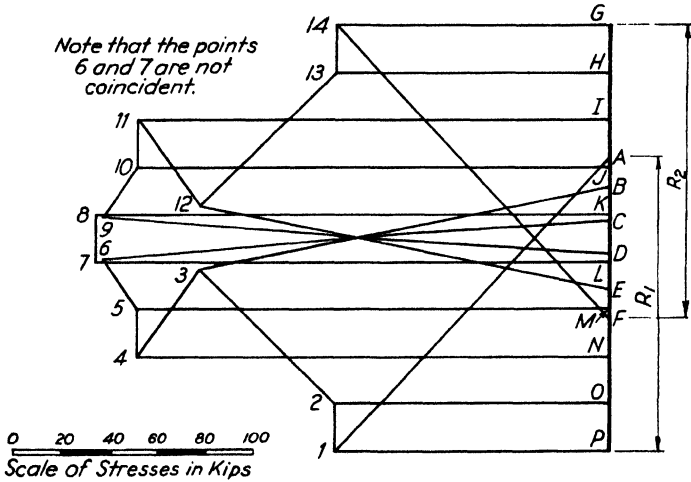
144. Graphical Dead Load Analysis of Curved-Chord Trusses. The analysis of a parallel-chord truss by graphics would seem unwarranted because of the simplicity of the algebraic methods. Graphics may be used, however, in the analysis of curved-chord bridge trusses. Two algebraic methods have been studied, the method of joints and the method of moments. These correspond to two methods of graphical analysis, the use of the stress diagram and the use of the moment diagram. The procedure used in drawing a stress diagram has been discussed fully in connection with the analysis of roof trusses. This method of analysis is illustrated by the stress diagram for the curved-chord Warren truss in Fig. 123.

The Graphical Method of Moments. The use of the moment diagram in stress analysis follows directly from the algebraic method of moments. We draw a force diagram and an equilibrium polygon for the dead loads. The bending moments at the joints of the truss are found by multiplying the corresponding vertical intercepts of the equilibrium polygon by the pole distance from the force polygon. These moments are divided by *scaled lever arms* to determine the stresses in the chord members.

Web Stresses. Moments about points outside of the truss, for determining the stresses in web members are found graphically as shown in Fig. 124. In this illustration the equilibrium polygon is drawn in the usual manner and then the strings from R_1 to P_1 , from P_1 to P_2 , and from R_1 to R_2 are extended to the left. The moments about the point N of R_1 and of P_1 are $R_1 \times 3$ and $P_1 \times 8$ as shown. Their difference is the value of M_N produced by

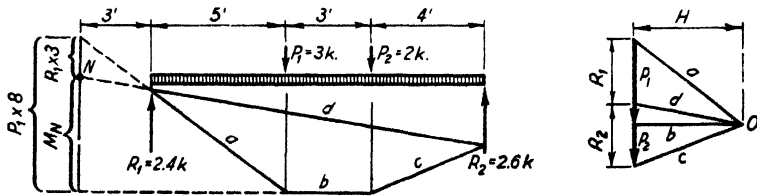


(a) Truss Diagram



(b) Stress Diagram

FIG. 123. DEAD-LOAD STRESSES IN A WARREN TRUSS BY GRAPHICS.



(a) Equilibrium Polygon

(b) Force Diagram

FIG. 124. USE OF THE EQUILIBRIUM POLYGON IN STRESS ANALYSIS.

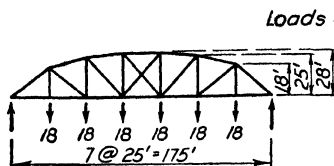
these forces. Note that the value of M_N is obtained from an intercept between the two strings b and d that form components in the force polygon of the algebraic sum (or numerical difference) of R_1 and P_1 , the loads under consideration.

PROBLEMS

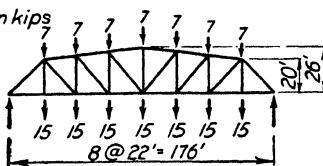
104. Calculate the stresses in this curved-chord Pratt truss for a panel dead load of 18 kips. Use the method of moments and check the stresses in the diagonals by the method of modified shears.

105. Calculate the stresses in this truss with sloping chord for a panel dead load of 15 kips at the lower chord and 7 kips at the upper chord. Check three stresses by the graphical method of moments.

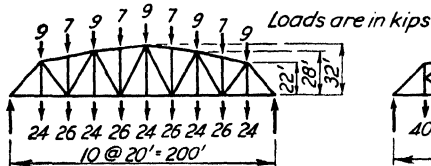
Ans. Stress in center member of upper chord = $-149,500$ lb.



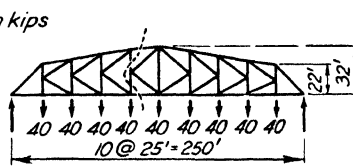
PROBLEM 104.



PROBLEM 105.



PROBLEM 106.



PROBLEM 107.

106. Draw a stress diagram for this curved-chord Warren truss. Check three stresses near the center of the truss by the method of moments.

Ans. Stress in center member of upper chord = $-266,000$ lb.

107. Use any method that you prefer to obtain the dead-load stresses in this K-truss with sloping top chord. The K divides each vertical into two equal lengths.

SUGGESTION. Study the condition of equilibrium for the joint at the mid-height of each vertical. Make use of swing sections that cut each vertical member at two points.

Ans. Stress in center panel of upper chord = $-408,000$ lb.

108. Draw a stress diagram for the truss of Problem 104. Check three stresses by the graphical method of moments.

145. Live Loadings of Highway and Railway Bridges. In this chapter on dead load stresses we have found no essential difference between the analysis of the stresses in highway or railway trusses. However, their live loadings are sufficiently dissimilar that we will treat them separately. Of course, the problem to be solved for any live loading is exactly how to place

the movable loads on the bridge to produce the greatest possible stress in a given member. Once the positions of the loads are determined, the computation of a live load stress is essentially the same problem as the calculation of a dead load stress.

A highway bridge carries several lanes of random motor cars and trucks. A railway bridge carries the locomotive and train loading. The rear wheel concentrations of the heaviest truck influence the design of the floor system and of the truss members. However, we place only one of these heavy trucks in each lane of motor traffic. The heaviest locomotive wheels are repeated on several successive axles. Hence the analysis of a railway bridge truss for a locomotive loading (without the aid of special tables or charts) is more complex than the live load analysis of a highway bridge truss because there are several rather than one axle loading whose position on the bridge may control the maximum stress in a member. However, after the principles of analysis for locomotive loadings have been established we will find that standard charts and tables greatly simplify the problem.

CHAPTER 6

TRUSS BRIDGES — HIGHWAY LOADINGS

146. Live Loads for Highway Bridges. Bridges of the different classes may be designed for the following loadings as specified by the American Association of State Highway Officials (A.A.S.H.O.).

CLASS OF BRIDGE	DESCRIPTION	TRUCK LOADING	TRUCK-TRAILER
A A	Bridge located on arterial highway	H-20	H-20, S-15
A	Bridge on main country highway	H-15	H-15, S-12
B	Bridge on secondary country road	H-10	

The *H-20* loading consisting of a 20-ton truck (Fig. 125) preceded and followed by a line of 15-ton trucks is shown in Fig. 126. The *H-15* and the *H-10* loadings are respectively 75% and 50% of the *H-20* loading. One of these truck trains is to be used for each lane (10-ft. minimum width). Hence two 20-ton trucks can be placed side by side on the floor but not in a line lengthwise of the bridge. The assumption that two fully loaded trucks or truck trains will pass each other on a bridge at the proper speed to produce maximum impact is a very stringent requirement for design. The justification for such a severe loading may be in the added life that such a requirement will produce for the structure.

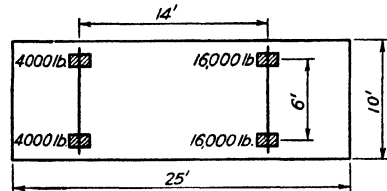


FIG. 125. 20-Ton Truck.

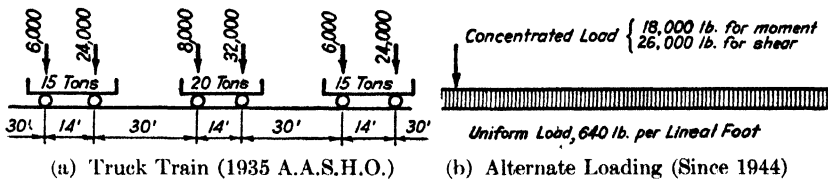


FIG. 126. *H-20* HIGHWAY BRIDGE LOADING. (A.A.S.H.O.)

Alternate Loading. The A.A.S.H.O. specifications suggest the use of an alternate or *equivalent loading* to take the place of a truck train. This alternate loading for the *H-20* truck train is shown in Fig. 126. The corresponding

loadings for the *H*-15 and *H*-10 truck trains are 75% and 50% as great. The alternate loading consists of a uniform load with a *floating concentration* because no single uniform load can represent the effect of a set of wheel concentrations on spans of all lengths. The alternate loading gives a larger moment for spans over 56 ft. and a larger end shear for spans over 33 ft., but the truck wheels must be used for shorter spans. Since bridge spans are seldom less than 60 ft., it follows that the floor system commonly is designed for the effect of the two 20-ton trucks placed side by side and the trusses or girders are designed for the alternate loading when the specified loading is *H*-20 or *H*-15.

H-S Loadings. Although highway wheel loads are limited by law in most states to a maximum value, the semi-trailer truck of Fig. 127 does produce a second rear axle load about 14 ft. behind the usual rear axle load of

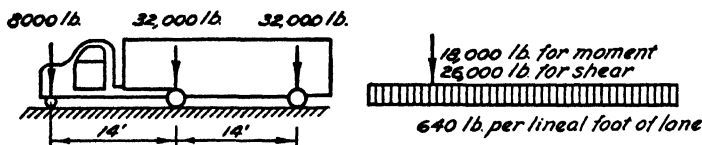


FIG. 127. *H*-20, *S*-16 SEMI-TRAILER TRUCK AND ALTERNATE LOADING.

According to A.A.S.H.O. specifications the spacing of 14 ft. on the trailer may be lengthened up to 30 ft. if a spacing greater than 14 ft. will increase the calculated stress.

the *H*-truck. This extra load may increase stresses in all members of the bridge. For loaded lengths of 140 ft. or less the semi-trailer truck shown in Fig. 127 may be used as the live load; above 140 ft. the alternate loading of Fig. 127 is used since it produces a greater moment. The *H*-15, *S*-12 loading is 75% of the *H*-20, *S*-16 loading. The semi-trailer loadings should be selected for the design of bridges on superhighways where more than an occasional semi-trailer truck is to be expected.

147. Impact Formulas. A standard increment for impact of 30% of the live-load stress has frequently been specified for the floor system. Highway-bridge specifications offer impact formulas that vary considerably in appearance, but they give about the same allowance. In these formulas, *L* is the length of bridge in feet that is loaded to produce the maximum live-load stress in the member, and *I* is the impact percentage increment of the live-load stress.

- (1)
$$I = \frac{50}{L + 125}$$
 (American Association of State Highway Officials, 1949).
Maximum Value Limited to 30%.
- (2)
$$I = \frac{L + 250}{10L + 500}$$
 (American Association of State Highway Officials, 1924).
- (3)
$$I = \frac{100}{L + 300}$$
 (Ketchum and American Society of Civil Engineers).
- (4)
$$I = \frac{75}{L + 200}$$
 (Iowa Highway Commission).

PROBLEMS

109. Analyze the factors that might affect impact and decide why the impact allowance reduces rapidly with increase in the loaded length.

110. Determine the length of a simple-span beam such that the maximum moment at the center caused by the entire weight of one 20-ton truck (Fig. 125) will exactly equal the moment caused by a uniform load of 1250 lb. per ft. Place the truck parallel to the beam.

SUGGESTION. The maximum moment at the center of a beam caused by the truck loading will occur with the rear wheels over the center of the span. *Ans.* 57.8 ft.

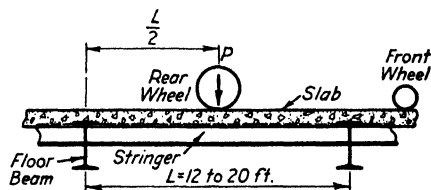
111. Same as Problem 110, except change to one 15-ton truck and use a uniform load of 900 lb. per ft. *Ans.* 60.3 ft.

112. Determine the length of a simple-span beam such that the end shear caused by the entire weight of one 20-ton truck will exactly equal the shear due to the alternate loading of Fig. 126. *Ans.* 33.0 ft.

113. Repeat Problem 112 for the *H-20*, *S-16* loadings of Fig. 127.

Ans. About 120 ft.

148. Loads on Stringers. Maximum moment in a stringer ordinarily occurs when a rear wheel of the truck is placed directly above the stringer span and at the middle of the stringer span, as shown by Fig. 128. When the wheel load P is placed in this position, there is a considerable deflection of

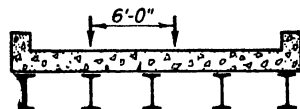


(a) Rear Wheel at Center of Span

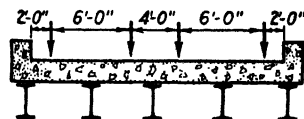


(b) Moment Diagram

FIG. 128. MOMENT IN A STRINGER, TRUCK LOADING.



(a) Single Truck



(b) Two Trucks

FIG. 129. LOAD DISTRIBUTION TO STRINGERS

the stringer directly under the load. Assume that the stringers are joined together laterally by a thick concrete or wood-strip floor. The thick floor forces the adjacent stringers to deflect also, which relieves the stringer under the load of a part of its stress.

Distribution Factors for One Truck. If the floor system is designed for a single truck (which used to be quite common), the closest spacing of adjacent wheel is 6 ft., as shown in Fig. 129(a). The load per stringer in (a)

will be nearly a full wheel load P if the stringer spacing is 6 ft. or more. For a stringer spacing of less than 6 ft., the live load per stringer is found by multiplying P by a distribution factor. This factor is a fraction whose numerator is the *stringer spacing in feet*, and whose denominator is 6 ft. where a *concrete slab* or steel grid of 4-in. thickness is used. In other words, only 50% of the load P will be carried by one stringer if the stringer spacing is 3 ft. The denominator of this fraction is specified as 4.5 ft. for a *wood-strip floor* 4 in. in thickness. A wood floor is not as effective as a concrete or steel grid floor in distributing a concentrated load.

Distribution Factors for Two Trucks. The average distance between the center load and the two adjacent loads in Fig. 129(b) is 5 ft. for passing trucks. Accordingly, the denominator of the fraction by which we multiply P to obtain the load per stringer should be reduced to 5 ft. for a 4-in. steel grid floor or a *concrete floor slab* designed to carry *two trucks*. The denominator of the fraction for a 4-in. *wood-strip floor* is also reduced. For two trucks it is set at 4.0 ft.

Economic Spacing of Stringers. The load-distribution factor is also intended to be applied for a stringer spacings greater than the specified value of the denominator of the distribution ratio. Hence the design load per stringer varies directly with the stringer spacing. The use of a few stringers of depth about one-twelfth of the panel length will be more economical than the use of small stringers closely spaced. Since the minimum 6-in. slab will span from $4\frac{1}{2}$ to 6 ft. between stringers, there is little justification for a closer spacing.

149. Moments and Shears in Stringers. When the load per stringer has been determined from the distribution factor, the bending moment is computed for the rear wheel load placed at the center of the panel. The *maximum moment* is equal to $PL/4$ where P denotes the *effective wheel load per stringer* in pounds, and L is the panel length in feet. The panel length in ordinary highway bridges seldom exceeds 25 ft., so that the front wheel load will be in the adjacent panel. This live-load moment must be increased for impact (§ 147) and must then be added to the dead-load moment of $wL^2/8$. The unit dead load w is the weight per lineal foot of the stringer itself plus the weight per foot of the floor surface for a width equal to the stringer spacing.

Maximum Shear. The large rear wheel load is placed at the end of the stringer to produce maximum shear. No allowance is made for lateral distribution because the load is at the face of the floor beam where no appreciable deflection of the stringer is possible. Accordingly, a *full rear wheel load*, increased for impact, plus the dead-load reaction equals the maximum shear at the end of the stringer. There will be a small *carry-back reaction* from the front wheel of the truck (for spans in excess of 14 ft.), which should be dis-

tributed laterally as explained in § 148. The maximum shear is used to check the stresses in the rivets of the standard end connection* which will be used between the stringer and the floor beam if it is found satisfactory. A welded connection may be used if desired. The unit shear in the stringer web usually is quite low.

The Outside Stringer. The maximum loading of the stringer placed under the curb ordinarily is less than for the other stringers, but this outside stringer often is made equally strong because of the *high impact stress* to which it may be subjected if a truck should strike the curb. The live load carried to an exterior stringer is found by placing the truck wheel as close to the curb as is allowed by the clearance diagram. Then the reaction of the slab on the outside stringer is determined as for a simple span between the first two stringers. The outside stringer carries the entire weight of the curb in addition to its tributary share of the weight of the road slab.

150. Loads on Floor Beams. *Live Loads.* Floor beams are always over 6 ft. apart. Hence they do not receive help from adjacent floor beams but must be designed for a live load equal to the full value of the rear wheels of two trucks. The reactions of the wheels are considered as concentrated loads applied to the top flange of the floor beam. If the panel length exceeds 14 ft., there will be small carry-back reactions from the front wheels of the trucks. See Fig. 130. These reactions would be distributed laterally, but they are usually simply added to the reactions of the rear wheels. For the *H-S* semi-trailer truck both P_1 and P_2 in Fig. 130 would become full rear wheel loads and the front wheel load P_0 would occur in the span to the left of P_1 . In that case, $R_1 = P_1 + (P_0 + P_2)(a/L)$.

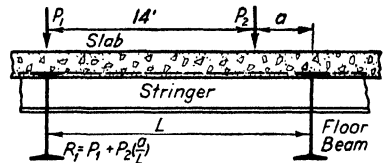


FIG. 130. REACTION ON FLOOR BEAM.

Dead Loads. The dead load acting on the floor beam is uniform where stringers are not used, but it is concentrated mainly at the stringer connections when stringers are used. In *either* case, it is common practice in design to consider it as a uniformly distributed load on the floor beam. This total dead load consists of one panel of the combined weight of slab and stringers for an interior floor beam, and approximately one-half of a panel load for an exterior or end floor beam. The floor beam must also carry its own weight. The depths of interior and exterior floor beams must be kept the same, and many engineers prefer to use the same weight of beam for each to allow for the possibility of a high impact stress in the end floor beam due to a low approach slab.

* *Standard end connections* for each depth of beam are given in all structural steel handbooks. See handbook for *Steel Construction*, A.I.S.C.

151. Maximum Moment and Shear in a Floor Beam. The problem of placing loads *laterally* on a floor beam to produce maximum moment and maximum shear, and at the same time not to overstep the bounds of reason is a difficult one to solve. Wheel loads from two trucks of the maximum size are placed on the bridge. It is assumed that (1) both of these *fully loaded trucks* are moving at (2) the *proper speed* to produce maximum impact. Even this hardly seems probable, but it appears absurd to assume further (3) that these trucks will not only pass each other when their rear wheels are *exactly over a floor beam*, but also to assume (4) that the trucks will crowd each other into the *exact position laterally* for absolute maximum moment or absolute

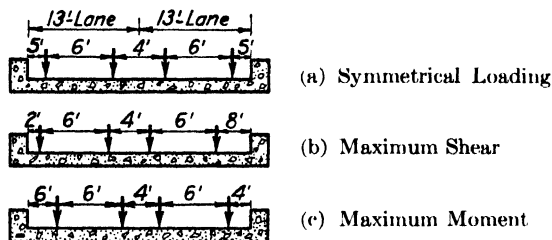


FIG. 131. POSITION OF LOADS FOR SHEAR AND MOMENT IN A FLOOR BEAM.

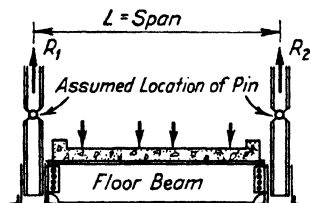


FIG. 132. EFFECTIVE SPAN OF A FLOOR BEAM.

maximum shear in the floor beam. It would seem a little *more reasonable* to place the trucks *symmetrically* in their own traffic lanes, Fig. 131(a), and to compute shear and moment for this position of the live load. Even that condition requires the improbable combination of the first three assumptions mentioned above.

Trucks Placed for Maximum Shear and Moment. However, if *absolute maximum shear* is desired, the trucks are moved as far as possible to one side of the beam and are placed as close together as their clearance diagrams will allow, that is, 4 ft. between wheels, as shown in Fig. 131(b). The position of rear wheels to produce *absolute maximum moment* in a floor beam will be controlled by an algebraic criterion that will be derived in the following section. This position will be shown to conform to (c) of Fig. 131 when the width of roadway is 26 ft., the minimum width by A.A.S.H.O. specifications.

The Span of the Floor Beam. An important point to remember is that the bending moment in a floor beam should always be calculated for a span equal to the distance from center to center of the trusses. This assumes that the vertical members are incapable of resisting moment, as shown by Fig. 132. The vertical post of a low truss, although a stiff member itself, is not supported laterally at its top except by the lateral resistance of the top chord and the adjacent verticals. A high truss has cross bracing at the top of the

verticals, but the posts themselves are long and slender. Hence, although the assumption that the verticals can not resist moment possibly is somewhat overconservative, it is always adopted.

152. Criterion for Placing Floor-Beam Loads for Maximum Moment.

It is desired to determine the proper placing of a group of concentrated loads fixed at given distances apart in order to produce the largest possible moment in a simple beam. It is possible to make the criterion derivation apply to any group of loads, although the case under consideration (maximum moment in a floor beam) is a special one in which there are two or four equal concentrated loads. The placing of the loads is shown in Fig. 133 where the wheel load P is to be placed so that the greatest possible moment will occur under P at some unknown distance x from R_1 . G , the resultant of all the loads on the beam is at the distance a to the right of P , and G_1 , the resultant of loads to the left of P is located at the distance b from the center of moments under the load P . The value of the left-hand reaction is $G(L - a - x)/L$. Whence we obtain the following expression for the bending moment under the wheel load P :

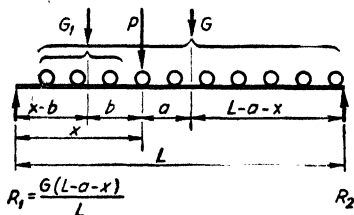


FIG. 133. MAXIMUM MOMENT UNDER A GIVEN WHEEL, P .

$$(5) \quad M = G \left(\frac{L - a - x}{L} \right) x - G_1 b,$$

The loads may be shifted back and forth by varying the value of x ; at one position of the loads, the value of M will be a maximum. The value of x for a maximum value of M can be determined by differentiating the expression for M with respect to x , setting the derivative equal to zero, and solving for x . Thus we find

$$(6) \quad \frac{dM}{dx} = \frac{G}{L} (L - a - 2x) = 0.$$

But G/L , the average load on the beam, can not be equal to zero. Therefore

$$(7) \quad L - a - 2x = 0, \quad \text{or} \quad L - a - x = x.$$

For a maximum moment under the load P , this load and the center of gravity of all the loads must be located at equal distances from the ends (or from the center) of the beam.

Use of the Criterion. In order to apply this criterion to the determination of absolute maximum moment, either we must place successive wheels near the center of the group in the proper position for producing maximum moment and then select the largest value of the moment produced, or else we must

decide from a study of the loads which one of the wheels will produce maximum moment. For the case of four equal wheel loads, the maximum moment will occur under one of the interior wheels.

PROBLEMS

114. Determine the maximum live-load moment produced in an interior beam of a 35-ft. beam bridge by the *H-20* loading. There are eight 35-ft. longitudinal beams spaced 4 ft. apart covered by a 6-in. concrete slab. Try the truck wheels first and then the alternate loading. *Ans.* 1,420,000 in-lb.

115. Repeat Problem 114 for the *H-20*, *S-16* loading.

116. Determine the maximum live-load moment produced in a transverse floor beam by the *H-15* loading for the following data: width of roadway, 22 ft.; panel length, 9 ft.; spacing of trusses, 25 ft. This bridge has no stringers. *Ans.* 182,000 ft-lb.

117. Calculate the maximum bending moment and maximum shear, including dead load, live load, and impact, for a floor beam of the bridge of Problem 92. Use the A.A.S.H.O. (1949) impact formula.

118. Calculate the maximum bending moment and maximum shear, including dead load, live load, and impact, for a floor beam and stringer (interior) of the bridge of Problem 93. Use the A.A.S.H.O. (1949) impact formula. Assume that there is an exterior stringer under the center of each curb.

119. Find the uniform load per foot of span that will produce the same maximum moment in a simple beam 57.6 ft. long as is produced by the 20-ton truck of Fig. 125. Place the truck parallel to the beam. Repeat for shear.

Ans. 1260 and 1320 lb. per ft.

120. Find the span length of a simple beam so that the maximum moment in the beam caused by the entire weight of one 15-ton truck will just equal the moment caused by a uniform load of 900 lb. per ft. Place the truck parallel to the beam.

SUGGESTION. Solve the cubic equation by trial.

Ans. 60.3 ft.

121. A highway bridge has 14-ft. panels, a 24-ft. roadway. Its trusses are spaced 27 ft.-6 in. apart. There are six lines of stringers spaced 4 ft.-3 in. apart. The loading is *H-15* and impact is controlled by the A.A.S.H.O. (1949) formula. Determine the design moments in an interior stringer and floor beam. There is a concrete floor slab 6 in. thick. Assume that a stringer weighs 45 lb. per ft.

INFLUENCE LINES FOR PLACING LIVE LOADS FOR MAXIMUM STRESS

153. Statement of the Problem. The analysis of stresses for a given placing of the live load on the bridge is essentially the same problem as the analysis of dead-load stresses. The only difference is that the panel concentrations may not be identical and may not be placed symmetrically on the truss, but this simply makes necessary the calculation of stresses on both sides of the center line and does not require any new tools of analysis. The only *new* problem is to learn how to place the live load on the bridge to produce maximum stress in a given member. It will be found that all live loads, whether uniform or concentrated, must be placed differently to produce

maximum stresses in different members. Hence, for a live-load analysis, the method of sections is more satisfactory than the method of joints.

Equivalent Loading. As has been discussed in § 146, the common live loading for the design of highway bridge trusses is a uniform load with a *floating concentration* to represent the excess load on the rear axle of a heavy truck. Uniform and concentrated loads are varied to make them equivalent to ($H-20$), ($H-20, S-16$) or a proportionately lighter truck-train loading. See Fig. 126 or 127. The placing of this simple conventional loading to produce maximum stress in any member is readily determined from the *shape* of the *influence line* for stress in the member. The development of influence lines will be discussed in the following sections.

154. Influence Lines for Reactions. The usefulness and the significance of influence lines will become clear from the study of a few simple examples. Figure 134(a) shows a simple beam with a rolling 1-lb. load acting on its top surface. The change in the reaction at the point a will be studied as the load moves along the beam. When the load is placed at a , the value of R_a is 1 lb. As the load rolls toward b , the reaction at a decreases and is always equal to $(1 \text{ lb.}) \times (x/L)$. The equation $R_a = x/L$ is *linear in x* and represents a straight line. Accordingly, Fig. 134(b) represents the variation in the reaction R_a as the 1-lb. load moves across the beam. This diagram is called an influence line for the reaction R_a .

SUMMARY. It is well to stop here to summarize the important points developed in the preceding analysis:

- (1) An influence line for the reaction R_a shows the variation in this reaction caused by a movement of a 1-lb. load across the span.
- (2) The ordinate to the influence line at any point x is equal to the actual value of the reaction R_a produced at the point a by the 1-lb. load placed at the point x . The ordinate at the point a is equal to the load of 1 lb.
- (3) The influence line for the reaction of a simple beam is a straight line.

155. Influence Lines for Stress Functions in Beams. *Shear.* We shall now develop an influence line for shear at any point of a simple beam. In Fig. 134(c), as the load moves from the point d to the right, the shear at d is always equal to the reaction at a . Therefore, between the points d and b the influence line for shear at d must be identical with the influence line for R_a . As the 1-lb. load passes from right to left across the point d , the change in shear at d must be 1 lb. The complete influence line is therefore as shown in (c). It should be clear that the negative part of this influence line will be identical with the left end of the influence line for R_b , and that the *two sloping lines* of the shear influence line are *parallel*.

Moment. The influence line for moment at a point d of a simple beam is shown in Fig. 134(d). The maximum moment at the point d occurs when the 1-lb. load is placed at d ; it is equal to $(1 \text{ lb.}) (K/L)(L - K)$. As the load

moves from d toward the right, the moment at d is always equal to the reaction at a times the constant distance $(L - K)$. Therefore, since the reaction influence line is a straight line the moment influence line must be a straight line from the point a to the point d . By considering the equation $M_d = R_b K$, where K is the distance from b to d , we see that the right-hand part of the influence line for M_d is also a straight line. Hence we conclude that the moment influence line must be a triangle, as shown in Fig. 134(d).

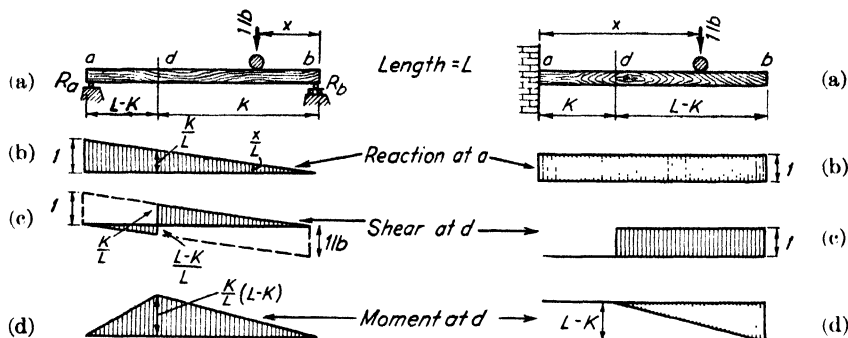


FIG. 134. INFLUENCE LINES FOR SIMPLE BEAM.

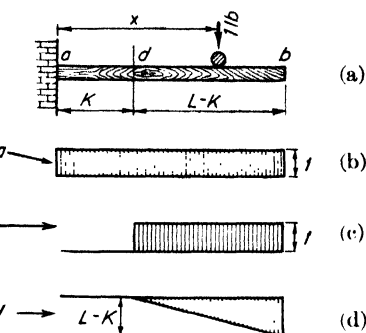


FIG. 135. INFLUENCE LINES FOR A CANTILEVER BEAM.

Beams that are not Simply Supported. Influence lines for functions of a cantilever beam are shown in Fig. 135. An understanding of the influence lines for the simple beam and the cantilever is sufficient background for a study of the influence lines for any statically determinate beam. Methods for obtaining influence lines for fixed and continuous beams will be developed later. Incidentally, influence lines for statically indeterminate structures are curved, while all influence lines for statically determinate structures are made up of straight lines.

156. Influence Lines for Truss Members. Influence lines for the reactions of trusses are identical with influence lines for the reactions of beams. The influence line for stress in a chord member of a truss must be of the same shape as the influence line for moment about its moment center, because the chord stress can be calculated by dividing the moment about its moment center by the constant lever arm from this moment center to the member. Influence lines for moment at a joint of a truss and for stress in the corresponding chord member are shown in Fig. 136(b) and (c).

The stress in a web member may be thought of as caused by the panel shear whenever the chords are horizontal. The influence line for shear in the panel cd of the truss considered above is shown in (d). The influence line for stress in the diagonal of this panel is illustrated by (e). The signs on the

diagrams signify positive and negative shear, or tension and compression if the influence line refers to stress. The same reasoning that was applied to

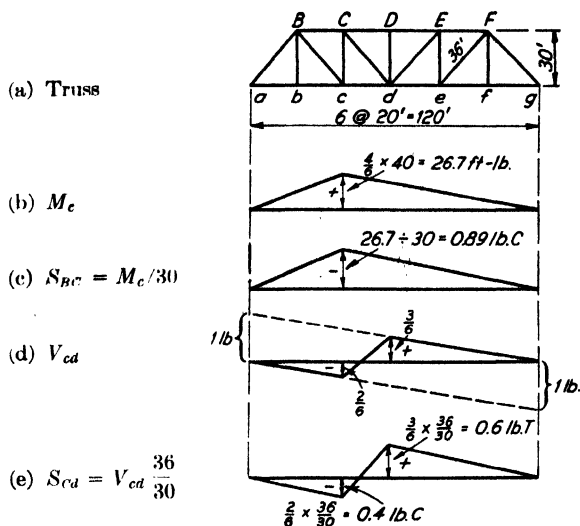


FIG. 136. INFLUENCE LINES FOR A TRUSS WITH PARALLEL CHORDS.

beams, in regard to the correspondence of reaction and shear, may be used to show that the outer lines of the influence line for panel shear are *parallel*. When these lines are extended as in Fig. 136(d), they intercept ordinates of unity at the reactions. Hence they are in reality the influence lines for the end reactions.

157. The Influence Line between Panel Points. A special proof is needed to show that the part of the influence line crossing the panel cd in Fig. 136(d) is a straight line. The sketch, Fig. 137(a), is an enlarged view of the influence line in the panel cd , while (b) represents any general case. The stringer is shown spanning between floor beams and carrying the concentrated load of 1 lb. acting through the floor slab.

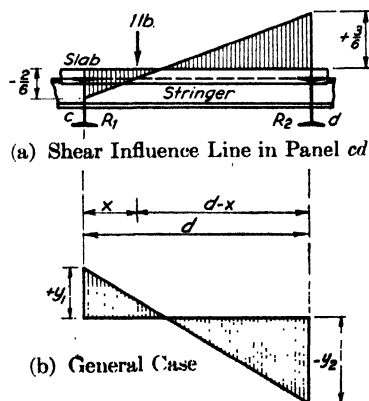


FIG. 137. INFLUENCE LINE BETWEEN JOINTS.

PROOF. The load of 1 lb. is carried to the panel points where the influence ordinates, y_1 and y_2 , are known. The value of the panel shear is found by multiplying each end reaction

by the corresponding value of the influence ordinate for shear. Hence the shear is

$$(8) \quad V_1 = R_1 y_1 - R_2 y_2 = \left(\frac{d-x}{d} \right) y_1 - \left(\frac{x}{d} \right) y_2.$$

Since this equation is *linear in x*, the influence line which it represents must be a *straight line* between the ordinates y_1 and y_2 , or between panel points.

158. Influence Lines for Curved-Chord Trusses. Influence lines for stresses in chord members and web members of curved-chord trusses have the same general shape as similar influence lines for trusses with parallel chords. The main difference is that the ordinates for web members of panels with

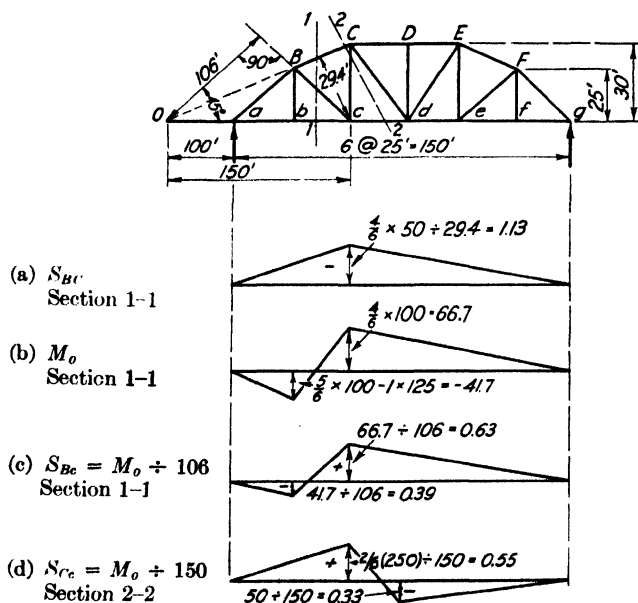


FIG. 138. INFLUENCE LINES FOR A CURVED-CHORD TRUSS.

sloping chords are controlled by the *modified panel shears*. The standard procedure in obtaining an influence line for a web member is to place the 1-lb. load first to one side and then to the other of the panel whose shear controls the stress in the member. The stress in the member is obtained for the load in each of these two positions, usually by the method of moments, and these stresses are plotted as ordinates over the corresponding load points. These positive and negative ordinates may be joined to the reaction points and to each other by straight lines to form the influence line desired. The influence lines for three members of a truss with sloping top chord are given with the

necessary computations in Fig. 138. Note for the influence line for S_{Cc} that the 1-lb. load is to be placed successively at c and d because the section 2-2 cuts through the panel cd . The stress in the vertical Cc is then found from $M_o \div 150$ for each position of the 1-lb. load.

159. Area under an Influence Line. The area under an influence line has an extremely important significance. It is known already that each ordinate represents some stress function, such as reaction, shear, moment, or bar stress, caused by a 1-lb. load placed in the same relative position on the truss. The area between any two given ordinates on the influence line will be shown to equal the value of the stress function produced by a *uniform load of 1 lb. per ft.* covering the length of the structure included between the two ordinates chosen.

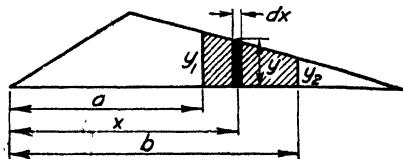


FIG. 139. INFLUENCE-LINE AREA.

PROOF. In Fig. 139, the average ordinate y over the differential length dx represents the value of a stress function for a 1-lb. load placed in the same relative position on the structure. The load over a length dx produced by a load of 1 lb. per ft. is $(1)dx$, and the corresponding value of the stress function must be $(y)dx$. However, the load of 1 lb. per ft. extends from $x = a$ to $x = b$; hence the total value of the stress function S will be found by an integration. We may write

$$(9) \quad S_{1 \text{ lb. per ft.}} = \int_a^b y dx = \int_a^b dA = \text{Area under influence line from } x = a \text{ to } x = b.$$

The total value of the stress function S produced by a load of w lb. per ft. will be wA , the product of the load per unit length and the area under the influence line for the length covered by the uniform load.

160. Use of Influence Lines for Placing Live Loads. The chief value of influence lines is that they afford a *picture* of the variation of any stress func-

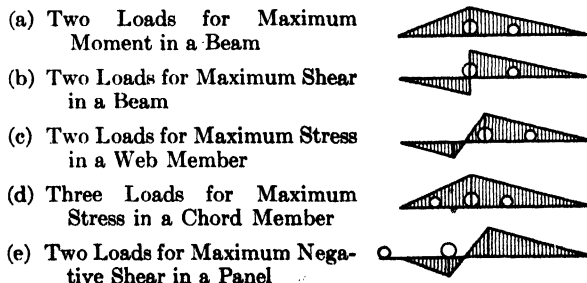


FIG. 140. PLACING LOADS OF FIXED SPACING FOR MAXIMUM STRESS.

tion as a concentrated load moves across the structure. We should be able to draw on our imagination sufficiently to let us actually visualize the move-

ment of the rolling load and the variation of stress from the picture of the influence line.

Concentrated Loads. An influence line shows at once where to place a *single* concentrated load on the structure to cause a maximum positive or a maximum negative (minimum) value of the stress function. Of course, the load is to be placed at the largest positive or negative ordinate on the influence line. Almost as clear is the indication of how to place *two or three concentrated loads* that move at a fixed distance apart, as the front and rear wheels of a truck. Place the largest load over the greatest positive or negative ordinate on the influence line, and place the other load or loads to one side or the other by observing the following rules:

(1) If the large load is placed over the positive ordinate, do not choose to place the small loads over negative ordinates.

(2) If you can accomplish this result and still are free to place the small loads either to the left or right of the large load, choose to place them on the side of the influence line which has the flatter slope.

Wheel loads placed on several influence lines in positions for maximum moment, maximum shear, or maximum stress are shown in Fig. 140.

Uniform Loads. The information for placing a uniform load to produce maximum positive or negative stress also is obtained from the influence line. You must load all the length of the structure which corresponds to positive or negative area of the influence line. In other words, you load the structure from one end up to the point where the influence line crosses the base line and changes sign. The entire span is loaded if the area is all above or all below the base line, as is true for reaction and moment in a simple beam and for chord stress in an ordinary truss. The span is to be loaded over only a part of its length for maximum shear in a simple beam (interior point) or for maximum stress in a web member of a truss.

161. Use of Influence Lines for Stress Calculation. Influence lines may be used in two ways. We may simply *sketch the general shape* of an influence line and make use of this sketch to determine the proper placing of the loads to produce maximum stress in a given member. Then we place the loads on the structure in this position and determine the stress by statics. The other procedure involves the *actual calculation of critical ordinates* to the influence line. Then, when concentrated loads are placed for maximum stress, the value of each concentrated load is simply multiplied by the value of the influence ordinate directly under it. These products finally are summed to obtain the actual value of the stress caused by the several concentrated loads.

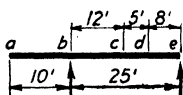
Uniform Loads. A uniform load can be handled in a similar manner. *The area under the influence line* for the length covered by the uniform load of w lb. per ft. is computed and is then multiplied by the value of w . Since

the area has been shown to equal the stress caused by a load of 1 lb. per ft., this area when multiplied by w becomes the value of the stress caused by a load of w lb. per ft. Therefore, the total area of the influence line (negative areas being subtracted from positive areas) when multiplied by the dead load per foot of span, gives the total value of the dead-load stress. This is a useful check upon our calculations of the dead-load stresses. Such a check also assures us that the influence ordinates are themselves correct.

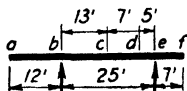
PROBLEMS

122. Determine critical ordinates for influence lines for reaction at e , shear at c , and moment at b of the overhanging beam.

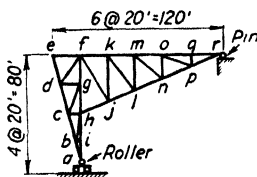
123. Calculate influence ordinates for reaction at b , shear at c , and moment at d and e of the overhanging beam.



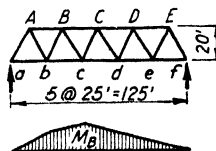
PROBLEM 122.



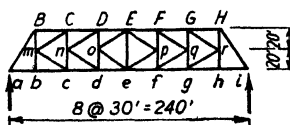
PROBLEM 123.



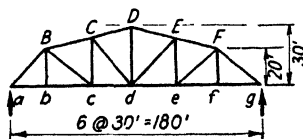
PROBLEM 124.



PROBLEM 125.



PROBLEM 126.



PROBLEM 127.

124. Draw influence lines for stress for the bars jl and hi of this frame and determine the maximum stresses in these bars for a rolling 60-ton load on the chord $e-r$.

Ans. 144 kips for hi .

125. Obtain the influence line for the lower-chord member bc of the Warren truss and check the area against the computed DL stress. ($DL = 1200$ lb. per ft.)

126. Draw influence lines to scale for the bars nd , nd , and od of the K-truss.

127. Obtain influence lines for web members Cd , Cc and Bc of this truss and find the maximum bar stresses for a live load of 1350 lb. per ft. BCD is a straight line.

Ans. $S_{Cd} = -42,100$ lb. or $+26,300$ lb.

128. Adjust the truss of Problem 127 by increasing the length of Dd to 35 ft. without changing the level of C or E . Then repeat the calculation of the stresses in Cd by use of an influence line.

ANALYSIS OF BRIDGE TRUSSES FOR UNIFORM LIVE LOADS

162. Conventional and Exact Methods of Loading. *Chord Members.* The study of influence lines that has just been completed has shown that any downward load on a simple through-truss bridge always causes compression in the top chord and end post, and tension in the lower chord. We arrive at the same conclusion by analogy of a truss and a simple beam. In any event, it is clear that the designer should place all possible load on the bridge in order to obtain the maximum chord stresses.

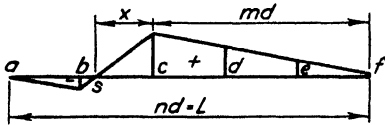


FIG. 141. INFLUENCE LINE FOR STRESS IN A WEB MEMBER.

Therefore, if the trusses are designed for a uniform live load, this load must cover the entire span. Hence, the maximum chord stresses caused by a uniform live loading can be obtained from the dead-load chord stresses by *direct proportion*.

Exact Loading for Web Members. The general shape of the influence line for a web member is as shown in Fig. 141. The letters from a to f indicate locations of the panel points. In order to obtain a maximum live-load stress in this member, which is tension for the influence line shown, the truss should be loaded with the uniform live load from the reaction at f to the point of crossing at s. Since the uniform loading is brought to the truss as panel concentrations by the floor system, full panel concentrations occur at e and d,

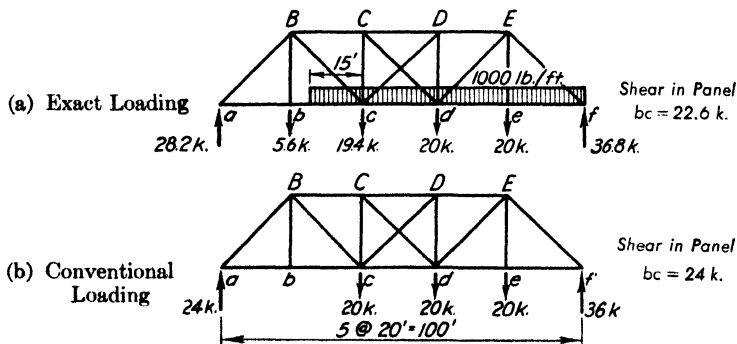


FIG. 142. COMPARISON OF THE EXACT AND CONVENTIONAL LOADINGS.

but only partial concentrations occur at c and b. This is the exact method of loading a truss for maximum stress in a web member. A typical example is shown in Fig. 142(a), where a truss is loaded with a live load of 1000 lb. per ft. to produce maximum shear in the panel bc. The distance x out from the joint c to the point of crossing on the influence line in this particular case

is three-quarters of a panel length, or 15 ft. The panel concentrations are as shown, and the panel shear is 22.6 kips.

Conventional Loading for Web Members. The same span is shown in Fig. 142(b) loaded by the conventional method of loading, which is to place full panel concentrations at all joints to the right of the panel for which maximum shear is desired. The shear as calculated by the conventional method is 24 kips. This shear is 7% higher than the exact maximum live-load shear. The conventional live-load shear always will be found to be *larger* than the exact shear; hence the approximate method is safe for use in design. You should demonstrate that no possible position of a uniform loading applied on a floor system could ever produce the conventional loading shown in Fig. 142(b). That is why such loading is called *conventional loading*. It does not represent an actual situation, but is a rather conservative approximation.

163. Analysis of Parallel-Chord Trusses. *Conventional Live Loading.* Influence lines are seldom needed for trusses with parallel chords. Chord stresses reach their maximum values when the truss is loaded over its entire length with uniform live load represented by equal live-load panel concentrations. They can be obtained by direct proportion from the dead-load stresses. Live-load stresses in the web members are often obtained by the *conventional method of loading*. The simplest procedure is to start loading panel points from one end of the bridge. As each additional panel concentration is placed on the truss without removing previously placed loads, we calculate the stresses in the diagonal and vertical in the panel just beyond the final concentrated load. When the loading has been extended panel by panel across the entire structure, all maximum live-load stresses for the web members and also for the counters will be known.

Criterion for Exact Panel Shear for Parallel-Chord Trusses. If the *exact method of loading* is to be employed, the following information will simplify the work. In each panel the load must be run out a distance x , which is determined from the crossing point on the influence line for panel shear.

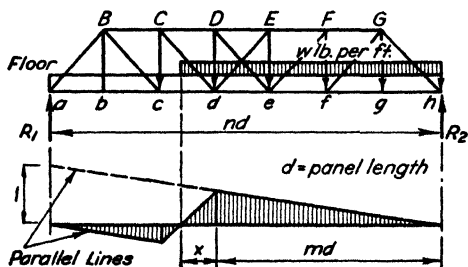


FIG. 143. MAXIMUM PANEL SHEAR.

See Fig. 143. However, it is known that the two sloping outside lines of the diagram are parallel. Therefore, the influence ordinates at the ends of the panel under consideration bear the same relation to each other as do the corresponding distances from these ordinates to the ends of the truss. A study of proportions from two pairs of similar triangles in Fig. 143 shows that

the distance from one end of the panel to the point of crossing is to the panel length as the number of panels from that end of the panel to the end of the truss (m) is to the sum of the number of panels from the one under consideration to the two ends of the bridge ($n - 1$). By making use of the terminology from Fig. 143, where d is a panel length, we obtain the expression

$$(10) \quad \frac{x}{d} = \frac{md}{(n-1)d} = \frac{m}{n-1} \quad (\text{for parallel-chord trusses}).$$

If instead, x is measured from c , md becomes the distance $a-c$.

CALCULUS DERIVATION. A direct calculus derivation of this rule may be preferred. The notation of Fig. 143 will be followed. The shear in the panel cd is equal to the value of R_1 minus the partial panel load at c , or

$$V = \frac{w(md + x)^2}{2nd} - \frac{wx^2}{2d}.$$

The variable is the distance x . The maximum value of the shear will occur at some value of x which can be determined by differentiating the expression for V , setting the derivative equal to zero, and solving for x . We have

$$\frac{dV}{dx} = \frac{w}{2nd} (2md + 2x) - \frac{wx}{d}.$$

Setting the value of the derivative equal to zero, we obtain the expression

$$\frac{w}{nd} (md + x) - \frac{wx}{d} = 0.$$

But the values of w , n , and d can not be equal to zero; therefore, multiplying by $\frac{nd}{w}$

$$md + x - nx = 0.$$

Solving this expression for x , we have

$$x(1 - n) = -md,$$

or

$$(10) \quad \frac{x}{d} = -\frac{m}{1-n} = \frac{m}{n-1}.$$

A knowledge of this simple proportion makes the calculation of influence-line ordinates unnecessary for the parallel-chord truss even where the web stresses are calculated by the exact method. The uniform load is placed for maximum by this criterion, and the corresponding stress in the diagonal or vertical web member is calculated from the panel shear.

164. Reversal of Stress and Use of Counters. An important study in live-load stress analysis is the investigation of possible reversal of stress in the diagonals and the use of counters or members connecting the corners of the panel not connected by the main diagonals. No reversal of stress is possible in a chord member or in an end post. The stresses in the web members of trusses with horizontal chords are dependent directly upon the panel shears. Accordingly, when the panel shear changes sign from passage of the live load, the live-load stress in the web member also changes sign. This reversal of stress is made clear by the shape of the influence line (Fig. 141).

A common cross-section for a diagonal of a standard Pratt truss is composed of two angles separated by tie plates. This member is reasonably stiff for a tension member, but it is entirely incapable of acting as a column or compression member. Hence it is necessary that the stress in such a member shall never reverse into compression. *In order to prevent such reversal it is necessary to place a counter diagonal in the panel if the calculated live-load compression in the first diagonal is equal to or greater than its dead-load tension.* The tension from dead load exists in the member as long as it is in operation. The live-load compression simply *reduces* the dead-load tension until the live-load compression becomes greater than the dead-load tension, at which time the member *goes out of operation* and the resultant negative panel shear due to combined dead and live loading is resisted by the counter diagonal. Impact must be considered as a part of the live load when reversal is being investigated. Should a light counter be used in a panel where the live-load compression comes within a few hundred pounds of reversing the stress in the diagonal to compression? Yes, to provide a factor of safety.

Shear Distortion of a Truss Panel. Panels distorted by positive and negative shear are shown in Fig. 144(b) and (c), respectively. It is important to visualize these distortions and the corresponding stresses in the diagonals, because a clear understanding of reversal can be developed only in this way. It must be understood that a tension diagonal will buckle under the effect of a slight resultant compression, but it can resist live-load compression up to the value of its dead-load tension. However, when the main diagonal bd goes out of operation (Fig. 144(c)) the counter diagonal ac must resist the combined dead- and live-load shear for the panel.

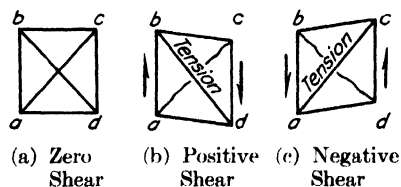


FIG. 144. SHEAR DISTORTION OF A PANEL.

165. Effect of Counter Action upon Stresses in Adjacent Members. It is to be noted in Fig. 145 that the dead-load stress in the lower-chord member cd is increased from 40 to 45 kips when the live load forces the counter into action. However, the live-load stress in the member cd will be very much less for this placing of the live load (joints b and c only loaded to produce negative shear) than when the entire truss is loaded; hence the increase of the dead-load stress in cd does not influence the design of the truss.

When the counter cD goes into operation from the effect of live loading to the left, the dead load stress in the vertical changes to tension. Actually, this member never can receive a resultant tension because the counter cD can never have a resultant $DL + LL$ compression. A study of the joint D

of Fig. 145 confirms this view. Since the diagonal or counter cD is a tension member, it can never push up at the joint D , which would be necessary to

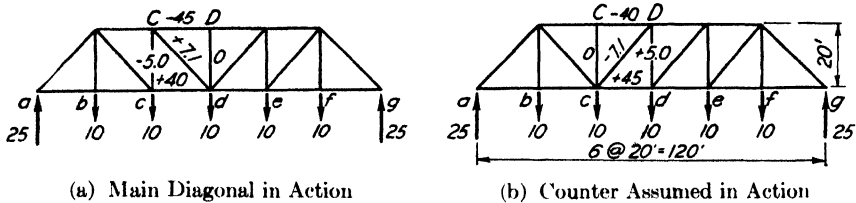


FIG. 145. DEAD-LOAD STRESSES CHANGED BY COUNTER ACTION.

balance a downward pull of the member dD . Instead, for a live and dead loading that would tend to produce compression in cD and tension in dD , the main diagonal cD would go back into operation.

166. Stiff Diagonals that Undergo Reversal. In some trusses, notably the Warren truss, the diagonals are made capable of resisting compression. Counters are not used. Such members are known as *reversal members*. The specifications governing their design are very stringent. Naturally, they must meet the requirement in regard to slenderness that is imposed on *any* column. In addition, a reversal member is designed to resist two combined stresses which usually are obtained by increasing both the maximum possible tension and the maximum possible compression by 50% of the smaller. This rigid requirement is considered necessary because repeated reversal of stress is the most severe treatment that any member can receive.

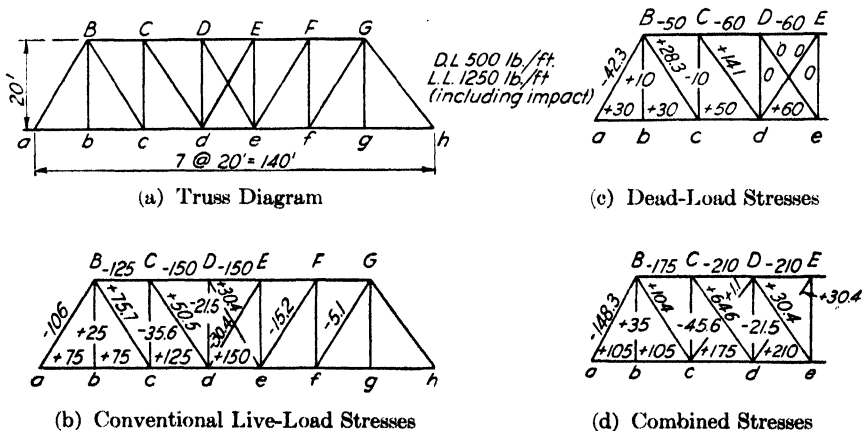


FIG. 146. EXAMPLE OF COMBINED STRESSES FOR A PRATT TRUSS—CONVENTIONAL LOADING.

167. Combined Stresses for Design of Parallel-Chord Trusses. *Conventional Loading.* The combined stress considered here is simply the algebraic

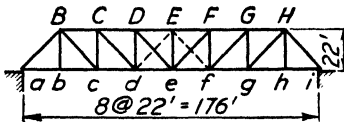
sum of the stresses caused by dead load, live load, and impact. Wind stresses will be considered in a later section. Chords, end posts, verticals, and diagonals of the Pratt truss each have but a single combined stress, which is the arithmetical sum of the dead-load, the live-load, and the impact stresses. The diagonals of a Warren truss have both a maximum and a minimum stress corresponding to a maximum tension and a maximum compression.

EXAMPLE. An example of combined stresses for a standard Pratt truss is given in Fig. 146. The procedure adopted here is recommended, that is, each set of stresses is placed on a truss diagram. This method is equally convenient in the design office, for the designer can pick stresses off of a truss diagram much more rapidly than from a stress table. The only members for which the table is preferable are those that have a reversal of stress. None of these occur in the Pratt truss with counters. For a reversal member (when a stress table is not used) the combined stress should be placed with the other maximum stresses on the truss diagram; then below the diagram, the combination of the dead-load stress, the live-load stress, and the impact stress, which produced the reversal stress, should be given.

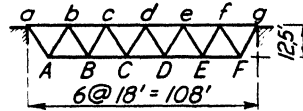
PROBLEMS

129. Find the maximum stresses in the diagonal De and in the counter dE of the Pratt truss by the exact and conventional methods of loading. The dead load is 1300 lb. per ft. of truss and the live load is 2000 lb. per ft. of truss. Take the impact factor from the A.A.S.H.O. (1924) impact formula § 147 using L as 4 panels, or 3 panels for reversal loading.

Ans. 117 and 39 kips (conv.); 108 and 30 kips (exact).



PROBLEM 129.



PROBLEM 130.

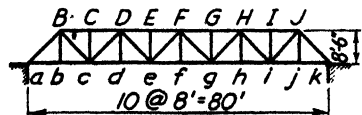
130. Find the maximum and minimum combined stresses for the diagonal cC of the Warren deck truss by the exact and by the conventional methods of loading. The dead load is 800 lb. per ft. of truss and the live load is 1500 lb. per ft. Use the A.A.S.H.O. (1949) impact formula and take L as the panel length times the number of load concentrations when using the conventional method. Answers are given without impact.

Ans. + 42.2 and - 7.8 kips (conv.); + 38.9 and - 4.4 kips (exact).

131. Obtain all maximum stresses for the Pratt truss of Problem 129 if the live loading is H -20. See Fig. 126(b). Use the exact method of loading.

132. Obtain all maximum and minimum stresses for the Warren deck truss of Problem 130 if the live load is H -20. Use the conventional loading, and follow Fig. 126(b).

133. Analyze for all maximum and minimum stresses in this low Warren truss for H -20 live loading (Fig. 126(b)). The dead load is 1280 lb. per ft. of truss. Allow for a constant impact factor of 30 per cent. Use either the exact or the conventional method of loading. Fit all data and calculated stresses into a stress table.



PROBLEM 133.

168. Influence-Line Analysis of Curved-Chord Trusses for Live Load.

The live-load analysis of a curved-chord truss is handled quite simply by the aid of influence lines. In fact, the analysis differs only slightly from the analysis of a truss with parallel chords. The live-load stresses in the *chords* can again be obtained by proportion from the corresponding dead-load stresses, provided that the entire dead load was assumed to be concentrated at the panel points of the loaded chord.

The Diagonal and the Counter. The maximum stress in a diagonal for uniform live load may be calculated either by the exact or by the conventional method. By the exact method it is perhaps simpler for a curved-chord truss to compute the actual values of the influence ordinates and to obtain the maximum live-load stress from the positive or tension area. It is important to note that the counter in a panel of a curved-chord truss does not have the same slope as the main diagonal. Hence the influence line for the stress in the counter will have different ordinates than the influence line for the stress in the main diagonal. Actually, since the same moment center may be used for calculating both stresses (Fig. 147), the difference is simply one of proportion, and the location of the point of crossing will be the same for the two influence lines.

Influence Line for Moment. We can avoid the necessity of drawing two influence lines for the diagonal and the counter by use of an influence line for moment about the point of intersection of the upper and lower chords.

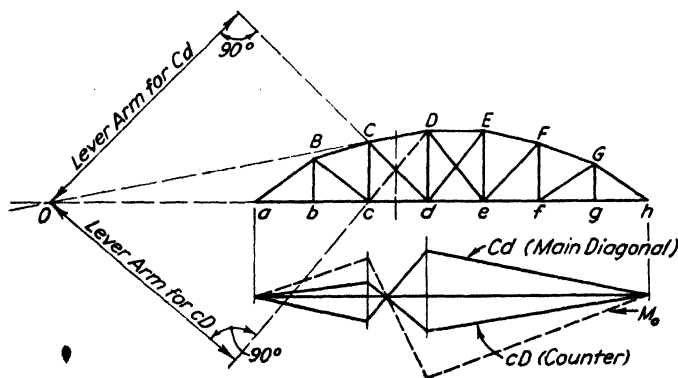


FIG. 147. INFLUENCE LINE FOR DIAGONAL AND COUNTER.

M_0 of Fig. 147. This influence line crosses the base line at the same point as the other influence lines discussed. In making the final computation for the maximum stress in the counter, we must realize that a dead-load stress exists in the counter whenever it is in operation under dead and live loading. We must also understand that this stress will not be the same as the dead-load

stress calculated for the main diagonal because of the different slope or different lever arm from the center of moments. See Fig. 147.

Maximum Compression in the Vertical. A study of Fig. 148 shows that the compression in the vertical Cc must be nearly a maximum when the tension in the diagonal Cd is a maximum ($\Sigma V=0$ at joint C). Accordingly, the maximum live-load stress in the vertical Cc is calculated by the conventional method for the same loading as the maximum stress in the diagonal Cd . For exact loading, use can be made of the negative area of the influence line for the stress in the vertical, as shown in Fig. 149(d). However, the points of crossing for this influence line and for the influence line of the main diagonal Cd are usually so nearly the same that there is little error in calculating the maximum stresses for both members from the same position of the live load.

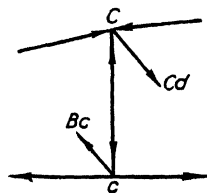


FIG. 148. MAXIMUM COMPRESSION IN A VERTICAL

169. Stress Reversal in the Verticals of Curved-Chord Trusses. One major point of difference between the analysis of trusses with curved chords and trusses with parallel chords is that a vertical of the curved-chord truss may reverse its stress from compression to tension under the passage of the live load. The tension stress usually is small, but it is of importance because the vertical then must be *designed for stress reversal* in the same manner as the diagonal of a Warren truss. This problem will be solved by the use of influence lines.

In Fig. 149 are shown the influence lines for two main diagonals, a counter, and a vertical of a curved-chord Pratt truss. Note that the influence ordinates for the vertical Cc were computed from the section 1-1 or upon the assumption that the main diagonals, Bc and Cd , were acting. Evidently the section 1-1 would cut four members if the counter cD were acting. Place a unit load at d . The ordinates to the influence lines for Bc and Cd are positive and these diagonals are in action. The same statement holds true for unit loads at e , f , and g . Hence the compressive side of the influence line for Cc represents properly the maximum live-load compression in this vertical since for this loaded length the assumption as to the action of Cd is fulfilled. However, if a unit load is placed at c , the stress in Bc is positive, but the negative ordinate at c for stress in Cd represents a compression. With live load extending from c toward the left in Fig. 149, Cd will not be in action at all, but, instead, the counter cD will be working unless the live load is too small to reverse the modified shear due to dead load. The conclusion is that the left-hand or positive area of the influence line for the vertical, which was drawn upon the assumption that the counter cD would not be in action, is usable only for loadings that keep the main diagonal in action and does not represent directly the maximum live-load tension in the vertical. As has

been explained the negative area of this influence line *does* represent the maximum live-load compression in the vertical.

Maximum Tension in the Vertical. In order to determine the maximum tension in the vertical Cc , it is necessary to observe in Fig. 148 that any tension in the main diagonal Cd produces compression in the vertical ($\Sigma V = 0$ at joint C), and, therefore, it is desirable to reduce the stress in Cd to zero. Moreover, the stress in Cd will be zero whenever the counter cD is in action.

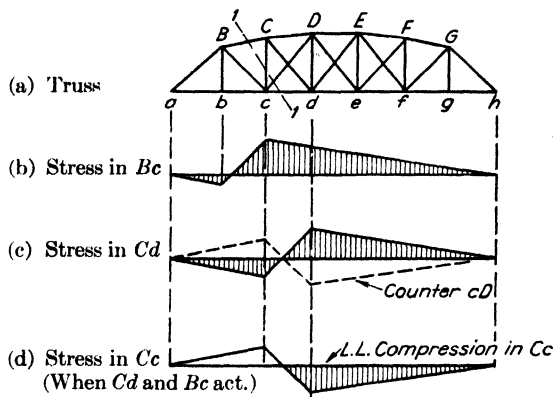


FIG. 149. EXACT MAXIMUM STRESS IN THE VERTICAL.

A further study of the joint C indicates that tension in the vertical can be caused *only* by the upward thrust of the resultant of the chord stresses, because the diagonal, a flexible tension member, can not push upward. Therefore, the chord stresses must be made as large as possible, consistent with zero stress in the diagonal. The criterion from maximum tension in the vertical follows:

RULE. *Load as much of the span as possible with the uniform live load and still keep the main diagonal Cd out of action.*

170. Balancing Areas for Maximum Tension in the Vertical. The procedure for finding the proper placing of the live load to cause maximum tension in the vertical is illustrated in Fig. 150 which represents an influence line for the main diagonal Cd of the truss of Fig. 149. This diagonal carries a known tensile stress from dead load. If the live load is moved onto the bridge from the left, there will be a point r such that the area $a-r-s$ times the uniform live load per foot will equal the dead-load stress in Cd . Since this live-load stress is compression, the resultant $DL + LL$ stress in the diagonal Cd is thereby reduced to zero. It is assumed that impact is included as a part of the live load.

The Second Condition. This loading from a to r in Fig. 150 meets one requirement of the criterion. However, the live load can be extended farther to the right toward the point c and Cd will be brought into compression. (Actually, the counter Dc is then in action.) A further extension to the right of the live load beyond the point c will tend to introduce tension into the main diagonal Cd . When this tension equals the compression represented by the area $r-s-b-c$, the main diagonal will return into operation. Hence the greatest length of span that can be covered with the live load without bringing the main diagonal Cd back into action is the length x , as shown in Fig. 150. This length is determined by making the areas $r-s-b-c$ and $c-d-t-u$ equal. Incidentally, it is not necessary that these areas be balanced with great precision. The determination of the distance x to the nearest foot is quite satisfactory. A cut-and-try process is quick and convenient. The area of the quadrilateral $c-d-t-u$ can be obtained as the difference of the two triangles $c-d-e$ and $u-t-e$.

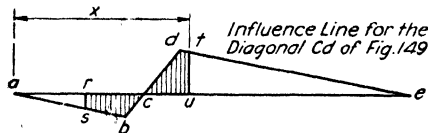


FIG. 150. BALANCING AREAS.

Computation of Tension in the Vertical. When we have found the position of the live load for maximum tension in the vertical, we could place this live load on the truss along with the full dead load and compute the final combined stress in the vertical. Since the stresses in both the diagonal and counter for the panel to the right of the vertical are zero, we will be cutting only three stressed members by passing a section through this panel and through the vertical, that is, the section 1-1 of Fig. 149. We can then determine the stress in the vertical from an equation of moments about the intersection of the two chord members cut by the section.

If the live-load stress is to be determined *separately* from the dead-load stress, as is more common, we must make the live-load analysis consistent with the dead-load analysis by assuming that the main diagonal will act in compression under live load. This assumption is necessary because this diagonal was found to be in tension under dead load, and the combined stress in the diagonal must be zero. For the same reason, the live-load tension in the vertical can be found from the net area taken from an influence line for the vertical such as Fig. 149(d) for a length x determined as in Fig. 150.

171. Special Cases of Tension Verticals. An unusual case occurs at times. It is found that the stress in the main diagonal can not be reduced to zero, and yet there is a possible tension in the vertical. For such a case, it is necessary to load the entire negative side of the influence line for the main diagonal. This loading will come as near as possible to reducing its dead-load stress to zero. Then, with this live loading, the live-load tension in the vertical can be calculated and can be combined with its dead-load compression.

Dead-Load Tension. The vertical member at the center of a truss with an *even number of panels* and a sloping top chord may reverse to tension, but the tension can not exceed a panel of live load plus dead load; and it may not be that large. This maximum tension occurs when the bridge is fully loaded with live load and dead load. A vertical next to the center panel of a truss with an *odd number of panels* acts similarly to the center member considered above. Its maximum tension also occurs under full live load and dead load. The reader can verify these statements by a study of the vertical components of the forces meeting at the top and at the bottom of these vertical members, for example the members *Dd* in the curved-chord trusses of Problems 127 and 135.

172. Example of the Analysis of a Curved-Chord Truss for Uniform Live Load. The stresses from live loading are worked out on the calculation sheet, Fig. 151, for a simple curved-chord Pratt truss of 150-ft. span. The truss used is not a practical type because it is too deep for the span, but it illustrates the analysis of curved-chord trusses in a simple manner. The stresses from dead load are given without comment. The assumed loads are 500 lb. per ft. of truss for the dead load, to be applied entirely to the lower chord, and 1250 lb. per ft. of truss for the live load, including impact.

Maximum stresses from live load are obtained by the conventional and by the exact methods. The reversal stress (tension) in the vertical is obtained from an influence-line analysis. The determination of combined stresses is left to the reader, since it is simply a matter of algebraic addition. Note the unusual tension stress in *Cc* caused by dead load. Another unusual stress is found in the diagonal *Cd* of the truss of Problem 127 where the *DL* stress in *Cd* is compression due to the reversal in direction of the modified panel shear because of the steep slope of the upper chord.

173. Graphical Analysis of a Curved-Chord Truss for Live Loading. The graphical analysis of a curved-chord truss for dead load was discussed in § 144 and was illustrated by the analysis of a Warren truss in Fig. 123. The live-load analysis for trusses having web members of many different slopes is also conveniently performed by the graphical method. The live-load analysis of the web members of a Pegram truss by graphics is illustrated in Fig. 152. The stress diagram for dead load is also shown. The stresses in chord members produced by the live loading can be obtained by proportion from the corresponding dead-load stresses.

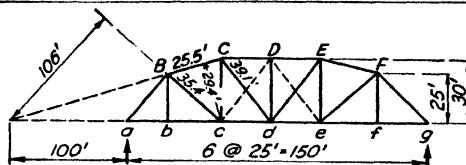
Conventional Loading. The maximum stress in any web member* is obtained for the conventional method of loading by the use of full-panel live loads at all joints to the right of the member under consideration. Therefore the stress in the member itself may be thought of as produced by a *single negative load to the left of the section*, i.e., the left-hand reaction of the truss. We will draw a stress diagram for the truss as caused by a reaction of 1000 lb.

* Here, as elsewhere, we are discussing web members on the left-hand side of the truss.

CALCULATION SHEET — L.L. ANALYSIS OF WEB — CURVED CHORD TRUSS

Loading —

Dead Load: 500 lb./ft.
Live Load: 1250 lb./ft.
(including impact)



Maximum L.L. Stresses in Web Members Conventional Loading —

Members Dd and De (Loads at e and f)

$$Dd = R_1 = \frac{31.2(1+2)}{6} = -15.6 \text{ k}$$

$$De = 15.6 \frac{39.1}{30} = +20.3 \text{ k}$$

Member Cd (Loads at d, e, and f)

$$R_1 = \frac{31.2(1+2+3)}{6} = 31.2 \text{ k}$$

$$Cd = 31.2 \frac{39.1}{30} = +40.7 \text{ k}$$

Member Cc (Loads at d, e, and f)

$$\Sigma M_0 = 31.2 \times 100 = 3120$$

$$Cc = \frac{3120}{150} = -20.8 \text{ k}$$

Member Bc (Loads at c, d, e, and f)

$$\Sigma M_0 = 100R_1$$

$$= 100 \frac{31.2(1+2+3+4)}{6} = 5200$$

$$Bc = 5200/106 = +49.1 \text{ k}$$

Exact Maximum L.L. Stresses in Web from Influence-Line Areas —

$$Bc = 1.25 \times 36.3 = +45.3 \text{ k}$$

$$Cd = 1.25 \times 29.3 = +36.6 \text{ k}$$

$$De = (-Cd) = 1.25 \times 13.1 = +16.4 \text{ k}$$

$$Dd = -16.4 \times \frac{30}{39.1} = -12.6 \text{ k}$$

$$Cc = 1.25 \times 14.1 = -17.6 \text{ k}$$

Maximum Tension in Vertical Cc — The D.L. stress of +8.1 kips in Cd is balanced by an equal L.L. compression when the L.L. extends from a to r (Area a-r-s on influence line for Cd). Then area r-s-b-c is balanced by an equal area c-d-t-u. The stress in Cd under full D.L. and with L.L. extending from a to u is zero, and the tension in Cc is a maximum. The distance a-u is 77.5 ft.

$$\Sigma M_0 = (77.5 - 15.6)100$$

$$= 31.2(125 + 150) = 2970$$

$$Cc = \frac{2970}{150} = +19.8 \text{ k (L.L. only)}$$

Check from area of influence line for Cc.

$$Cc = 1.25(18.3 - 2.4) = 19.8 \text{ k}$$

Maximum tension (D.L. + L.L.) for

$$Cc = 19.8 + 2.1 = +21.9 \text{ k}$$

(a) Truss Diagram

(b) Dead-Load Stresses

(c) Live-Load Chord Stresses by Proportion

(d) Maximum Live-Load Web Stresses (Conventional Loading)

(e) Maximum Live-Load Web Stresses (Exact Loading)

(f) Influence Line for Stress in Bc

(g) Influence Line for Stress in Cd

(h) Influence Line for Stress in Cd

(i) Live Loading for Maximum Tension in the Vertical Cc

(j) Influence Line for Cc when Cd acts

Comment — Conventional stresses are from 3 to 4 kips larger than the exact stresses for the web members.

FIG. 151.

at the left end. Then the true maximum stress in any particular web member produced by the live loading will be the stress taken from this diagram when it is multiplied by the value of the reaction in *kips* produced by the conventional live loading for this particular member. Reversal is handled as usual by an investigation of the stress in the corresponding member on the right-hand side of the truss with live loading to the right of the section.

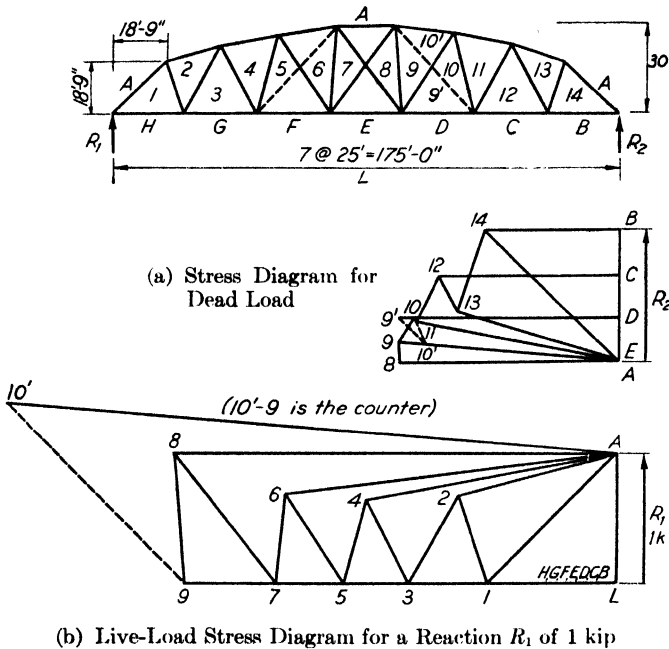


FIG. 152. GRAPHICAL ANALYSIS OF A PEGRAM TRUSS FOR CONVENTIONAL LOADING.

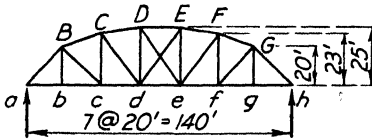
EXAMPLE OF THE PEGRAM TRUSS. The layout of the Pegram truss of Fig. 152 is of interest. The panel points of the upper chord lie on the arc of a circle whose radius is 270 ft. The distance horizontally from the reaction to the hip joint is made three-quarters of a panel length or 18 ft.-9 in. The members of the upper chord are all of the same length. The radius of curvature of the upper chord is chosen so as to make the posts practically of the same length. In this particular truss the height was set at 30 ft.

PROBLEMS

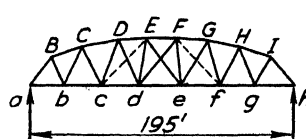
134. Investigate the need for a counter in panel *cd* of the truss of Fig. 122. Use a dead load of 800 lb. per ft. of truss and a live load of 1500 lb. per ft. including impact. Use the exact method of loading. *Ans.* Combined stresses are +15,000 and -42,200 lb.

135. Calculate by means of the conventional method all maximum stresses for the web members of this curved-chord Pratt truss. The dead load is 1000 lb. per ft. of truss and the live load is 2000 lb. per ft. including impact. Study possible reversal and the need for a counter in the panel cd .

Ans. Combined stresses for Cd are $+71,300$ and $-16,200$ lb.



PROBLEM 135.



PROBLEM 138.

136. Determine the maximum tension in the verticals Cc and Dd of the truss of Problem 135. Make use of influence lines.

Ans. The maximum stresses for Dd are $+28,800$ and $-12,300$ lb.

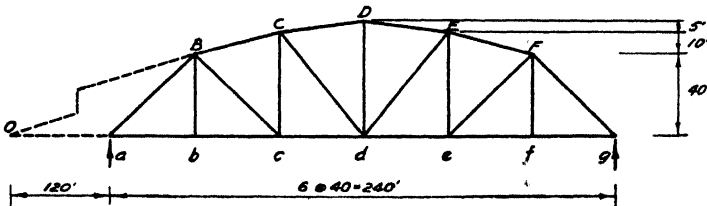
137. Use the conventional method to compute the maximum live-load stresses in the truss of Fig. 123. The live load is 1200 lb. per ft. of truss. Check your work by recalculating the stresses graphically.

138. Analyze graphically for the maximum conventional live-load stresses and also for the dead-load stresses in this Pegram truss which is laid out as follows: The span of 195 ft. is divided equally into seven panels. The horizontal distance from the reaction to the hip joint is three-quarters of a panel length. The end post has a slope of 45° . The upper-chord panel lengths are equal. The panel points lie upon the arc of a circle of 300-ft. radius. The dead load is 1200 lb. per ft. of truss and the live load is 1800 lb. per ft. including impact.

139. Write a brief discussion explaining how to place a single concentrated wheel load on the curved-chord Pratt truss of Problem 135 to produce maximum stress successively in each member. Include counters, and the minimum stress or tension in Cc and Dd . Illustrate by sketches of influence lines where desirable.

140. Investigate the truss of Fig. 151 and find the maximum tension in the vertical Cc caused by the $H-15$ loading. Use the loading as specified for moment, that is, a concentrated load of 13,500 lb., plus a uniform load of 480 lb. per ft. Neglect impact.

Ans. $+16,700$ lb.



PROBLEM 141.

141. Compute stresses in member Cc of the truss illustrated assuming that there is no counter in the panel cd . $DL = 2000$ lb. per ft.; $LL + I = 3000$ lb. per ft. plus a 30-kip floating concentration.

Ans. DL stress $= 24k.T$; $LL + I$ stress $= 114k.T$ or $69k.C$.

TRUSSES WITH SUBDIVIDED PANELS AND MULTIPLE WEB SYSTEMS

174. Trusses with Subdivided Panels. For reasonable economy, the slope of a diagonal should be only a little more than 45° with the horizontal. In order to maintain this slope, the panel length must be increased as the span and depth of the truss are increased.



(a) Baltimore Truss with Sub-Ties



(b) Pettit Truss with Sub-Struts



(c) Pettit Truss with Broken End Post

FIG. 153. TRUSSES WITH SUB-DIVIDED PANELS.

However, the *cost of the floor system* becomes excessive when the panel length considerably exceeds 25 ft. The use of a truss with subdivided panels, as shown in Fig. 153, overcomes both of these difficulties.

Truss Types. The three trusses shown in Fig. 153 include the Baltimore type and the Pettit types. Actually, they are respectively parallel-chord and curved-chord Pratt trusses with subdivided panels. These trusses may be built either with sub-ties, as shown in (a), or with sub-struts, as shown in (b). The Pettit truss of (c) has a broken end post, which gives

the end member a sharper slope and reduces its stress.

175. Dead-Load Stresses in the Baltimore Truss. The analysis of the Baltimore truss will be studied first. See Fig. 154. The action of the sub-members is explained easily. Members *Bb*, *Dd*, and *Ff* simply act as *hangers* to bring the panel loads at *b*, *d*, and *f* to the truss joints *B*, *D*, and *F*. The sub-struts *Bc*, *Dc*, and *Fe* prevent the floor-beam loads from bending the main diagonals *aC*, *Ce*, and *Eq*. Sub-ties from *D* to *E* and from *F* to *G* would perform the same function as the sub-struts, but the sub-ties would act in tension.

EXAMPLE. Sub-Strut and Sub-Tie. From the isolated joint *D* shown in Fig. 154(b), it can be shown that the vertical component of the stress in the sub-strut *Dc* is one-half of the panel load at *d*. Take moments of all forces about *e*. Two forces, the stresses in the members *DC* and *De*, pass through the center of moments. Also, the horizontal component *H* of the stress in the sub-strut *Dc* passes through *e*. It follows that the vertical component of the stress in the sub-strut *V* must be one-half of the load *P_d*, because the two forces must produce equilibrium by equal and opposite moments about *e*, and their lever arms are in the ratio of two to one. The proof that the stress in a sub-tie is a tension that has a vertical component equal to one-half of the stress in the hanger is nearly identical. See Fig. 154(c). Note that this proof assumes that the top chord is horizontal.

Diagonals and Posts. The analysis of the other members of the truss is as follows: The diagonal members *aB*, *De*, and *Fg* must have vertical components equal to the respective panel shears. Note that the shear in the panel *ef* is not the same as in the panel *fg*. The

stresses in the members BC , CD , and EF must be calculated from *modified panel shears*. The calculation of the stress in CD is illustrated by (d). The total shear is $R_1 - P_1 - P_2$. The vertical component of the stress in CD has been shown to be $P_3/2$. Accordingly, the modified shear in the panel, which must be resisted by CD , is equal to $R_1 - P_1 - P_2 - P_3/2$. The stress in CD is this modified shear times the length Ce divided by the height of the truss. Attention is called to the fact that the modified shear in the panel bc for use in calculating the stress in the diagonal BC is greater than the true shear, because the stress in the member Bc acts up as shown in (e). The stresses in the posts Cc , Ee , and Gg are obtained from the equation $\Sigma V = 0$ at corresponding upper-chord joints.

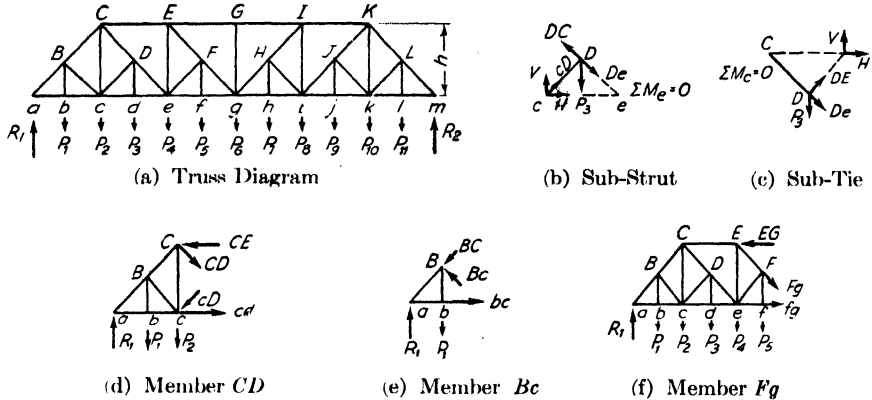


FIG. 154. DEAD-LOAD ANALYSIS OF A BALTIMORE TRUSS.

Chord Members. The dead-load stresses in the chord members of the Baltimore truss are calculated just as for any truss with horizontal chords. For the stress in CE , cut a section through the panel de and take moments about e . Similarly, the point g is the center of moments for the member EG . The stress in fg is equal to the moment about E , for the forces shown in Fig. 154(f), divided by the height of the truss. One force, P_6 , is to the left of the section, but to the right of E . It must be included when the moment about E is computed. The stress of ef is equal to the stress in fg (horizontal equilibrium at joint f).

176. Conventional Analysis of the Baltimore Truss for Live Loading. The truss is loaded over its entire length for maximum chord stresses. The stresses in a sub-vertical and a sub-strut are obtained from a single panel load applied at the bottom of the sub-vertical. The maximum stress in the lower half of any diagonal, such as aB , De , Fg of Fig. 155, is obtained as for any ordinary diagonal by loading all panel points to the right of the section. Similarly, the stress in the counter GH is a maximum when all panel points from h to l inclusive are loaded. The member HI then operates as a sub-tie and gH is not operating. The maximum live-load stress in a vertical such as Ee is the V component of the maximum live-load stress in the adjacent diagonal. The maximum tension in the end vertical Cc is equal to two panel loads. This stress occurs under full live loading. $\Sigma V = 0$ at c .

EXAMPLE. The Upper Halves of the Main Diagonals. The stresses in these members are controlled by the *modified panel shears*. The total shear in the panel ef , for instance, is reduced by the vertical component of the stress in the sub-strut eF to obtain the modified shear. However, if no load is placed at f , there is no stress in eF , and the member EF must resist the entire shear in the panel. Shall the truss be loaded with full panel loads from m to f , or only to g , for maximum live-load tension in EF ? Place a panel load at f . The shear in the panel ef is $\frac{1}{2}$ of the panel load. The V component for the sub-strut eF is $\frac{1}{2}P$. The modified shear is $\frac{1}{2}P - \frac{1}{2}P$ or 0 of a panel load. Hence the load at f increases the modified panel shear and increases the tension stress in EF . Full panel loads should be placed at the joints l to f inclusive. We can now show that all panel points must be loaded for maximum stress in both aB and BC .

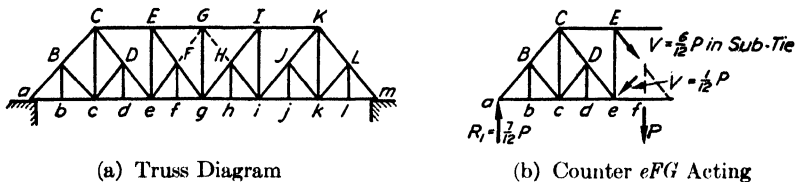


FIG. 155. LIVE-LOAD ANALYSIS OF A BALTIMORE TRUSS.

Counter Diagonals. The action of the counters deserves separate consideration. When the counter $G-F-e$ is in operation, the sub-strut becomes a part of the counter. The member Fg is a tension member that cannot act as a sub-strut. Instead the member EF must act as a sub-tie. The panel loads to produce maximum stress in the upper half of the counter or FG clearly should be placed from b to f inclusive, because this member resists the entire shear in the panel fg when the member Fg is out of operation. Shall the joint f also be loaded for maximum tension in eF when the member EF acts as a sub-tie? Place a panel load at f and isolate the section to the left of f , as shown in Fig. 155(b). The shear in the panel ef is $\frac{1}{2}$ of a panel load. The stress in the sub-tie EF has a downward vertical component of $\frac{1}{2}$ of a panel load. The result is to produce a compression in the member eF having a downward vertical component of $\frac{1}{2}$ of a panel load; hence, joint f is not to be loaded.

RULE. An easy rule to remember is to load the sub-panel and all joints to the right with live load to produce maximum tension in the upper half of the main diagonal, and to load *all other joints* to produce maximum tension in the lower half of the counter. The lower half of the counter then becomes a *reversal member* and must be designed as such.

177. The Pettit Truss. We must combine our ideas of analysis of the curved-chord Pratt truss and the subdivided Baltimore truss to accomplish the analysis of the Pettit truss. In reality, the Pettit truss is a subdivided Pratt truss with a curved or rather a polygonal upper chord. The Pettit truss will be studied with sub-ties, as shown in Fig. 156(a). The stress in the sub-tie does *not* have a V component equal to one-half of a panel load. By taking moments about E of Fig. 156(b), we find this V component to be $Pa_1/(a_1 + a_2)$ where a_2 is not a panel length.

EXAMPLE. Chord Members. The chord stresses are calculated for full dead and live loading as for any curved-chord truss. As an example, the stress in cd is found by cutting the section 1-1 through the panel cd and by taking moments of all forces to the left of the

section about the joint C . The stress in the upper-chord member CE is found from the same isolated part of the truss by taking moments about the joint e . Note particularly that the load P_3 lies to the *left* of the center of moments at e , but it still must not be considered when taking moments, for it lies to the *right* of the section 1-1.

Web Members. After computing the stresses in all chord members, we can obtain the dead-load stresses in the web members by the method of modified shears. In the panel cd , for instance, the shear that causes stress in CD is the panel shear ($R_1 - P_1 - P_2$) minus the vertical component of the stress in the upper-chord member CE . The shear resisted by the diagonal member De is the shear in the panel de ($R_1 - P_1 - P_2 - P_3$) minus the vertical component of the stress in the upper-chord member CE plus the vertical component of the stress in the sub-tie DE . The stresses in the verticals may be obtained with least difficulty by use of the equation $\Sigma V = 0$ at panel points of the lower chord.

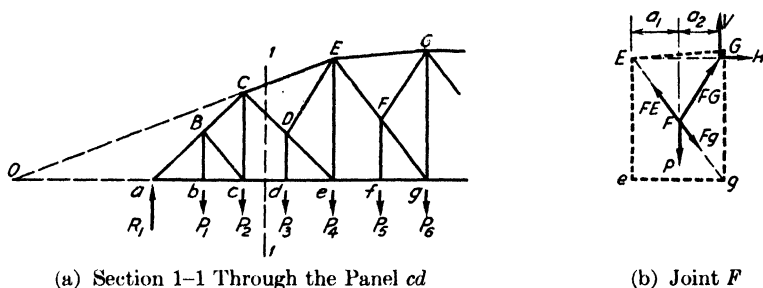


FIG. 156. ANALYSIS OF A PETTIT TRUSS.

The stresses in the web members from live load may be calculated by the conventional method for the same positions of the loads that gave maximum stresses for the Baltimore truss. The method of modified shears is replaced by the method of moments with centers of moments at points such as O in Fig. 156(a). Influence lines can be obtained with little difficulty if an exact analysis is desired. There is less possibility of tension occurring in the vertical of a Pettit truss with sub-ties, because the downward pull of the sub-tie tends to keep the vertical in compression. However, the procedure of determining the maximum tension in the vertical would be essentially the same as the procedure developed in § 169.

178. Trusses with Multiple Web Systems. The early attempts to lay out diagonals of an *economical slope* (about 40° with the vertical) and still to retain short panel lengths for long spans, resulted in trusses with multiple web systems as shown in Fig. 157. Such trusses are seldom if ever used as bridge structures today, but occasionally the engineer must analyze one of these old structures in order to make recommendations for strengthening it. Many old trusses have been strengthened at small expense since the development of structural welding.* The lattice frame is used as a light welded truss in modern building construction and also for aircraft.

* Frank B. Doolittle, *Bridge capacity increased from 12 to 30 tons*, Engineering News-Record, December 7, 1933, p. 683.

W. R. Roof, *Welded material strengthens old railway truss bridge*, Engineering News-Record, August 4, 1927, p. 176.

Subdivision. All trusses shown in Fig. 157 are in fact indeterminate structures. However, their analysis by statics is made possible by certain reasonable assumptions. As will be explained, the common assumption is that the truss may be divided up into two or more *component trusses* with the same chords but with different web systems. The loads also are divided and placed upon these component trusses. Then the stress in a web member is determined as its stress calculated in the truss of which it is a part. The chords are a part of all component trusses; hence the stress in a chord member is obtained by adding its *partial stresses* from each component truss.

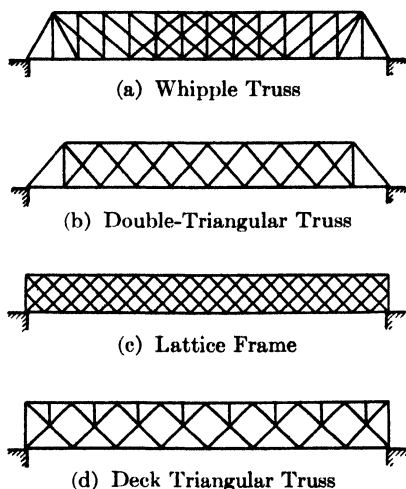


FIG. 157. MULTIPLE-WEB TRUSSES.

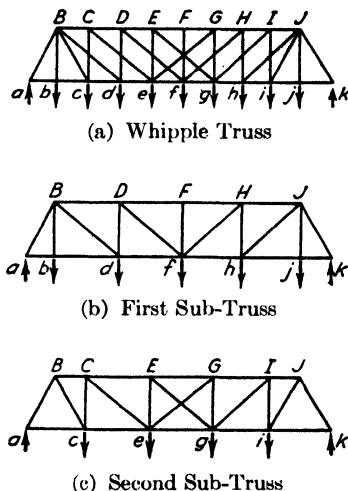


FIG. 158. THE WHIPPLE TRUSS.

179. The Whipple Truss. This truss was introduced by Squire Whipple about 1850 and was widely used for many years. The loads in Fig. 158 are shown distributed to the panel points of the component trusses. We can calculate the stresses in each component truss and combine these partial stresses for members that form a part of each sub-truss. A slight ambiguity exists, for we are not certain whether the member Bb and the load at b belong to the truss of (c) or whether they should be shown on the truss of (b). Perhaps the most reasonable solution is to show the member Bb in both trusses and to divide the load at b equally between the two trusses. The final stresses will not be changed greatly.

180. The Lattice Truss or Girder. This is the only multiple-web structure that is commonly used today. As a light welded truss for carrying relatively small loads around industrial shops, the lattice girder is both stiff and economical. For this reason it is also used in aluminum for airplane construction.

The usual form of lattice girder has the double web system shown in Fig. 157(d), but the quadruple web system shown in (c) is also used. The analysis of the latter truss is illustrated by Fig. 159. The quadruple-web girder is shown in (a) and the component trusses are shown in (b) to (e) inclusive. This figure shows that each component part is a Warren truss with diagonals of the same slope (frequently 45°).

EXAMPLE. Full Loading. If loads occur at each panel point of Fig. 159(a), it is a simple matter to place the proper loads on the component trusses, as shown in the sketches, and to compute all stresses. The member Ac , for instance, has its stress computed from (c), while the stress in the member aC must be found from (b). All web stresses are handled in the same manner. The stress in the short diagonal OB is the same as that of the main diagonal Bd . The same relationship exists between Ob and bD . The stress in a chord member such as ab is found by summing the proper stresses from the component trusses. As an example, the stress in ab is found by adding the following stresses: ae from (b), ac from (c), ad from (d), and ab from (e). The same reasoning is applied to the determination of stresses in other chord members.

Partial Loading. If loads do not occur at all panel points of the lattice truss, it is difficult to determine how to load each of the component trusses. Concentrated static loads frequently occur on building trusses. An analogy between the lattice girder and the rolled beam or the plate girder is of use in developing a reasonable method of analysis. The web system of the lattice girder replaces the web of the plate girder, while the chords serve the same purpose as the flanges of the plate girder. It follows that the vertical component of the stress in a web member is the vertical shear at the section divided by the minimum number of diagonals cut by a vertical or sloping section. (Diagonals are assumed able to resist compression as well as tension.) The chord stress between diagonal connections is approximately equal to the moment about the adjacent panel point toward the center of the truss (larger moment) divided by the depth between the chords.

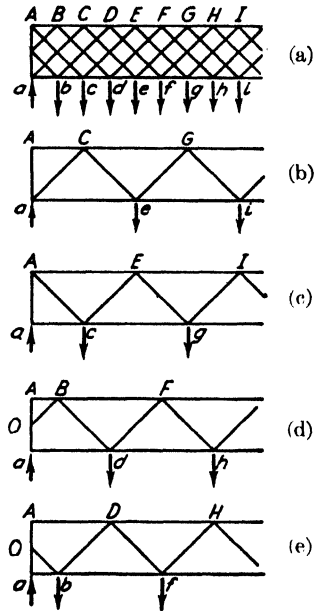


FIG. 159. REDUCTION OF A LATTICE FRAME.

181. Example of the Analysis of a Double Triangular Truss for Dead Load.

This truss may be broken down into two Warren-type trusses, as shown in Fig. 160. Panel dead loads at lower-chord panel points will be taken as 25 kips with 12 kips at each panel point of the upper chord. The loads at B and H (hip joints) will be divided equally between the two component trusses. The method of sections will be used for the analysis, although the method of joints is equally convenient.

TRUSS OF FIG. 160(b).

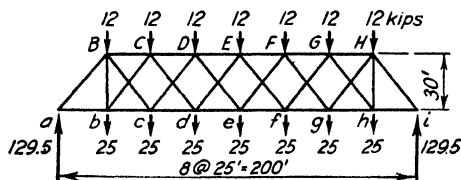
$aB = 55.5 \times 1.3 =$	72.1 kips
$Bc = 49.5 \times 1.3 =$	64.3
$cD = 24.5 \times 1.3 =$	31.9
$De = 12.5 \times 1.3 =$	16.3
$BD = (55.5 \times 50 - 6 \times 25) \div 30 =$	87.6
$DF = (55.5 \times 100 - 25 \times 50 - 6 \times 75 - 12 \times 25) \div 30 =$	118.3
$ac = (55.5 \times 25) \div 30 =$	46.2
$ce = (55.5 \times 75 - 25 \times 25 - 6 \times 50) \div 30 =$	108.2

TRUSS OF FIG. 160(c).

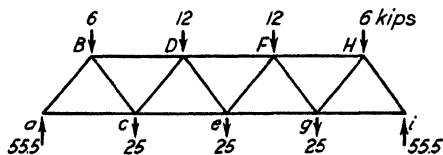
$aB = 74 \times 1.3 =$	96.2 kips.
$Bb = 68 \times 1.0 =$	68.0
$bC = 43 \times 1.3 =$	55.8
$Cd = 31 \times 1.3 =$	40.3
$dE = 6 \times 1.3 =$	7.8
$BC = (74 \times 25) \div 30 =$	61.6
$CE = (74 \times 75 - 31 \times 50 - 12 \times 25) \div 30 =$	123.3
$ab = (74 \times 25) \div 30 =$	61.6
$bd = (74 \times 50 - 31 \times 25) \div 30 =$	97.4
$df = (74 \times 100 - 31 \times 75 - 12 \times 50 - 25 \times 25) \div 30 =$	128.4

FINAL COMBINED STRESSES FOR CHORDS.

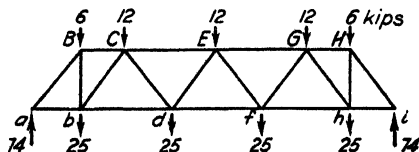
$aB = -(96.2 + 72.1) =$	-168.3 kips.
$BC = -(61.6 + 87.6) =$	-149.2
$CD = -(123.3 + 87.6) =$	-210.9
$DE = -(123.3 + 118.3) =$	-241.6
$ab = 61.6 + 46.2 =$	107.8
$bc = 97.4 + 46.2 =$	143.6
$cd = 97.4 + 108.2 =$	205.6
$de = 128.4 + 108.2 =$	236.6



(a) Truss with Loads



(b) First Sub-Truss



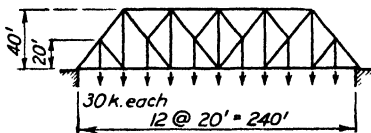
(c) Second Sub-Truss

FIG. 160. DEAD-LOAD ANALYSIS OF A DOUBLE TRIANGULAR TRUSS.

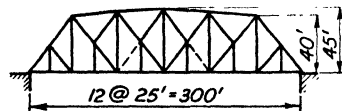
182. Live-Load Analysis of Multiple-Web Trusses. As has been shown, all trusses with multiple web systems can be reduced to two or more component trusses with simple web systems. Each of these component trusses can be analyzed for maximum live-load stresses in the web members for its proportionate part of the panel live-load concentrations. The stresses in the chords from live loading may be obtained from the corresponding dead-load stresses by proportion, if the entire dead load was placed at *panel points of the loaded chord*. If not, a separate analysis of the chord stresses for full live loading may become necessary. The exact live-load analysis of a truss with multiple web system by influence lines is rather complicated and is not widely enough used to be included here.*

PROBLEMS

142. Obtain the dead-load stresses in this Baltimore truss. The panel loads are shown on the drawing.



PROBLEM 142.

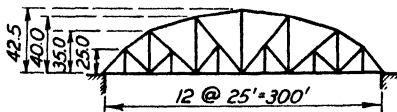


PROBLEM 143.

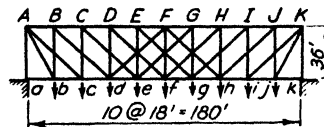
143. Obtain the dead-load and conventional live-load stresses for this truss and combine for maximum. $DL = 1500$ lb. per ft. of truss; $LL = 2000$ lb. per ft. of truss including impact.

144. Investigate by use of influence lines the need for a counter (not shown) in one of the center panels of the Pettit truss. The dead load is 1400 lb. per ft. of truss and the live load is $H-20$ increased for 30% impact. Check by the conventional method.

Ans. Upper counter stress (conv.) = +75 kips.



PROBLEM 144.



PROBLEM 145.

145. Find the maximum live-load stresses by the conventional method for the members CD , Ce , and Dd of the Whipple truss for a uniform live load of 1200 lb. per ft. of truss.

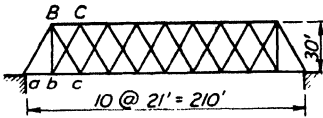
146. Find the maximum live-load stresses by the conventional method for all members of the first two panels of this double triangular truss. The live load is 1000 lb. per ft. of truss.

147. Find the stresses in the lattice frame for the loads as shown. Note that there are small intra-panel stresses to be added to the main stresses.

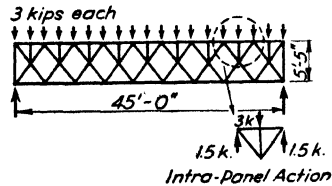
* An excellent discussion occurs in *Modern Framed Structures*, Part 1, Johnson, Bryan, and Turneaure (Wiley).

148. Find the important stresses in the lattice girder for the loads shown. Assume that the stiff web members are of one size and that the chords are changed in section at the quarter points of the span. Use the beam theory.

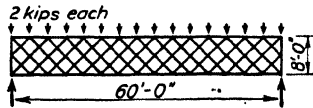
Approx. Ans. { Maximum web stress = 5 kips.
 Maximum chord stress = 28 kips.



PROBLEM 146.



PROBLEM 147.



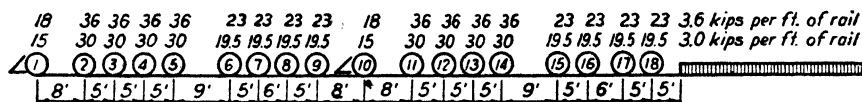
PROBLEM 148.

183. Moving Loads in other Fields. A study of the influence of wheel loads on highway and railway bridges may be useful in other fields. Many industrial structures such as cranes, loading frames and ore bridges carry moving loads. The principles developed here for placing live loads to produce maximum stresses in beams and trusses have direct applications to such industrial structures and machines. Another important application is to airplane structures. There the moving loads are produced by shifting centers of pressure as the plane is maneuvered, but the problem of obtaining the most severe load condition for each member is closely related to the problem of placing live loads on a bridge truss.

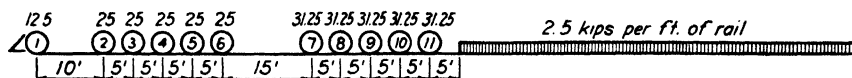
CHAPTER 7

TRUSS BRIDGES — RAILWAY LOADINGS

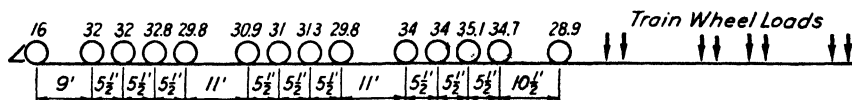
184. Locomotive Loadings. The actual loads on railway bridges are caused by the wheel reactions of one or two locomotives followed by the tender and train. Some railroads design their own bridges for the concentrations of their heaviest locomotives. This procedure is desirable if the railroad has unusually heavy or unusually long locomotives, as is true for certain roads



(a) Cooper's *E*-60 and *E*-72 Loadings



(b) Steinman's *M*-50 Loading



(c) Triple-Articulated Mallet—Erie R.R.

FIG. 161. LOCOMOTIVE LOADINGS.

Notice that the heavier wheel loads of the Mallet engine in (c) are equalized in part by the closer spacing of wheels in the conventional loadings of (a) and (b). However, the heavier wheel loads would influence directly the design of stringers and floor beams.

that still operate over heavy grades. The more common procedure is to design railway bridges for a conventional system of locomotive concentrations followed by a uniform load to represent the train.

Conventional Loadings. Two types of conventional locomotive loadings are in use, the Cooper *E*-loading and the Steinman *M*-loading. The Cooper *E*-60 loading is practically equivalent to the Steinman *M*-50 loading, as shown in Fig. 161. The *E*-72 loading is specified by the American Railway Engineering Association for the design of railway bridges. The *E*-50, *E*-60, or *E*-72 loading may be obtained by direct proportion according to the number of the loading. *The wheel spacing remains fixed.* The same is true of the Steinman *M*-loadings.

Actual Locomotive Loads. It is of interest to compare the conventional loadings with the heaviest modern locomotives and train loads. The double-header *E-72* loading, which has become the standard loading for railway-bridge design, weighs 1,022,000 lb. for an overall length of 117 ft., or 8750 lb. per ft. The *M-60* load is 705,000 lb. for an overall length of 78 ft. or 9040 lb. per ft. At the time when Steinman introduced his *M-loading** (1923), the heaviest locomotives were as follows:

RAILROAD	ENGINE TYPE	WEIGHT, LB.	LENGTH, FT.	WEIGHT, LB. PER FT.
Pennsylvania	2-10-2 + tender	651,000	96	6780
Virginian	2-10-10-2 + tender	875,000	109	8030
Erie	2-8-8-2	864,000	105	8230

Since 1923, heavier locomotives have been built for freight service over mountain grades or as passenger diesels, as, for instance, the following:

Southern Pacific ^a	4-8-8-2 + tender	906,000	120	7550
Northern Pacific ^a	2-8-8-4 + tender	1,116,000	125	8920
Union Pacific ^b	4-10-10-4 + tender	1,209,000	133	9100
Electromotive Diesel	4 coupled units	1,320,000	267	4950

^a Engineering News-Record, February 28, 1929, p. 351.

^b For Rocky Mountain service, built 1941.



FIG. 162. MODERN DIESEL LOCOMOTIVE.

The Electromotive diesel locomotive of *EA7* classification has 6 wheel trucks with 27.5 kips load on each wheel. Wheel spacing for four units in feet is: 7-7-29-7-7: 13: 7-7-29-7-7: 13: 7-7-29-7-7: 13: 7-7-29-7-7. The four units are 267 ft. in length for a total weight of 1,320,000 lb. or only 4950 lb. per ft. of track.

Equivalent Conventional Loadings.

The data presented above show that the *E-60* loading is inadequate to represent the heaviest modern locomotives because the average load of 7280 lb. per ft. is too light. The *E-75* loading at 9100 lb. per ft. would seem indicated. The *M-60* loading actually approaches closer to the wheel arrangement and heavier wheel loads of the modern locomotive, and for short spans its use is recommended. However, it covers too short a length (about 78 ft.) to represent adequately the effect of a modern long locomotive (125 ft.) or of a double header (150 to 180 ft.) upon a span of com-

* D. B. Steinman, *Locomotive loadings for railway bridges*, Transactions, A.S.C.E., 1923.

parable length. Hence, the use of *E-72* or *E-75* loading seems preferable. If the diesel locomotive completely replaces the steam locomotive, bridges might be designed for an average load of only 5000 lb. per foot as is clarified by the caption to Fig. 162.

Train Loads. The maximum train load is produced by ore cars, gun cars, or by special cars around steel mills. *Ore trains* are the most common heavy train loads. They seldom exceed 6000 lb. per ft. of track, but they have been known to approach 7000 lb. per ft. There seems to be no reason to anticipate



FIG. 163. HEAVY RAILWAY-AXLE LOADING.

The Industrial Brownhoist wrecking crane shown weighs 460,000 lb. It can pick up a 210,000-lb. load at either end. When the crane is lifting this load, its reaction on eight wheels at one end is about 520,000 lb. or 130,000 lb. per axle. This load might be serious in its effect upon a bridge floor designed for the *E-60* loading which is 60,000 lb. per axle. However, the crane would not produce a heavy impact stress. A.R.E.A. (1949) specifications require two axle loads of 90,000 lb. 7 ft. apart to represent such loadings for design purposes.

greater loads than these except in special locations, such as within the privately owned yard of a steel mill, where cars of unusual capacity may be found in use. A large axle concentration is produced by a locomotive crane (Fig. 163).

185. Simplified Loadings — Locomotive Excess. Many suggestions have been made for a simpler loading to replace the Cooper loadings. One suggestion has been to use the uniform train load plus one, two, or three *floating concentrations*. These concentrations would account for the large floor-beam reactions produced by the locomotive. In order to represent exactly the effect of the locomotive, the excess concentrations are found to vary with the span; hence they are not particularly convenient.

186. Equivalent Uniform Live Loads. Uniform live loads may be used to reproduce the effect of an *E*-loading, an *M*-loading, or of any particular locomotive loading upon *any part of the structure*. Naturally, the value of the uniform load would vary with the type of locomotive loading, but we might expect to find a single uniform load that would replace completely a particular engine loading. It will be shown that even this is impossible.

Maximum Moment at the Mid-Span. Suppose that the *E-72* loading is correctly placed upon a beam of *given length* to produce maximum moment at the center. (The method of placing wheel loads to produce maximum moment will be studied later.) This maximum moment M_c can then be calculated and set equal to the center moment produced by a uniform live load. This gives

$$(1) \quad M_c = \frac{1}{8} w_c L^2, \quad \text{or} \quad w_c = \frac{8M_c}{L^2}.$$

Since the heavy wheel loads will be relatively more severe on a short span (where they may cover nearly the entire span) than upon a longer one, it is clear that the value of w_c will vary with the span length.

Maximum Moment at the Quarter Point. Again, the *E-72* loads may be placed upon the span to produce maximum moment at the quarter point, M_q . This moment will be set equal to the quarter-point moment as produced by uniform load. Thus we obtain

$$(2) \quad M_q = \frac{3}{32} w_q L^2, \quad \text{or} \quad w_q = \frac{32M_q}{8L^2}.$$

Similarly, every span length within the normal range will have a value of w_q corresponding to the *E-72* loading. If this process is repeated for the $\frac{1}{8}$ point and the $\frac{3}{8}$ point of each span, sufficient data will be accumulated to develop a table of equivalent uniform loads, or to plot curves if they are preferred. Table 3 is a highly abbreviated table of equivalent uniform loads for the *E-72* locomotive loading. However, the variation between successive lines or columns of Table 3 is so small that interpolation cannot cause a serious error. In the final column is the value of the uniform loading that will produce the same end reaction as the *E-72* locomotive loading.

187. Use of Equivalent Uniform Loads in Stress Analysis. The data collected in Table 3 furnish all of the information necessary for determining the maximum live-load moments and shears in beams and the maximum live-load stresses in the trusses of railway bridges. Equivalent uniform loads for moments and for reactions in beams may be taken directly from Table 3. Chord members of trusses also have their stresses determined from moments. The use of the equivalent uniform load for moment (chosen from the table according to the fraction point of the span represented by the corresponding panel point of the truss) simplifies the computation of the live-load chord stresses.

Equivalent Uniform Load for Shear in a Beam. A study of the darkened parts of the influence lines in Fig. 164 will indicate how the data of Table 3 can be used to calculate the maximum live-load shear in a beam or the maximum live-load stress in a web member of a railway-bridge truss. In (a), the *placing* of wheel loads would be the same for maximum reaction in the

TABLE 3

EQUIVALENT UNIFORM LOADS FOR THE *E-72* LOCOMOTIVE LOADING IN KIPS PER FOOT OF TRACK

SPAN IN FEET	E. U. L. FOR MOMENT — FRACTION POINTS OF SPAN				E. U. L. FOR END REACTION
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$	
40	12.4	12.1	12.0	11.9	13.6
60	10.9	10.7	10.4	10.4	11.8
80	10.3	9.9	9.7	9.7	11.2
100	10.0	9.7	9.5	9.3	10.8
120	9.8	9.4	9.5	9.2	10.4
140	9.5	9.2	9.1	9.1	10.1
160	9.3	9.0	8.9	8.9	9.8
180	9.1	8.8	8.7	8.7	9.6
200	9.0	8.7	8.6	8.6	9.4
250	8.7	8.4	8.3	8.1	9.0
300	8.3	8.2	8.0	7.8	8.8
400	8.0	7.8	7.7	7.6	8.4

NOTE. These uniform loads are in kips per foot of track. Divide by two for load in kips per foot of rail.

beam *ab* at *a* and for maximum shear in the beam *cde* at *d*. The areas of these triangles are to be multiplied by the equivalent uniform loads to obtain *reaction* or *shear*. These unequal areas bear the same ratio to each other as do their corresponding vertical ordinates. Since these ordinates are to be

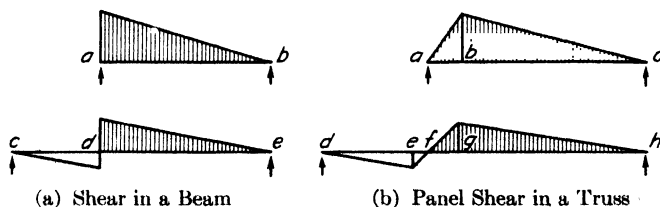


FIG. 164. COMPARISON OF INFLUENCE LINES.

multiplied by identical wheel concentrations to obtain this *same reaction* or *shear*, we necessarily conclude that the equivalent uniform loads in the two cases are identical. Accordingly, we determine the E. U. L. for beam shear from the column in Table 3 which is headed E. U. L. FOR END REACTION.

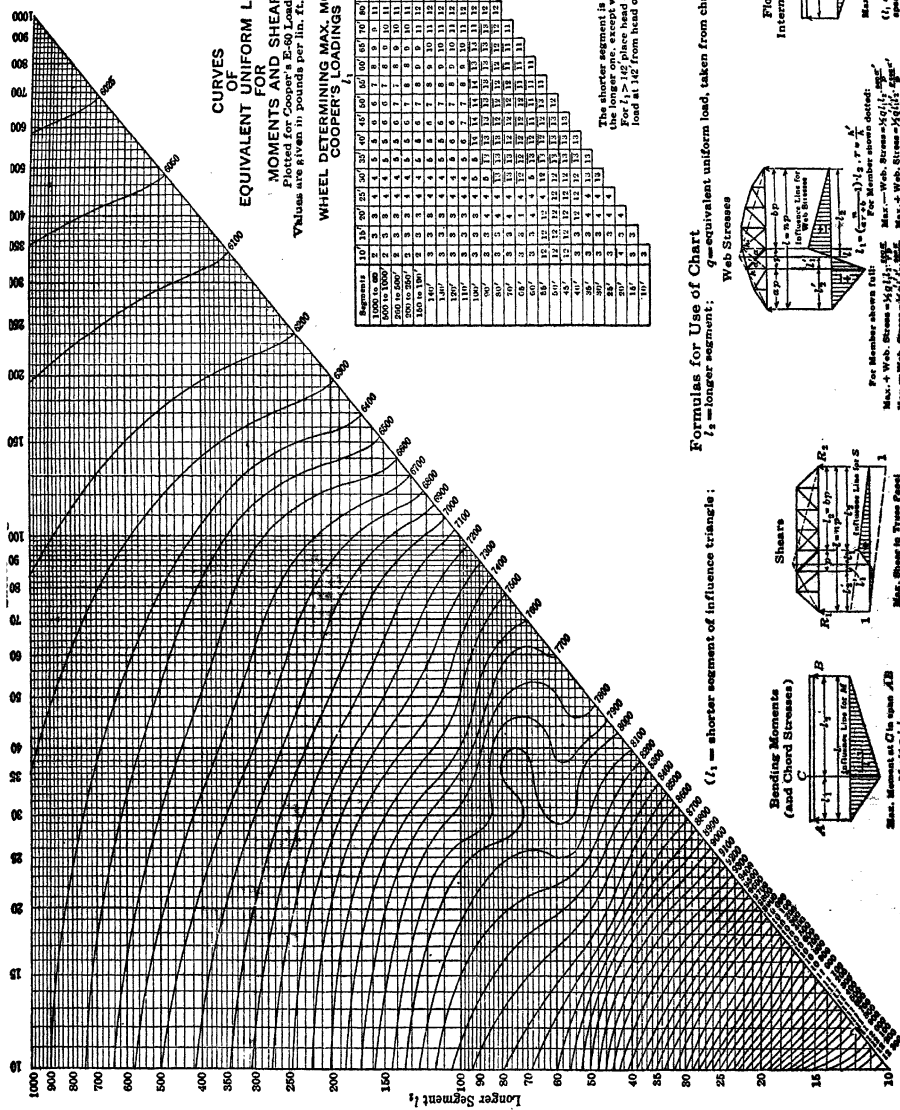
CORRECTION. Actually, there should be a correction made for the fact that the *wheel 1 usually will be placed to the left of the section over a negative ordinate of the influence line*. This correction is seldom made since it is negligibly small in its effect upon the large shear near the end of the span, and the smaller shears near the middle of a beam or girder are of little importance anyway. The calculated shear is always larger than the true shear. The proper correction is suggested in the lower right-hand corner of Fig. 165.

Equivalent Uniform Load for Panel Shear in a Truss. An identical parallelism exists between the two influence lines shown in (b) of Fig. 164, namely, the influence line for moment at b in the beam abc and for shear in the panel eg of the truss of length $d-h$. It follows that the equivalent uniform load for maximum panel shear or for maximum stress in a web member of a truss may be obtained from the table of equivalent uniform loads for moment. The length of span should be taken as the length of the segment $f-h$ of the influence line, and the fraction point of the span is found by dividing $f-g$ by $f-h$. The negative part of the influence line may be used in the same manner to obtain negative shear.

188. Charts of Equivalent Uniform Loads. Figure 165 is the Steinman chart of equivalent uniform loads for $E-60$ loading and Fig. 166 is a similar chart for $M-60$. The use of the chart is dependent upon an understanding of influence lines. The lengths of the shorter and longer segments l_1 and l_2 of a typical triangular moment influence line are used as known coordinates to locate a point on the chart. Then with a pencil we follow out the nearest contour line until it intersects the diagonal boundary line of the chart where the equivalent uniform load may be read. Since the chart is plotted on logarithmic paper, a value of $l_1 = 0$ can not be shown. This explains why there is a table of values of E. U. L. for reaction or shear in beams to the right of the chart. Critical wheels also are indicated. The equivalent uniform loads for $E-72$ loading may be obtained from the chart of Fig. 165 by multiplying by the constant factor 1.2.

Importance of Equivalent Uniform Loads. The use of equivalent uniform loads places the analysis of railway bridges upon the same basis as the analysis of highway bridges. Influence lines clarify the proper placing of the live load. However, the engineer may be called upon to analyze a truss for the stresses caused by a particular locomotive. It is unlikely that a chart of equivalent uniform loads would be available for such a study. Further, the knowledge of how to analyze a truss for locomotive loadings is a part of the basic training of a structural engineer. Without such training he would be handicapped by a lack of judgment of the effect of moving loads on structures of *all kinds*. Hence, the study of the effects of wheel loads will be continued.

189. Impact in Railway Bridges. The impact allowance in percentage of the live load is much higher for railway bridges than the corresponding allowance for highway bridges. Anyone who has clung to the top chord of a railway truss while a locomotive rushed across the structure will appreciate the need for a heavy allowance for impact and vibration stresses. The impact allowance for main truss members decreases as the loaded length increases. This is explained by the fact that the time necessary to cover the greater length with the moving load increases, and the load is applied less suddenly.



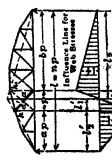
CURVES OF EQUIVALENT UNIFORM LOADS
MOMENTS AND SHEARS
 Values are given in pounds per lin. ft. per Track
WHEEL DETERMINING MAX. MOMENT
COOPER'S LOADINGS

Segments	10	15	20	25	30	35	40	45	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300	310	320	330	340	350	360	370	380	390	400	410	420	430	440	450	460	470	480	490	500	510	520	530	540	550	560	570	580	590	600	610	620	630	640	650	660	670	680	690	700	710	720	730	740	750	760	770	780	790	800	810	820	830	840	850	860	870	880	890	900	910	920	930	940	950	960	970	980	990	1000
1000 to 1500	10	15	20	25	30	35	40	45	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300	310	320	330	340	350	360	370	380	390	400	410	420	430	440	450	460	470	480	490	500	510	520	530	540	550	560	570	580	590	600	610	620	630	640	650	660	670	680	690	700	710	720	730	740	750	760	770	780	790	800	810	820	830	840	850	860	870	880	890	900	910	920	930	940	950	960	970	980	990	1000

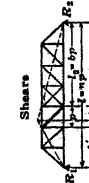
The shorter segment is ahead followed by the longer one, except wheel is overhauled. For l_1 and l_2 the shorter segment train load at $1/2$ from head of span.

Formulas for Use of Chart
 l_1 = longer segment;
 g = equivalent uniform load, taken from chart for l_1 and l_2

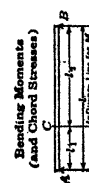
Web Stresses



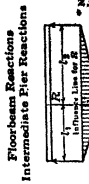
For Member shown full:
 Max. + Web Stress = $\frac{M}{I} \cdot \frac{y}{I}$
 Min. - Web Stress = $\frac{M}{I} \cdot \frac{y}{I}$
 For Member shown dotted:
 Max. + Web Stress = $\frac{M}{I} \cdot \frac{y}{I}$
 Min. - Web Stress = $\frac{M}{I} \cdot \frac{y}{I}$



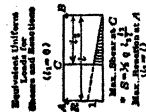
Max. Shear in Truss Panel
 $S = \frac{M}{I} \cdot \frac{y}{I}$
 $(I_1 = \frac{M}{I} \cdot \frac{y}{I})$



Max. Moment at G in span AB
 $M = \frac{M}{I} \cdot \frac{y}{I}$
 $(I_1 = \frac{M}{I} \cdot \frac{y}{I})$



Max. Intermediate Reaction
 $R = \frac{M}{I} \cdot \frac{y}{I}$
 $(I_1 = \frac{M}{I} \cdot \frac{y}{I})$



Equivalent Uniform Loads and Moments	1000	1500	2000	2500	3000	3500	4000	4500	5000	5500	6000	6500	7000	7500	8000	8500	9000	9500	10000	10500	11000	11500	12000	12500	13000	13500	14000	14500	15000	15500	16000	16500	17000	17500	18000	18500	19000	19500	20000	20500	21000	21500	22000	22500	23000	23500	24000	24500	25000	25500	26000	26500	27000	27500	28000	28500	29000	29500	30000	30500	31000	31500	32000	32500	33000	33500	34000	34500	35000	35500	36000	36500	37000	37500	38000	38500	39000	39500	40000	40500	41000	41500	42000	42500	43000	43500	44000	44500	45000	45500	46000	46500	47000	47500	48000	48500	49000	49500	50000	50500	51000	51500	52000	52500	53000	53500	54000	54500	55000	55500	56000	56500	57000	57500	58000	58500	59000	59500	60000	60500	61000	61500	62000	62500	63000	63500	64000	64500	65000	65500	66000	66500	67000	67500	68000	68500	69000	69500	70000	70500	71000	71500	72000	72500	73000	73500	74000	74500	75000	75500	76000	76500	77000	77500	78000	78500	79000	79500	80000	80500	81000	81500	82000	82500	83000	83500	84000	84500	85000	85500	86000	86500	87000	87500	88000	88500	89000	89500	90000	90500	91000	91500	92000	92500	93000	93500	94000	94500	95000	95500	96000	96500	97000	97500	98000	98500	99000	99500	100000
--------------------------------------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	--------

FIG. 165. STEINMAN'S E-60 CHART OF EQUIVALENT UNIFORM LOADS IN POUNDS PER FOOT OF TRACK. FOR E-72 MULTIPLY BY 1.2.

Courtesy D. B. Steinman

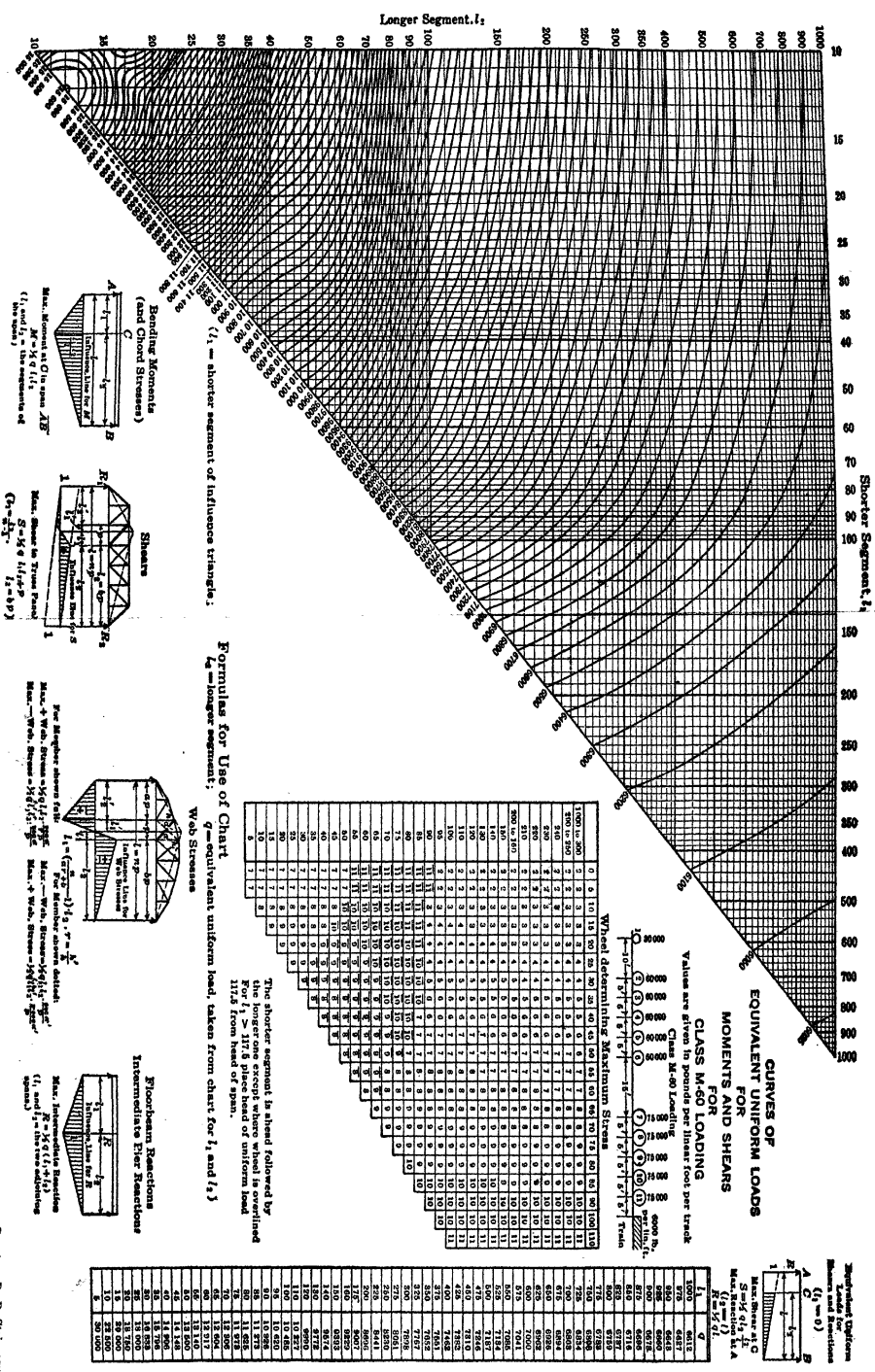


FIG. 168. STEINMAN'S M-60 CHART OF EQUIVALENT UNIFORM LOADS IN POUNDS PER FOOT OF TRACK. TRANSACTIONS A.S.C.E., Vol. 86.

Courtesy D. B. Steinman

It is well understood that an instantaneously applied load produces 100% impact, or twice the stress produced by a static load.

Mass. Impact is reduced by the great mass of a long span. Accordingly, a concrete floor slab, as used on most highway bridges and on some railway bridges, would be expected to reduce vibration and also impact. However, published tests are hardly conclusive on this point. The weight of the floor slab may make necessary a deeper and more rigid floor system. This increase in rigidity may produce an increase in impact because the force of a blow is decreased by any flexibility existing in the members receiving the blow. On the other hand, if the impact stress must travel some distance through the concrete of the floor slab before reaching the truss members, the elasticity of this material will damp the stress vibrations and reduce the impact. It is the author's opinion, based upon tests made at the University of Illinois in 1924 (unpublished), that in isolated cases a concrete floor slab may increase the impact stresses in the floor system, but that it will reduce the impact stresses in the main truss members.

A.R.E.A. Impact Studies. Committees of the A.R.E.A. have been investigating bridge impact for decades. A report published in 1945, A.R.E.A. Bulletin 450 pp. 189-434 gives data on the impact effects of steam and diesel locomotives on short steel bridges with open and ballasted decks. Measured static strains were 80% for ballasted decks and 90% for open decks of the usual calculated strains. Impact at any speed was defined as the excess over the stress produced at a speed of 5 mph.

<i>Impact Effect of Roll of Locomotive</i>	STEAM LOCOMOTIVE	DIESEL LOCOMOTIVE
Open Deck	23%	17%
Ballasted Deck	19%	17%
<i>Total Impact Effect</i>		
Open Deck	30% at 10 mph to 75% at 50 mph or more	20% at 10 mph to 50% at 30 mph or more
Ballasted Deck	30% at 10 mph to 75% at 50 mph or more	20% at 10 mph to 50% at 50 mph or more

The difference between the impact from roll or sway of the locomotive and the total impact effect is due to (1) the condition of the rolling stock and the track and (2) the action of the counterbalance of a steam locomotive. The most significant results of these tests is the reduction of impact from use of the balanced diesel locomotive.

Impact Measurements. A typical strain diagram obtained from a recording extensometer placed on the vertical of a railway bridge is shown in Fig. 167(a). These vibrations were caused by the passage of a railway train of ordinary

weight at full speed. Below, in Fig. 167(b), is reproduced the experimentally determined strain diagram for the vertical of a highway truss. The load here was a truck of average weight driven at about 25 miles per hour over a 2×4 placed on the concrete floor at the foot of the vertical hanger. Since ordinates in these diagrams are proportional to stresses, the reader can form some idea as to the relative impact stresses existing in the two structures. It is not intended to convey the impression that (b) represents a normal condition of impact for a highway bridge, but the result is interesting.

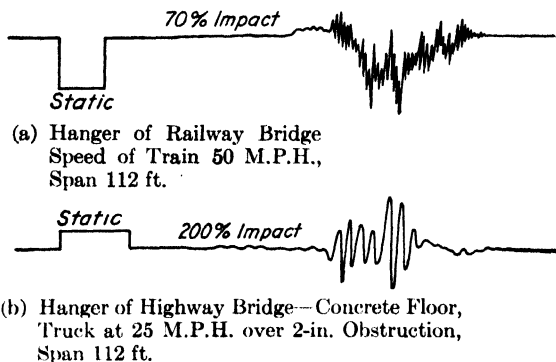


FIG. 167. EXTENSOMETER DIAGRAMS FOR MOVING LOADS.

190. Impact Formulas for Railway Bridges. Many impact formulas have merit, but it is thought best to reproduce here only those that have been used by the American Railway Engineering Association.

$$(3) \quad I = \frac{300}{L + 300} \quad (\text{A.R.E.A., 1910}).$$

$$(4) \quad I = \frac{100}{l + \frac{20,000}{l^2}} \quad (\text{Recommendation of the A.R.E.A. Committee on Impact}).$$

$$(5) \quad I = \frac{300}{300 + (L^2/100)} \quad (\text{A.R.E.A., 1921 and 1925}).$$

$$(6) \quad I = \frac{400 - (L/2)}{400 + L} \quad (\text{A.S.C.E.-A.R.E.A. Committee, 1929}).$$

For beams, girders, floor beams and hangers:

$$(7) \quad I = 100 - 0.60l \quad \text{for } l \text{ less than 100 ft.}$$

$$(8) \quad I = \frac{1800}{l - 40} \quad \text{for } l = 100 \text{ ft. or more.}$$

For truss spans:

$$(9) \quad I = \frac{4000}{l + 25} + 15 \quad \text{for standard steam locomotives.}$$

$$(10) \quad I = \frac{600}{l - 30} + 16 \quad \text{for diesels or locomotives without unbalanced loads.}$$

(A.R.E.A., 1949)

In all formulas, L is the loaded length in feet, l is the span in feet, and I is the impact percentage by which the live-load stress is multiplied to obtain the impact stress. Note the change of opinion with the years as to whether L or l should control the percentage of impact for a truss member. This question is still not settled. When considering the effect of *lateral* and *longitudinal* forces on railway bridges, we should realize that specifications do not require an increase of such forces for impact. These forces will be considered in Chapter 8.

PROBLEMS

149. Plot on a single sheet all of the formulas for impact as given in § 190. Plot percentages of impact as ordinates and lengths up to 1000 ft. as abscissas. Might impact be neglected on a suspension bridge 3000 ft. long?

150. Select the E.U.L. for moment at 100 ft. from the end of a 400-ft. span for $E-72$ loading, and then compute the bending moment. Use Table 3 and check by Fig. 165. Repeat for end reaction. *Ans.* 118,500,000 ft.-lb.; 1,680,000 lb.

151. Repeat Problem 150 for moment and shear at a point 110 ft. from the end of a 275-ft. girder.

STRESSES IN RAILWAY BRIDGES FROM MOVING LOADS

191. Objectives and Methods. The problem may be stated as follows: It is necessary to learn to place any engine loading, of which $E-60$ is typical, for maximum shear or moment at any point of a girder, and for maximum stress in any truss member. Since the bar stresses in a truss are determined from the panel shears and the joint moments, the methods of handling the truss and the beam or girder are nearly identical. Four methods of determining the exact maximum live-load stresses are in common use:

- (1) *Use of Equivalent Uniform Live Loads.* This method has already been discussed.
- (2) *Trial Method by Use of Influence Lines.* By this method we place the loads in such a manner that the large loads occur in conjunction with the large ordinates on the influence line. Two or more possible positions may suggest themselves and the stress is computed for each position. The largest value of the stress is accepted as the maximum. See § 194.
- (3) *Trial Method by Use of the String Polygon.* This is a graphical procedure that is particularly useful for finding the maximum moment in a simple span. See Fig. 171.
- (4) *Use of Algebraic Criteria.* An algebraic criterion can be derived for each important case by the use of calculus or from the influence line. Then at any subsequent time these criteria may be used to determine exactly how to place the engine loads to produce maximum stress. See §§ 197–207.

192. The Moment Table. A typical moment table is illustrated by Table 4. This particular table was produced for $E-60$ loading. Lines 1, 2, and 3, respectively, give the wheel spacing, the wheel number, and the corresponding weight in kips. Line 4 gives the total distance from any wheel to the wheel 1. Line 5 gives the total load up to any selected wheel and line 6

TABLE 4
MOMENT TABLE FOR COOPER'S E-60 LOADING FOR ONE RAIL

1	Spacing (ft.)	← 8'	← 5'	← 5'	← 5'	← 9'	← 5'	← 6'	← 5'	← 8'
2	Wheel	①	②	③	④	⑤	⑥	⑦	⑧	⑨
3	Weight (kips)	15	30	30	30	30	19.5	19.5	19.5	19.5
4	Distance* (ft.)	0	8	13	18	23	32	37	43	48
5	Load† (kips)	15	45	75	105	135	154.5	174	193.5	213
6	Moment‡ (kip-ft.)	0	120	345	720	1240	2460	3230	4280	5240
7	Distance		0	5	10	15	24	29	35	40
8	Load		30	60	90	120	139.5	159	178.5	198
9	Moment		0	150	450	900	1980	2680	3630	4520
	Distance			0	5	10	19	24	30	35
	Load			30	60	90	109.5	129	148.5	168
	Moment			0	150	450	1260	1810	2580	3320
	Distance				0	5	14	19	25	30
	Load				30	60	79.5	99	118.5	138
	Moment				0	150	690	1090	1680	2270
	Distance					0	9	14	20	25
	Load					30	49.5	69	88.5	108
	Moment					0	270	520	930	1370
	Distance						0	5	11	16
	Load						19.5	39	58.5	78
	Moment						0	97	330	620
	Distance							0	6	11
	Load							19.5	39	58.5
	Moment							0	117	310
	Distance								0	5
	Load								19.5	39
	Moment								0	97
	Distance									0
	Load									19.5
	Moment									0
	Distance									
	Load									0
	Moment									0

The E-60 loading consists of two 213-ton engines followed by a uniform load of 6000 lb. per ft. of track; loads in kips, moments in kip-feet. All values are given per foot of rail. Multiply by two to obtain values per foot of track. For E-72 multiply loads, shears and moments by 1.2.

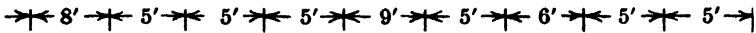
* Distance to right of wheel ① in ft.

† Wheel load above plus all loads to left in kips.

‡ Moment of all wheel loads to left in kip-ft.

Below double line an increasing number of the first wheels are omitted in succeeding lines.

TABLE 4—Continued
 FOR E-72 LOADING MULTIPLY LOADS AND MOMENTS BY 1.2

									
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	
15	30	30	30	30	19.5	19.5	19.5	19.5	3 kips per ft. of rail
56	64	69	74	79	88	93	99	104	109
228	258	288	318	348	367.5	387	406.5	426	426
6950	8770	10060	11500	13090	16220	18060	20380	22420	24550
48	56	61	66	71	80	85	91	96	101
213	243	273	303	333	352.5	372	391.5	411	411
6110	7810	9030	10390	11910	14900	16670	18900	20860	22910
43	51	56	61	66	75	80	86	91	96
183	213	243	273	303	322.5	342	361.5	381	381
4670	6130	7200	8410	9780	12500	14120	16170	17980	19880
38	46	51	56	61	70	75	81	86	91
153	183	213	243	273	292.5	312	331.5	351	351
3380	4600	5520	6580	7800	10250	11720	13790	15250	17000
33	41	46	51	56	65	70	76	81	86
123	153	183	213	243	262.5	282	301.5	321	321
2240	3220	3990	4900	5970	8150	9470	11160	12670	14270
24	32	37	42	47	56	61	67	72	77
93	123	153	183	213	232.5	252	271.5	291	291
1250	1990	2610	3370	4290	6200	7370	8880	10240	11690
19	27	32	37	42	51	56	62	67	72
73.5	103.5	133.5	163.5	193.5	213	232.5	252	271.5	271.5
780	1370	1890	2550	3370	5110	6180	7570	8830	10190
13	21	26	31	36	45	50	56	61	66
54	84	114	144	174	193.5	213	232.5	252	252
410	840	1260	1830	2550	4120	5090	6360	7530	8790
8	16	21	26	31	40	45	51	56	61
34.5	64.5	94.5	124.5	154.5	174	193.5	213	232.5	232.5
156	430	760	1230	1850	3240	4110	5270	6340	7500
0	8	13	18	23	32	37	43	48	53
15	45	75	105	135	154.5	174	193.5	213	213
0	120	345	720	1240	2480	3230	4280	5240	6310

gives the moment about any wheel of all loads to the left. Lines 7, 8, and 9 repeat the data of lines 4, 5, and 6 but with the wheel 1 omitted. An increasing number of the first wheels are omitted in succeeding lines of the table.

The use of this table is extremely simple and will not be explained. Unless the engineer has practically produced a table or graph *for himself*, he is not in a position to use it intelligently. The reader should study and use this table until he is convinced that he could reproduce it if necessary. An excellent procedure would be to check several values of moment as given in the table. Table 5 is an abbreviated moment table for M -60 loading. It corresponds to the first six lines of Table 4. To obtain the moment of the wheels 4 to 10 about the wheel 10, for instance, we must subtract the moment of the wheels 1 to 3 about the wheel 10 from the total moment at the wheel 10 as given in the table. For example, $M_{10} = 8025 - (375 + 75(60' - 15')) = 4275$ kip-ft. Here, 8025 is the moment of wheels 1 to 10 about wheel 10, 375 is the moment of wheels 1 to 3 about wheel 3, 75 is the sum of the loads of wheels 1, 2 and 3, and $(60-15)$ is the distance between wheels 3 and 10.

TABLE 5
MOMENT TABLE FOR M -60 LOADING FOR ONE RAIL

Wheel Spacing (ft.)	10'	10'	5'	5'	5'	5'	15'	5'	5'	5'	5'	5'	5'
Wheel Numbers	1	2	3	4	5	6	7	8	9	10	11		
Wheel Loads (kips)	15	30	30	30	30	30	37.5	37.5	37.5	37.5	37.5	3 kips per ft. of rail 70 ft.	
Total Distance (ft.)	0	10	15	20	25	30	45	50	55	60	65		
Sum of Loads (kips)	15	45	75	105	135	165	202.5	240	277.5	315	352.5		
Total Moment (kip-ft.)		150	375	750	1275	1950	4425	5440	6640	8025	9600	11360	kip-ft.

193. Maximum Reaction by the Trial Method. The influence line for reaction in a beam or truss is shown in Fig. 168(a). The E -60 loading is shown on a beam of 40-ft. length in Fig. 168(b). The wheel 1 is placed at the left reaction. Consider a movement to the left of 8 ft. in order to bring the wheel 2 up to the reaction. This movement increases the reaction produced by every wheel load from 2 to 7, places a larger load directly over the reaction, and compensates for the loss of the wheel 1 by bringing the wheel 8 onto the span. The reaction has been increased considerably. A further movement to bring the wheel 3 over the reaction would reduce its value, for the result would be merely to replace the wheel 2 on the span by a smaller load, that is,

by the wheel 9. *Wheel 2 produces a maximum reaction.* Why must one wheel be placed over the reaction?

194. Maximum Shear in a Beam by the Trial Method. The influence line for shear in a beam is shown in Fig. 169(a). In (b) the *E-60* loading is placed on the beam of 60-ft. span. The wheel 1 is at the section where maximum shear is desired. A movement of 8 ft. to the left will bring the wheel 2 up to the point *b*, which will increase the left-hand reaction. Since the value

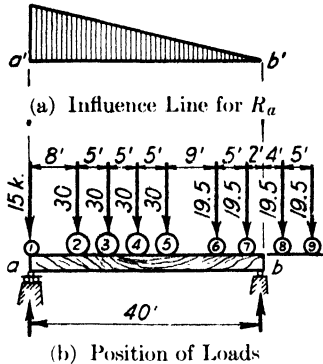


FIG. 168. REACTION OF A SIMPLE BEAM.

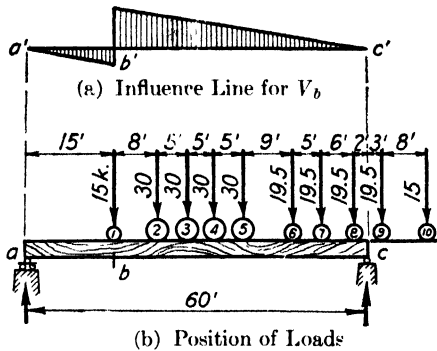


FIG. 169. SHEAR IN A SIMPLE BEAM FOR *E-60* LOADING.

of the wheel 1 must then be subtracted from the reaction to obtain the shear at the point *b*, a numerical calculation is necessary to determine whether the shear has or has not been increased. The shear at the point *b* will be determined for three cases, namely, wheels 1, 2, and 3 at *b* in Fig. 169.

CASE (1). With the wheel 1 at *b*, the moment about the point *c*, which is 2 ft. to the right of the wheel 8, is found by use of the moment table to be $4280 + 193.5 \times 2 = 4667$ kip-ft. The reaction at *a* is $4667 \div 60 = 78$ kips. This is also the shear at *b*.

CASE (2). With the wheel 2 at *b*, the moment about the point *c*, which is now 5 ft. to the right of the wheel 9, is $5240 + 213 \times 5 = 6305$ kip-ft. The reaction at *a* is $6305 \div 60 = 105$ kips. The shear at *b* is $105 - 15 = 90$ kips. This is the maximum shear at *b*.

CASE (3). With the wheel 3 at *b*, the moment about the point *c*, which is 2 ft. to the right of the wheel 10, is 7406 kip-ft. The reaction at *a* is $7406 \div 60 = 123$ kips. The shear at *b* is $123 - 45 = 78$ kips.

As found above, the wheel 2 or less commonly the wheel 1 at the section usually will produce maximum shear for the *E-loadings*. For the *M-loadings* it will be noticed that there is a gap of 15 ft. between the wheels 6 and 7. The wheel 7 can be placed at any point within 15 ft. of the end of the span and there will be no load to subtract from the left-hand reaction when the shear is being calculated. The wheel 7 produces the maximum shear in all such cases.

The reader should convince himself that any finite movement of the loads so as to place a wheel to one side or the other of the point b , will inevitably result in a decrease in the shear from its maximum value. Whenever mention is made of a wheel at the point b , it is considered that the wheel is placed an infinitesimal distance to the right of b , so that its value is not to be deducted in the calculation of the shear. When placed for negative shear, the wheel would be slightly to the left of b .

195. Maximum Moment in a Beam by the Trial Method. The influence line for moment at a point in a beam is shown in Fig. 170(a). In (b) the $E-60$

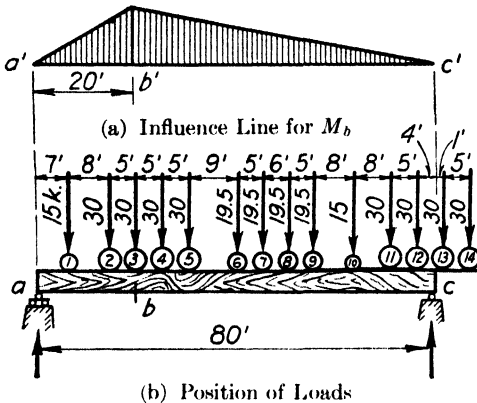


FIG. 170. MOMENT IN A SIMPLE BEAM.

the left end. Also, this movement brings a group of light wheels near the center of moments, which is not desirable. Each of these three cases will be tried out numerically for the beam of Fig. 170.

CASE (1). With the wheel 3 at b , the moment about a point 4 ft. to the right of the wheel 12 is found by the use of the moment table to be $10,060 + 288 \times 4 = 11,210$ kip-ft. The reaction at a is $11,210 \div 80 = 140$ kips. The bending moment at the point b is $140 \times 20 - 345 = 2455$ kip-ft.

CASE (2). With the wheel 4 at b , the moment about a point 4 ft. to the right of the wheel 13 is found to be $11,500 + 318 \times 4 = 12,770$ kip-ft. The reaction is $12,770 \div 80 = 160$ kips. The bending moment at the point b is $160 \times 20 - 720 = 2480$ kip-ft.

CASE (3). With the wheel 5 at b , the moment about a point 4 ft. to the right of the wheel 14 is found to be $11,910 + 333 \times 4 = 13,240$ kip-ft. The reaction at the point a is $13,240 \div 80 = 165$ kips. The bending moment at the point b is $165 \times 20 - 900 = 2400$ kip-ft. Case (2) produced the largest value of M_b .

This example is typical and shows that the *wheel 3 or 4 usually produces maximum moment in short-span beams*. However, the wheel 12 will be found to produce maximum moment in certain spans between 50 and 80 ft. For

spans exceeding 100 ft., when maximum moment is desired for a point to the left of the center, it is well to place the loading on the span with the wheel 1 from 10 to 20 ft. to the right of the left-hand reaction. Then successively move the loads to the left until the moment starts to decrease. The previous location will represent one position for maximum. A second possible maximum moment will occur when the large wheels of the second locomotive are moved up near the center of moments. Critical wheels for moment are listed in Fig. 165 and Fig. 166.

196. Maximum Moment Graphically. The special tool needed here is an equilibrium polygon for the concentrated load system for about the first 200 ft. of its length. Such a polygon is shown in Fig. 171. This polygon should be drawn to a large scale in ink on permanent material, so that it can

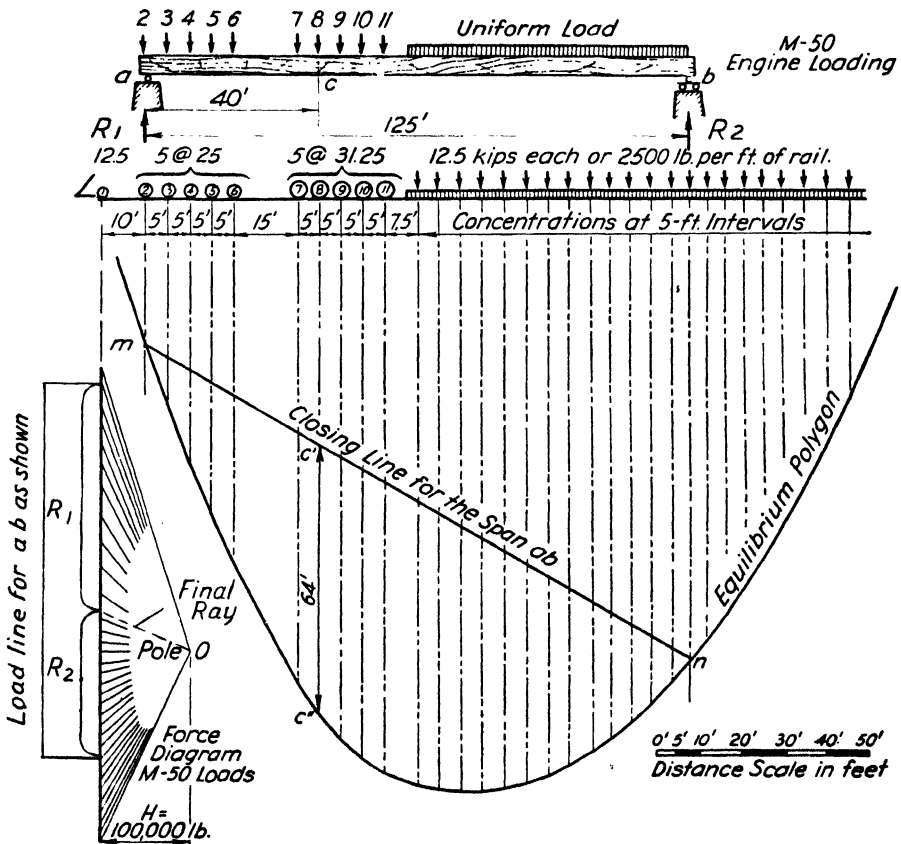


FIG. 171. GRAPHICAL DETERMINATION OF MAXIMUM MOMENTS FOR THE M-50 LOADING.

be used repeatedly. The pole distance used in Fig. 171 is 100,000 lb. Any part of the total polygon, such as mn , is a string polygon for the loads contained within the projection of mn on the horizontal load line.

PROCEDURE. In order to determine the bending moments in any simple span, the span is drawn on the sheet at its proper location relative to the loads. Then verticals are dropped from its ends, a and b , to intersect the polygon at m and n . Any vertical ordinate from the closing line $m-n$ downward to the polygon, when scaled in feet and multiplied by the pole distance of 100,000 lb., represents the bending moment in *foot pounds* in the span at the corresponding point. This relationship was developed in § 46. Note in the ray polygon how the proper group of loads and the final ray determine the end reactions.

Figure 171 shows the graphical determination of the moment at the point c , which is 40 ft. from the left-hand end of a 125-ft. span, when the wheel 8 of the $M-50$ loading is placed at the point c . The scaled value of the ordinate $c'-c''$ between the closing line and the polygon is 64 ft., which corresponds to a bending moment of $64 \times 100,000 = 6,400,000$ ft.-lb. The *maximum moment* at the point c can be found by sketching in several closing lines 125 ft. in length (corresponding to various positions of the span relative to the fixed position of the loads) and by choosing the largest value of $c'-c''$ for the calculation of moment. Note that the position of $c'-c''$ will not remain fixed in relation to the equilibrium polygon but only with respect to a . The maximum moments for other points of this span and of other spans can be obtained in the same manner.

PROBLEMS

152. Determine by trial loading the maximum end reactions for a 50-ft. and a 100-ft. beam due to one rail $E-72$ loading. *Ans.* 157 and 270 kips.

153. Determine by trial and check by use of E.U.L. the moments at the mid-span of 50-ft. and 100-ft. beams due to one rail $E-72$ loading. *Ans.* 1700 and 5800 kip-ft

154 and 155. Solve Problems 152 and 153 for the $M-60$ loading, Table 5.

156 and 157. Solve Problems 152 and 153 for the diesel locomotive loading described in the caption to Fig. 162.

PLACING WHEEL LOADS BY THE USE OF CRITERIA

197. Criterion for Maximum Moment at a Given Point in a Beam. When the loads in Fig. 172 are placed so that the moment at the point b is a maximum, a finite movement of the loads in *either* direction will necessarily produce a *reduction of moment*. The change in the bending moment at b produced by a unit (1 ft.) movement of all loads toward the left can be written by *inspection* as the change in the moment at b produced by the change in the reaction R_1 minus the change in moment at b caused by the 1-ft. movement of the loads G_1 that exist to the left of the center of moments. Thus, when the movement is *toward the left*, we may write

$$\Delta M_b = \frac{W \times 1}{L} k - G_1 \times 1.$$

Or, in general terms

$$(11) \quad \Delta M = \frac{Wk}{L} - G_1.$$

Similarly, when the unit movement is *toward the right*, we obtain

$$(12) \quad \Delta M' = -\frac{Wk}{L} + G_1.$$

Accordingly, the expression $(Wk/L) - G_1$ passes from positive to negative through zero as the load producing the maximum moment passes the center of moments b in its movement toward the left. When used in this form, the criterion is dependent only upon the sign of ΔM , which evidently is independent of the distance through which the loads are moved. Of course, the loads are actually moved, not by increments of one foot, but by the distance between successive loads.

Loads Rolling on or off the Span. Any movement of the loads in Fig. 172 that increases the bending moment at b in the beam (positive value of ΔM) is desirable. When a load moves onto the beam at the right, W is increased and ΔM is positive. Hence, movement to the left would be desirable and a load should not remain at c , the right-hand reaction. Similarly, $\Delta M'$ is positive when a load moves off at the right, because W is decreased. A movement to the left that permits a load to roll off at the left end decreases G_1 relatively more than it decreases W

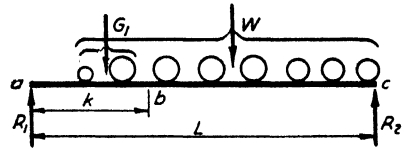


FIG. 172. MAXIMUM MOMENT AT THE POINT b .

and therefore *increases* ΔM . However, when a load passes b moving to the left, G_1 is increased and ΔM is decreased. We conclude then that a load *must* be placed at b , the center of moments, in order to satisfy the criterion. Loads can not occur at the ends of the beam a or c unless a load also occurs at the point b at the same time. Whenever the finite movement under consideration (distance between adjacent loads) will roll a load on or off of the span, there are two possible values of W to be considered in applying the criterion. Either value of W may satisfy the criterion and determine the loading for maximum moment.

Other Forms of the Criterion. If the movement of the loads is assumed to be an infinitesimal distance, the change in ΔM will be zero when the loads are in the exact position to produce maximum moment. Hence we may write

$$(13) \quad \frac{Wk}{L} - G_1 = 0,$$

from which we find

$$(14) \quad \frac{Wk}{L} = G_1,$$

or

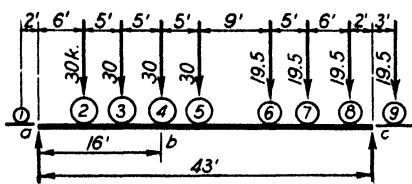
$$(15) \quad \frac{W}{L} = \frac{G_1}{k}.$$

RULE. This latter expression gives rise to the following rule: *For maximum moment at the point b of a simple beam, the average load to the left of b must equal the average load on the entire span.*

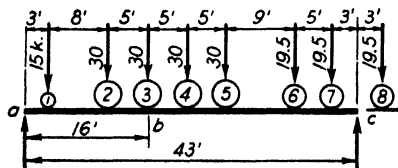
In the application of either equation (14) or equation (15), two possible values of G_1 are to be considered, depending upon whether the load at the point b is placed to the left or right of the point. The criterion is satisfied when the fixed value of the left-hand side lies between the two possible values of the right-hand side.

198. Example Using Criterion for Maximum Moment at a Given Point.

The problem is to obtain the maximum moment caused by the $E-60$ loading at a point 16 ft. from the left-hand end of a 43-ft. span. The wheel 4 is shown



(a) Wheel 4 at the Point b



(b) Wheel 3 at the Point b

FIG. 173. EXAMPLE OF MAXIMUM MOMENT.

at this point in Fig. 173(a). The total value of W is 178.5 kips. The value of Wk/L is $178.5 \times 16/43 = 66.4$ kips. The value of G_1 may include either the wheels 2 and 3, or the wheels 2 to 4 inclusive. These two values are 60 and 90 kips. Since the value of Wk/L lies between the two values of G_1 , the criterion is satisfied. The moment is found to be 1034 kip-ft.

The wheel 3 will now be tried, as shown in (b). The total load on the beam is 174 kips, and Wk/L is $174 \times 16/43 = 64.7$ kips. The two possible values of G_1 are the wheels 1 and 2 or the wheels 1 to 3 inclusive, that is, 45 or 75 kips. Since the value of Wk/L lies between the two values of G_1 , the criterion is

again satisfied. The value of the moment, determined by use of the moment table, is 1051 kip-ft.; a larger value than the one corresponding to the wheel 4 at the point.

EXAMPLE. It will be enlightening to study the change in moment at b as the loads are moved to the right by 1-ft. increments from the position (a) to the position (b), Fig. 173. For the first two units of movement, the change in the moment becomes

$$\begin{aligned} -\frac{Wk}{L} + G_1 &= -\frac{178.5 \times 16}{43} + 60 \\ &= -6.4 \text{ kip-ft.} \end{aligned}$$

The wheel 4 is now to the right of b . After a 2-ft. movement to the right, the wheel 8 passes off of the bridge at the right-hand end, and the wheel 1 moves onto the bridge at the left-hand end. The total load is now 174 kips, and the value of G_1 includes the wheels 1 to 3, or 75 kips. The change in moment for each foot of the remaining 3-ft. movement is therefore $-174 \times 16/43 + 75 = +10.3$ kip-ft. For the entire 5-ft. movement, the total change-in moment will be $-6.4 \times 2 + 10.3 \times 3 = +18.1$ kip-ft. Accordingly, the final moment should be $1034 + 18 = 1052$ kip-ft.; this checks with the value of 1051 obtained above.

Direction of Movement. The example above shows that the criterion can be made to furnish information as to the *proper direction of movement*. If the expression for the moment change corresponding to a movement toward the left, $Wk/L - G_1$, is positive, the proper movement is toward the left. Note, however, that when loads move on or off of the beam during this movement, there are two values of W to be considered. The two corresponding values of the change in moment may then be of opposite sign (as in the example above). In that case, the direction of movement is undetermined; hence we must try out the questionable positions of the loads to see which produces the maximum moment.

199. Absolute Maximum Moment in a Beam. In designing a beam or girder, the engineer is interested in the maximum moment at any given point and also in the greatest possible moment that can ever occur in the beam. The absolute maximum moment will occur near the center of the span under one of the wheel loads, but both the location of the point and the identity of the particular wheel are unknown. Note then that three studies are needed:

- (1) Criterion for maximum moment at a given point in a beam; $W/L = G_1/k$ (§ 197).
- (2) Criterion for maximum moment under a given wheel; $L - a - x = x$ (§ 152).
- (3) Criterion for absolute maximum moment in a span will be shown to be an extension of Case (2).

Maximum Moment under a Given Wheel. The location of the point on the span at which there is a maximum moment developed under a given wheel will be determined first. In Fig. 174, W represents the entire load on the span and G_1 is the total load to the left of the undetermined point at which moment is to be computed, which here is assumed to be continually under the particular load P . The general expression for the moment under P is

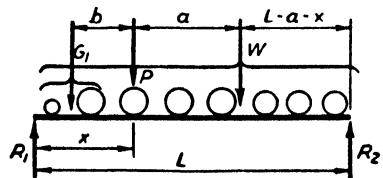


FIG. 174. MAXIMUM MOMENT UNDER THE WHEEL P .

$$M = \frac{W(L - a - x)}{L} x - G_1 b,$$

in which a and b , as shown in Fig. 174, are fixed distances from P to the resultant loads W and G_1 . This expression will become a maximum when its first derivative is zero. Hence we may write

$$\frac{dM}{dx} = \frac{W}{L} (L - a - 2x) = 0.$$

But W/L , the average load on the span, can not equal zero. It follows that

$$(16) \quad L - a - 2x = 0, \quad \text{or} \quad L - a - x = x.$$

Again we conclude as in § 152 that the particular load and the center of gravity of all the loads must lie equidistant from the ends or from the center of the beam when maximum moment occurs under the particular load.

Absolute Maximum Moment. Since the point of absolute maximum moment must lie near the center of the beam, *either the wheel that produces maximum center moment or an adjacent wheel* will be the particular wheel to produce absolute maximum moment in the beam. If *three wheels* are selected in this manner, then the correct wheel for producing absolute maximum moment should be the one of these three that lies nearest to the center of gravity of the loads, because this wheel can be placed nearer to the center of the beam than any other and still satisfy the criterion (equation 16).

200. Maximum Floor-Beam Reaction. A floor beam receives load from the wheel loads in two panels as shown in Fig. 175(a). The influence line for

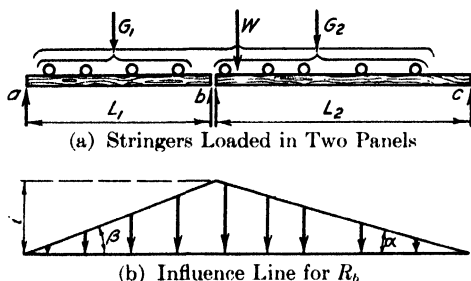


FIG. 175. MAXIMUM FLOOR-BEAM REACTION.

floor-beam reaction is shown in (b). A criterion for placing the loads will be derived from the information obtained from the shape of the influence line. Hence this criterion will apply to all stress functions where the influence line is a simple triangle as was suggested by W. S. Evans.

The loads will be moved a short distance x to the left, so that no wheel will move on or off the span or into another span. Then the increase in the stress function caused by the movement of the wheels in the span L_2 will be G_2 times the increase in the influence ordinate under G_2 , or $G_2(x \tan \alpha)$. Similarly, there is a decrease in the stress function of $G_1(x \tan \beta)$ produced by the movement of the loads in the span L_1 . But for a continuous movement past the position that produces maximum stress, it must happen that the increase in stress will first be larger than the decrease, then the two will become equal, and finally the decrease will become the larger. Hence we have

$$\begin{aligned}
 &\text{Increase} \qquad \qquad \text{Decrease} \\
 &G_2(x \tan \alpha) \geq G_1(x \tan \beta), \\
 &G_2 \times \frac{x}{L_2} = G_1 \times \frac{x}{L_1}, \\
 &\frac{G_2}{L_2} = \frac{G_1}{L_1},
 \end{aligned}
 \tag{17}$$

or, for equal spans,

$$G_2 = G_1.$$

If we adopt the simpler form of the criterion which uses only the equal sign, we arrive at the same expressions that were used in § 197, where $L_1 = k$, $L_1 + L_2 = L$, and $G_1 + G_2 = W$.

$$(18) \quad \frac{G_1}{L_1} = \frac{G_2}{L_2} = \frac{G_1 + G_2}{L_1 + L_2}, \quad \text{whence} \quad \frac{W}{L} = \frac{G_1}{k}, \quad \text{or} \quad \frac{Wk}{L} = G_1.$$

Conclusion. We conclude that the *form* of the criterion is dependent upon the *shape* of the influence line and that *a single criterion will serve in all cases where the loaded part of the influence line is a simple triangle*. When the angle β of the influence line is a right angle, both G_1 and L_1 are zero, the term G_1/L_1 or G_1/k becomes indeterminate, and the criterion can not be used.

PROBLEMS

158. Show that $Wx/L - P_1$ can be used as a criterion for shear at a point in a simple span where W is the total load on the span L , x is the distance from the first to the second wheel and P_1 is the weight of the first wheel load. Start by placing the wheel P_1 at the point where the shear is desired, and then consider a movement x to the left. Make use of this criterion to place the M -60 loading on a beam for maximum shear at a point 15 ft. from the left end of a 48-ft. span. Compute the shear for the loads on one rail. *Ans.* 92 kips.

159. Determine the maximum moments at the quarter point and at the center of a 140-ft. span for the E -72 loading. From the values obtained, check the corresponding equivalent uniform loads from Table 3. Repeat for maximum shears.

160. Determine the maximum moment at 40 ft. from the left end of a 110-ft. span as caused by the M -60 loading. Compute the corresponding equivalent uniform load.

Ans. 9700 lb. per ft. of track.

161. Find the absolute maximum moment in the stringer of a two-stringer bridge for the E -72 loading where the floor beams are spaced 30 ft. apart.

SUGGESTION. First try out wheel 3 at the center by use of the criterion.

Ans. 741 kip-ft.

162. Determine the maximum moments at the center of a 50-ft. span as caused by the M -60 and the E -72 loadings. Check by use of the equivalent uniform loads. Repeat for maximum shears.

Ans. 3690 kip-ft. for M -60.

163. Determine the maximum total live-load reaction produced on a floor beam per rail by the E -70 loading where the stringer length is 25 ft.

Ans. 132 kips.

201. Maximum Stresses in Truss Members. The chord members of ordinary trusses have their stresses determined from moments about the panel points. Evidently, the moment criterion as developed in § 197 governs the placing of the load for maximum chord stress. The stresses in the web members of the Pratt, Howe, and Warren trusses are controlled by the maximum panel shears. A criterion for maximum panel shear will be developed in § 202. A few special criteria will be needed for placing loads to produce maximum stresses in the web members of curved-chord trusses, etc.

202. Criterion for Maximum Shear in a Panel of a Truss or Girder. Loads are shown placed upon a truss in Fig. 176. The *change in shear* in the panel *bc*

(designated on the influence line) caused by a *unit movement* of the loads toward the left is seen by inspection to be $W/L - G_1/d$. The first term W/L is the change in the left-hand reaction and the term G_1/d is the change in the panel reaction to the left of the section. When the loads are properly placed for maximum shear, the sign of this expression representing change in shear will be *negative*. If the system of loads is moved the distance between adjacent loads, each term of the expression is multiplied by this distance and *the sign of their difference is unchanged*. In fact, the sign of this expression will not be changed by multiplying both of its terms by the panel length d , which simplifies the expression above to $W/n - G_1$, where n is the number of panels in the truss or girder. A unit movement toward the right produces a change in shear of $-W/L + G_1/d$, which reduces to $-W/n + G_1$ for sign determination.

A load passing c (the apex of the influence line in Fig. 176) moving to the left will increase G_1 and tend to make $W/n - G_1$ negative. Therefore, some load at c will satisfy the criterion. Similar reasoning shows that a load can not exist at d and produce a maximum shear unless a load also occurs at c . We conclude then that with some load just to the right of c , a movement to the left will change the expression $W/n - G_1$ from positive to negative through zero. Hence for exact maximum panel shear we may write

$$(19) \quad \frac{W}{n} - G_1 = 0, \quad \text{or} \quad \frac{W}{n} = G_1.$$

This latter form of the criterion will be satisfied if the single value of W/n lies between the two possible values of G_1 . (The load at c may be considered to be on either side of the point.)

RULE. Expressed in words, this latter form of the criterion states that *for maximum panel shear, the load in this panel must equal the average load per panel for the entire span.*

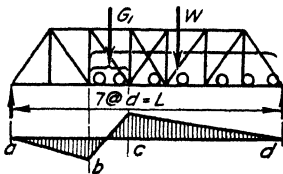


FIG. 176. MAXIMUM PANEL SHEAR.

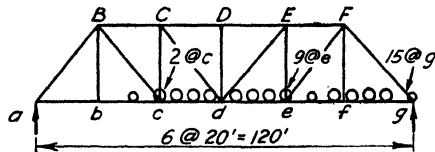


FIG. 177. MAXIMUM SHEAR IN THE PANEL bc .

203. Example of the Determination of Maximum Panel Shear. Maximum shear is desired in the panel bc of the truss shown in Fig. 177. The wheel 2 of the $E-60$ loading is shown at c . The wheel 15 will be found to be

exactly over g . The first form of the criterion will be used for this particular case.

CALCULATIONS. With wheel 2 at c try a movement toward the left; $W = 367.5$, $G_1 = 45$; $W/n - G_1 = 367.5/6 - 45 = +16.2$ kips. This suggests a movement toward the left.

Move the wheel 3 to c . The wheel 16 will be exactly over g . Consider a further movement toward the left; $W = 387$, $G_1 = 75$; $W/n - G_1 = 387/6 - 75 = -10.5$ kips. Therefore, the wheel 3 is the proper wheel to place at c for maximum shear in the panel bc . The panel shear is found to be 133 kips.

204. Maximum Moment at an Upper-Chord Joint of a Warren Truss.

The maximum moment about the joint D of the upper chord of the Warren truss, Fig. 178, which controls the maximum stress in the lower-chord member cd , may be taken conservatively as the *average of the maximum moments at c and d* . However, an exact criterion for determining a single placing of the loads is perhaps more satisfactory. From Fig. 178, the change in moment about the point D corresponding to a unit movement of the loads toward the left is seen by inspection to be

$$\frac{Wk}{L} - G_1 - \frac{G_2}{2}.$$

This gives rise to the criterion

$$(20) \quad \frac{Wk}{L} = G_1 + \frac{G_2}{2}.$$

It will be found that this criterion can be satisfied only when a load exists at c or d , the locations of the maximum ordinates on the influence line.

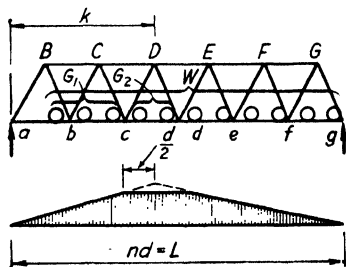


FIG. 178. MOMENT AT A JOINT OF THE UNLOADED CHORD.

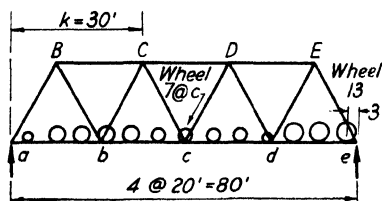


FIG. 179. MAXIMUM MOMENT AT THE POINT C.

EXAMPLE. It is desired to place the E-60 loading on the Warren pony truss of Fig. 179 for maximum moment about C or maximum stress in the member bc . Try the wheel 7 at the joint c . The total load (wheels 1 to 13) is 318 kips per truss. The value of G_1 (wheels 1 to 3) is 75 kips, and G_2 is either 99 or 79.5 kips, for the wheels 4 to 6 or 4 to 7 inclusive. Hence $Wk/L = 318 \times 30/80 = 119.4$ and the two values of $G_1 + G_2/2$ are $75 + 49.5 = 124.5$ and $75 + 39.7 = 114.7$. Since the value of Wk/L lies between the two values of

$G_1 + G_2/2$, the criterion is satisfied. The reader will find that the wheel 3 at b and the wheel 14 at c also satisfy the criterion. It is recommended that the reader calculate the moment at C for these three positions of the loads. Note that we must use the joint concentration at b from the panel bc when taking moments.

205. Maximum Stress in a Web Member of a Curved-Chord Truss. The criterion will be written by an inspection of the action of the loads in producing moment about the point O , the moment center for the member Cd of Fig. 180.

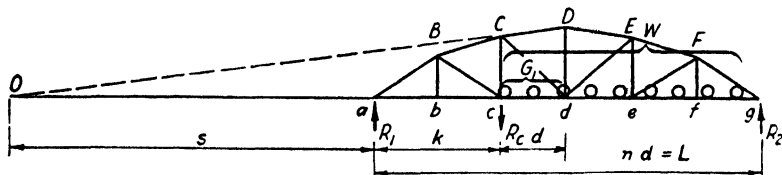


FIG. 180. MAXIMUM STRESS FOR THE WEB MEMBER Cd OF THE CURVED-CHORD TRUSS.

This moment is found to be $R_1s - R_c(k + s)$. The change in moment about O when the loads are moved a unit distance toward the left is therefore

$$\frac{W}{L}s - \frac{G_1}{d}(k + s),$$

or, more conveniently,

$$\frac{W}{L} - \frac{G_1}{d} \left(1 + \frac{k}{s}\right).$$

This expression may be equated to zero and multiplied by the panel length d to give the common form of the criterion

$$(21) \quad \frac{W}{n} = G_1 \left(1 + \frac{k}{s}\right).$$

This criterion is the same as the one for shear in a panel of a parallel-chord truss except for the factor $(1 + k/s)$ which accounts for the effect of the sloping chord. The criterion will be satisfied with a load at d .

206. Tension in the Vertical of a Curved-Chord Truss. This problem is readily solved by the use of influence lines and an equivalent uniform load which may be made dependent upon the shape of the negative part of the influence line for stress in the main diagonal Cd , Fig. 181. If wheel concentrations are to be used, the problem is best solved by trial. We must keep in mind that the desired conditions are zero stress in the main diagonal Cd and a heavy stress in the upper-chord member BC .

EXAMPLE. The $E-60$ loading is shown on the truss in Fig. 181. The dead load assumed was 1650 lb. per ft. of truss. The dead-load stress in the diagonal Cd is 26,800-lb. tension. With the wheel loads as shown, the live-load shear in the panel cd is $77.6 - 57.8 = 19.8$ kips. The corresponding stress in the diagonal is $19.8 \times 39.1/30 = 25.8$ kips com-

pression. Since this compressive stress is 1000 lb. less than the dead load tension, which it should reduce to zero, the wheel loads must be moved toward the left. A movement of 3 in. to the left would produce an exact balance, but such precision is not warranted. The maximum tension in Cc is found as the vertical component of the corresponding live-load plus

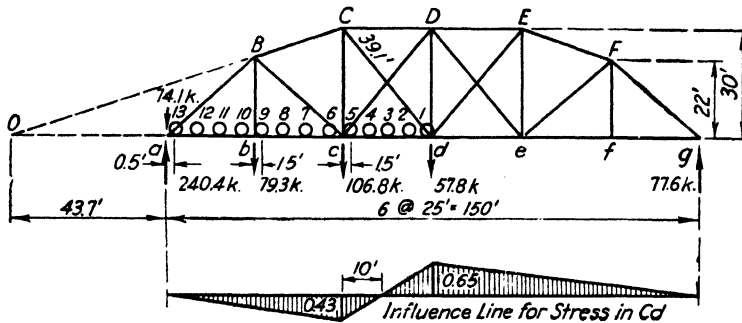


FIG. 181. MAXIMUM TENSION IN THE VERTICAL Cc , DUE TO E-60 LOADING.

dead-load stress in the sloping chord member BC . The exact value is + 112,000 lb. An approximate value of 122,000 lb. was found by the use of a uniform live load of 4500 lb. per ft. of rail corresponding to $l_1 = 10$ ft. and $l_2 = 50$ ft. for the negative part of the influence line in Fig. 181. Impact which was neglected here is properly introduced early in the analysis as an increase in the value of the live loading.

207. Live-Load Analysis of the Pettit Truss. Most members of the Pettit truss can be analyzed for maximum live-load stresses by placing the locomotive loading to satisfy the various criteria that have already been developed, for example, the following members from Fig. 182:

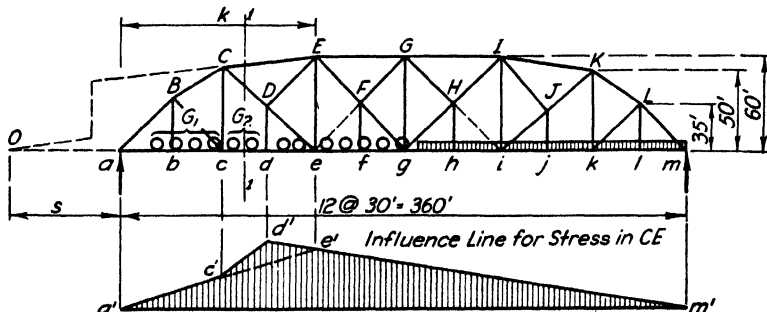


FIG. 182. MAXIMUM STRESS FOR THE UPPER-CHORD MEMBER CE OF A PETTIT TRUSS.

Upper-Chord Member, CE . The influence line for CE is shown in Fig. 182. The moment center for this member is at e . The unusual shape of the influence line is due to the fact that a load at d lies to the *right* of the section

1-1 but to the *left* of the center of moments e . Therefore that part of the influence line in the panel de must be an *extension* of $m'-e'$. Clearly, then, a load must be placed at d (the peak of the influence line) to produce maximum stress in the member CE . The change in moment about e caused by a unit movement of the loads toward the left is seen by inspection to be $Wk/L - G_1 - (G_2/d)(2d)$. From this expression, we obtain the more usual form of the criterion,

$$(22) \quad \frac{Wk}{L} = G_1 + 2G_2.$$

Lower-Chord Member, cd-de. Place loads to produce maximum moment about C so that the average load to the left of C is equal to or just less than the average load on the entire span.

Sub-Vertical and Sub-Tie, dD and DE. Place loads for maximum floor-beam reaction at d so that the average load in the panel cd is equal to or just less than the average load in the double panel $cd-de$.

Main Diagonal, CD-De. The member CD is a normal diagonal in a curved-chord truss acting in a panel of length cd . We may use the criterion $W/n = G_1(1 + k/s)$ to place the loads. Here G_1 is the sum of the loads in the panel cd as shown in Fig. 180 rather than Fig. 182. For the member De , n is the number of double panels of length $c-e$ in the bridge, and G_1 is the sum of the loads in the panel ce .

Counters, eF-FG. The counter is treated exactly as a main diagonal but with the loads extending out from the left reaction. The criterion for this member, $W/n = G_1$, is quite simple because of the horizontal top chord. Here again G_1 is not as shown in Fig. 182.

Vertical, Ee. The maximum compression for this member occurs with practically the same placing of the wheel loads that produces maximum tension in EF . Reversal of stress (§ 206) seldom occurs in the vertical of a Pettit truss.

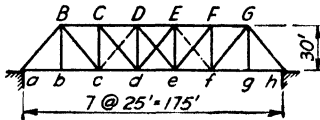
208. Equivalent Uniform Loads for Non-Triangular Influence Lines. The common charts of equivalent uniform loads are based upon the lengths of the segments of triangular influence lines. The irregular influence lines shown for the Warren truss in Fig. 178 and for the Pettit truss in Fig. 182 can not be used to obtain *exactly equivalent* uniform loads. However, the variation from the triangular influence line is not as great as is permitted in the analysis of indeterminate structures when such charts are used as a guide in the selection of equivalent uniform loads for use with *curved influence lines*. To assume that the influence line shown in Fig. 182 is a triangle with apex at d' , will not be sufficiently in error in its effect upon the selection of an equivalent uniform load to justify any uneasiness regarding its use.

PROBLEMS

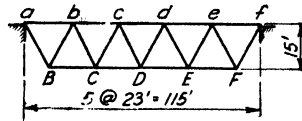
164. Obtain the design stresses for the diagonal, the counter, and the two chord members of panel cd of the Pratt truss for single-track E -50 loading. Use the impact formula $I = 300/(L + 300)$ (with loaded lengths taken from the influence lines) and a dead load of 1650 lb. per ft. of truss.

Ans. $CD = -828$; $cd = +690$; $Cd = +308$; $cD = +21$ kips.

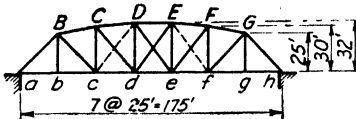
165. Obtain the design stresses for the members bc , Cc , CD , and bC of the deck Warren truss for single-track M -50 loading. Use the A.S.C.E. (1929) impact formula (6) and a dead load of 2750 lb. per ft. of bridge. Check your live-load stresses by use of equivalent uniform loads.



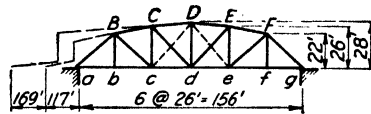
PROBLEM 164.



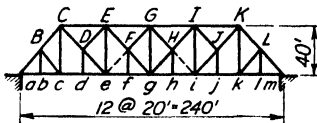
PROBLEM 165.



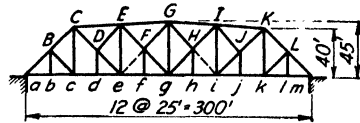
PROBLEM 166.



PROBLEM 168.



PROBLEM 170.



PROBLEM 171.

166. Compute the maximum live-load stresses in the diagonal, the counter, and the two chord members of the panel cd of the curved-chord truss. This is a double-track bridge designed for the E -72 loading on each track. Check the values of the live-load stresses by the use of equivalent uniform loads.

167. Determine the maximum resultant compression and tension in the vertical Cc of the truss of Problem 166. The dead load is 6000 lb. per ft. of bridge. The impact factor is 70%. Check your results by the use of equivalent uniform loads.

168. Compute maximum live-load stresses in the diagonal, the counter, and the two chord members of the panel cd of the curved-chord truss. This is a single-track bridge designed for the M -65 loading. Check your results by use of equivalent uniform loads.

169. Determine the maximum resultant compression and tension in the vertical Cc of the truss of Problem 168. The dead load is 3120 lb. per ft. of bridge. The impact factor is 80%.

170. Determine the maximum live-load stresses in all members of the panel ef - fg of the Baltimore truss. The bridge is to be designed for double-track E -72 loading.

171. Determine the maximum live-load stresses in all members of the panel ef - fg of the subdivided truss with sloping upper chord as shown. The bridge is to be designed for single-track E -75 loading. Obtain the final design stresses for these members based upon a dead load of 5000 lb. per ft. of bridge and the A.R.E.A. (1949) impact formula.

209. Practical Considerations in the Design of Railway Bridges. A considerable portion of this chapter has been devoted to a study of algebraic criteria. Probably the importance of criteria has been over-emphasized, since they are little needed by the practical railway-bridge engineer. After considerable experience with the use of standard engine loadings, the designer usually can place the loading for maximum by one or two trials. Accordingly, the trial method is perhaps the more important tool.

Most handbooks give tables which list the critical wheels for producing maximum moments in beams and maximum panel shears in trusses. Also, there are available tables and charts of equivalent uniform loads. These data have been checked and they are reliable. The author's view, however, is that every engineer who uses these tables or charts should be *able* to reproduce them if necessary. Consequently, considerable importance is attached to the study of locomotive loadings, a study that develops the engineer's judgment as to the effects of moving loads upon structures of all kinds.

The railroads are faced with the long-term probability of a radical change in equipment. Freight traffic must be greatly speeded up, but whether this will be accomplished by the introduction of new equipment constructed of light alloys or by the use of heavier and more powerful freight locomotives is a matter for conjecture. Full diesel equipment would greatly reduce locomotive wheel loads. The effect upon railway-bridge design is of serious import. This uncertainty of future loads might lead one to question the usefulness of wheel loadings and to wonder why a single uniform load plus one or two moving concentrations might not be quite as reasonable.

Other factors enter into the choice of the type of live loading: the proper allowance for impact, the importance of sway, the effect of traction, and the neglect of secondary stresses. Secondary stresses are considered commonly only by a reduction of the basic working stress. This procedure corresponds to an allowance for secondary stress based upon a constant percentage of the primary stress. Actually, the position of the loading for maximum primary plus secondary stress would not even be the same as for maximum primary stress alone. Even if the precision attending the use of wheel concentrations or their equivalent uniform loads is thought justified, most of this attempted precision is being wasted until some of the factors mentioned above are given greater consideration.

CHAPTER 8

LATERAL AND PORTAL BRACING OF BRIDGES

210. Lateral Bracing Systems. With the floor system and the trusses designed for the wheel loads or an equivalent simplified live loading, the bridge has adequate strength to perform its necessary function if it remains upright. It is the purpose of the lateral bracing and the portal bracing to keep the trusses upright in their proper vertical planes and to resist wind and other lateral forces. As is shown by Fig. 183 a through truss bridge has a horizontal or lateral truss at the level of each chord of the main trusses. In

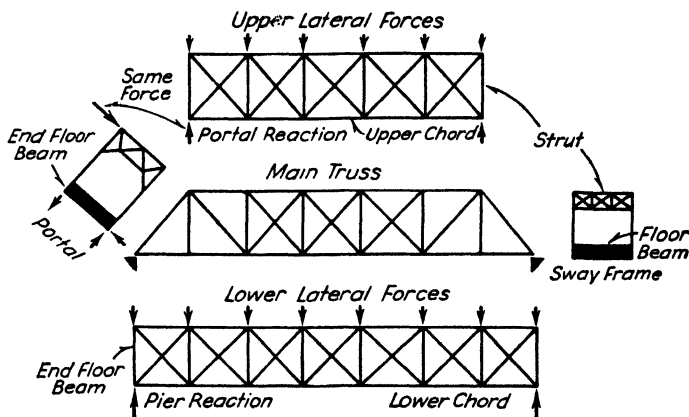


FIG. 183. LATERAL SYSTEM FOR A THROUGH-TRUSS BRIDGE.

fact, the chords of the main trusses also form the chords of the lateral trusses. The floor beams act as the cross-struts of the lower lateral truss so that this truss is formed merely by adding the diagonals shown in the lower view of Fig. 183. The upper lateral truss uses the upper chords of the main trusses and the cross-struts that are the upper horizontal members of the sway frames or end portals in conjunction with its own set of diagonals. Double diagonals are required because such members are often too long and slender to function in compression. The lateral forces may produce a positive or negative horizontal shear in any panel of either the upper or the lower lateral truss.

Function of Portals and Sway Frames. In Fig. 183 it is evident that the reaction to the upper lateral wind forces can be furnished by the end portal

if it is designed to perform this function. Then the path of stress would be through the upper laterals to the end portals and down the portals to the abutments. However, an alternate path is possible because of the need for a sway frame at each panel point to laterally stiffen the internal cross-sections of the bridge and to hold the trusses in their vertical planes. A sway frame as shown in Fig. 183 consists of a pair of vertical posts connected rigidly to the floor beam at the bottom and joined by a strut and by diagonal bracing at the top. The diagonals reach as far down the posts as is permitted by the clearance requirements. The result may be a sway frame that is nearly as stiff as an end portal. The alternate path for transfer of an upper lateral

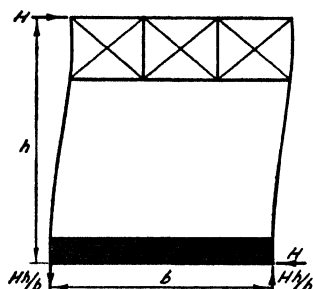


FIG. 184. SWAY-FRAME ACTION.

force to the abutments would be down the corresponding sway frame to the lower laterals and thence through the lower laterals to the abutments.

Stress Paths. The choice of path to the abutments of the upper lateral forces will depend upon the way the structure is designed to act. A sway frame or a portal can resist a horizontal force at the top only by horizontal sidesway as illustrated by Fig. 184. But remember that there are lateral forces acting on both the upper and the lower lateral trusses simultaneously as shown by Fig. 183. Hence,

upper and lower lateral trusses will deflect sideways together and the posts of the sway frames will tend to remain vertical and straight. Then the upper lateral forces must be largely transferred down the end portals whose bases are on the fixed abutments.

The argument given above must be reversed when a concrete floor slab or a welded steel grid floor is used. Such a floor forms a horizontal girder that is many times as stiff as the best lateral truss. Hence its horizontal deflection under the lower lateral forces is negligible, and the sway frame action depicted by Fig. 184 actually occurs. The upper lateral forces that are transferred down the sway frames must then be carried to the abutments by the floor system and not by the lower laterals which will be practically unstressed. Later a suggestion will be made for a division of the upper lateral forces between the portals and the sway frames of highway bridges.

LOADS AND STRESSES IN BRACING SYSTEMS

211. Lateral Wind Forces. The lateral wind pressure is the force that comes to mind immediately when we consider the design of lateral bracing. Although the specifications as given in Table 6, § (a) vary somewhat, it is

common to design for a lateral wind pressure of 30 lb. per sq. ft. on one and one-half times the side area of the trusses, the handrail, and the vertical projection of the floor and curb. This force should be divided between the top and bottom chords according to the area tributary to each. It is common practice, however, to place two-thirds of the wind force at the loaded chord. A moving force of 200 lb. per linear ft. applied 6 ft. above the floor is specified to represent the wind pressure acting on a line of motor vehicles. For a train this force is 300 lb. per ft. applied at 8 ft. above the top of the rail. See Table 6, § (b). An alternate wind loading of 50 lb. per sq. ft. acting on the unloaded bridge (§ (c)) is specified for combination with the dead load stresses wherever this alternate loading produces greater combined stresses than $DL + LL + Impact + Lateral Forces$ acting on the live load and on the loaded bridge as covered by §§ (a), (b), (e) and (f) of Table 6.

Action of the Floor System. Another purpose of the lateral bracing is to *stiffen* the structure and to prevent unwarranted lateral vibration. In order to assure the use of satisfactory bracing, it is usual to specify that a minimum moving lateral force of 150 lb. per ft. of bridge shall be applied to the unloaded chord and that a minimum force of 300 lb. per ft. shall be applied to the loaded chord for the design of the lateral bracing. See Table 6, § (d). In modern highway bridges, the reinforced concrete or welded-grid floor engages the upper flanges of the stringers and floor beams. This horizontal girder is so much stiffer than the lateral bracing that the diagonal bracing just below the plane of the floor will receive practically no stress *after the floor is completed*. The purpose served by the lateral bracing in the plane of the floor is simply to true up the bridge and to resist wind forces *during erection*. Specifications should allow these laterals to be designed of light sections to resist wind pressure on the trusses and erection equipment only. On the other hand, double diagonals preferably of stiff section should be used between the upper chords of through bridges.

212. Stresses in the Lateral Trusses. The usual forms of lateral bracing for a typical Pratt through-truss bridge are as shown in Fig. 183. The lower lateral system spans from abutment to abutment. The upper lateral system is two panel lengths shorter than the span of the bridge because of the sloping end posts.

The analysis of the stresses in the *diagonals* of the lateral trusses is carried out just as for any truss with parallel chords. If the double diagonals are both stiff members (where L/r does not exceed 140), they should both be considered to be acting. Then the lateral shear in the panel is divided equally between the two diagonals. For more slender diagonals, the tension member only is considered to be in action. The placing of the moving lateral load (all wind forces should be treated as live loads because they may be produced by gusts) for maximum shear in a particular panel is accomplished by loading

TABLE 6
LATERAL FORCES ACTING ON BRIDGES*

TITLE OF SPECIFICATION	A.A.S.H.O. SPECIFICATIONS	A.R.E.A. SPECIFICATIONS
(a) Wind on Loaded Bridge	Moving horizontal force of 30 lb. per sq. ft. on $1\frac{1}{2}$ times the area seen in elevation including floor and railings and on $\frac{1}{2}$ the area of trusses or girders in excess of two.	Moving horizontal force of 30 lb. per sq. ft. on $1\frac{1}{2}$ times the elevation of girder spans. For truss spans use area in elevation plus full area of that part of leeward trusses not shielded by floor.
(b) Wind on Live Load	200 lb. per lin. ft. at 6 ft. above roadway. 300 lb. per ft. for electric railway.	300 lb. per lin. ft. on one track applied 8 ft. above top of rail.
(c) Wind on Unloaded Bridge (Combine with <i>DL</i> only)	As an alternate wind loading take 50 lb. per sq. ft. on the areas specified in (a).	As an alternate wind loading take 50 lb. per sq. ft. on the areas specified in (a).
(d) Minimum Wind Force (Use if greater than (a) + (b))	For girders, not less than 300 lb. per lin. ft. For truss spans, not less than 300 lb. per lin. ft. for the loaded chord and 150 lb. per lin. ft. for the unloaded chord.	For girder and truss spans not less than 200 lb. per lin. ft. for the loaded chord or flange and 150 lb. per lin. ft. for the unloaded chord or flange. This Loading is unlikely to equal (a) plus (b).
(e) Nosing of Locomotive	Does not apply to highway bridges.	Apply a single moving lateral force of 20,000 lb. horizontally at top of rail. Neglect any vertical effect.
(f) Centrifugal Force with Curved Track	For electric railway track this force should be taken at 10% of the electric railway loading. It acts laterally at 4 ft. above the top of rail, (1944, A.A.S.H.O.)	A lateral force applied 6 ft. above the top of rail is given as a percentage of the wheel loading by the following formula where <i>S</i> is the speed in mph and <i>D</i> is the degree of curve $\%C.F. = 0.00117S^2D$.
(g) Stability against Over- turning	Consider the leeward lane live loaded with 400 lb. per lin. ft. without impact. Apply wind according to (a) and (b) or (d).	Consider the leeward track to be live loaded with 1200 lb. per lin. ft. without impact. Apply wind according to (a) and (b).

* Lateral and longitudinal forces are not to be increased for impact.

all joints from that panel to the farthest end of the truss. Counter action need not be considered because a heavier shear reversal occurs from wind blowing in the *opposite* direction.

The Chords and Struts of the Lateral Systems. The analysis of the stresses in the chords and struts of the lateral system of a highway bridge seldom needs to be made for the following reasons. The chords of the lateral system are also the chords of the main trusses, and the primary stresses in truss members may be increased 25% by wind before the areas of the chords are required to be increased. The wind stress in a chord member will not ordinarily amount to 25% of its primary stress except for spans over 150 ft. This effect will naturally become serious for narrow (single-track) railway bridges. The struts of the lower lateral system are the floor beams. A floor beam is of such large section that its average direct stress caused by wind will be negligibly small. However, if the lateral bracing is connected only to one flange of the floor beam, this flange will serve as the lateral strut and its resistance to lateral buckling should be checked. The struts of the upper laterals must be designed as columns. For such members the requirement of stiffness ($L/r < 140$) usually will make necessary a strut that could resist a stress several times as large as its wind stress.

The Upper Laterals of a Curved-Chord Truss. Since these members do not lie in a single plane, an exact analysis of stresses would be rather complicated. The usual assumption is that the lateral system can be straightened out into a horizontal plane for the purpose of calculating its stresses. The lateral truss then will be too long for the bridge, but the calculation of stresses will be satisfactory for design purposes. The common procedure is to transfer the real lateral joint forces to the new system without change.

213. The Portal. Since the end reactions of the upper lateral truss are formed by the portals, the maximum lateral force at the top of the portal can not exceed one-half of the entire lateral force tributary to the upper chord. Where the floor system carries a reinforced-concrete or welded-grid floor, the lateral deflections of the lower-chord panel points will be far less than the corresponding lateral deflections of the upper-chord panel points. For this typical case, perhaps as much as one-half of the upper-chord forces are transferred by the sway frames to the panel points of the lower chords. This transfer would not take place if the sidewise deflection of the lower chord was resisted by the lower laterals alone. Hence this load must be transferred to the abutments by the floor system and *not by the lower laterals*. Because of this probable transfer of load by the sway frames, it seems permissible to reduce the load on the portal from 25 to 33%, depending upon relative stiffness of portals and sway frames. This recommendation applies to highway bridges with concrete or steel-grid floors but does not apply to the typical railway bridge with open floor.

The portal actually has a small distributed load acting on its windward leg, caused by direct wind pressure, but this force is so small in comparison to the load at the top of the portal that it is as well to consider it to be concentrated at the top. Increasing the complexity of the portal analysis to take this small amount of distributed load into consideration seems an unjustifiable refinement.

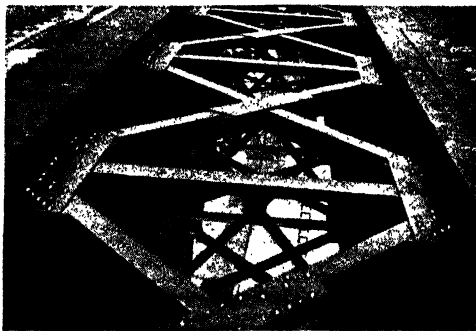
Column Base Restraint. Once the lateral force at the top of the portal is determined, the portal analysis is performed by the methods that were used in Chapter 4, § 110. Either algebraic or graphical analysis (§ 111) may be used. The only question is in regard to the proper location of the assumed points of contraflexure in the columns which are the end posts of the truss. Where a riveted connection of the end post of the end floor beam is used, it is usual to consider the lower end of the post as fixed and to locate imaginary pins half way up to the foot of the knee brace. In fact, the riveted connection may not be called upon to take any moment. Even in a pin-connected bridge, the pin connection is not in the plane of the portal, and, unless the wind moment at the base of the end post is large enough to overcome the dead load bearing on the pin, the end of the member actually will remain fixed. A reversal of bearing on the pin would produce free rotation at the base of the portal. The bending moment and direct stress in the end post as a member of the portal (see Fig. 88) is of importance since an increase in the size of its cross-section may be required for this reason.

214. The Sway Frames. Sway frames are used at all panel points. They are made as deep as possible consistent with the clearance requirements. They are of great importance in stiffening the structure. The diagonals of the sway frames may well be made of double angles riveted together and detailed short, so that they will have to be *drifted* into place with initial tension. Initial tension can also be used to advantage in the lateral bracing. It is satisfactory to detail all diagonals $\frac{1}{32}$ -in. short for each 15 ft. of length. This will introduce about 5000 lb. per sq. in. of initial tension since $(5000 \div 30,000,000)(15 \times 12) = 0.03$ in.

If the portal has been designed for less than one-half of the total lateral force acting on the top chord, the sway frames must be able to transfer the rest of this load to the floor system. At one time it was thought that heavy sway frames would equalize the effect of *unsymmetrically placed loads* upon the two main trusses, but this belief is no longer held. The sway frame and the lateral trusses on which it must depend for lateral support are so flexible as compared to the main trusses that their ability to equalize the effect of an unsymmetrically placed load must be small indeed. It will be found that the minimum angles commonly allowed by specification are satisfactory for the diagonals of sway frames, although heavier angles are frequently used for increased stiffness.

215. Centrifugal Force, Nosing and Overturning. As specified in Table 6 centrifugal force for a curved track must be treated as a distributed lateral force comparable to the wind force. Also the curvature of the track will place the locomotive or train loading off center and contribute an overturning moment. Nosing or sway of the locomotive is taken into account by placing one 20,000-lb. force at the top of the rail acting horizontally in either direction and at any point along the bridge. These horizontal forces along with wind are to be considered as affecting stresses in the lateral bracing of the loaded chord and in the other wind bracing as determined by the stress paths. Overturning of any bridge should be checked for the combination of lateral wind forces with dead load and the minimum live load of 400 lb. per lineal foot for a highway bridge or 1200 lb. per lineal foot for a railway bridge (without impact) placed on the leeward track or lane. Remember, also, that lateral forces are never increased for impact. There is room for engineering judgment here since some combinations of forces are impossible or at least improbable.

216. Lateral Systems for Deck Bridges. Railway bridges are often of deck construction, particularly for plate girders. The lateral system of a railway deck girder bridge was photographed before the laying of the ties in Fig. 185. In this case a cross frame is merely a single pair of diagonals reaching from one upper flange to the opposite lower flange of the girder. Such frames are so stiff that all lateral forces applied to the upper flange might pass down the cross frames and along the lower laterals to the abutments. This assumes that the supports are at the level of the bottom flanges of the girders. If instead, the girder is supported at the level of the upper flanges (more common for deck trusses than girders) the lower laterals might even be omitted and all lateral force transmitted to the abutments via the upper laterals. For highway deck girders having a well keyed concrete floor slab all laterals may be omitted if cross frames are used at a spacing of not over 25 ft. However, for railway bridges the laterals between the compression chords or flanges are considered of great importance and are designed for a cross shear computed from the lateral forces plus $2\frac{1}{2}\%$ of the sum of the stresses in the compression chords or flanges of the girder. Note the heavy top laterals in Fig. 185.

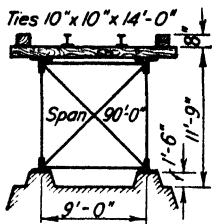


Courtesy American Bridge Co.

FIG. 185. UPPER AND LOWER LATERALS AND SWAY FRAMES OF A DECK GIRDER.

PROBLEMS

172. Lay out a complete lateral system for the truss of Problem 142. The trusses are spaced 24 ft.-6 in. apart to accommodate a 22-ft. plank floor. Determine the maximum stresses for the diagonals of the upper laterals, lower laterals, and portal caused by a lateral force of 250 lb. per ft. at the upper chord and 600 lb. per ft. at the lower chord. The sway frames are considered to be inactive.



PROBLEM 173.

173. Determine the factor of safety against overturning for the deck-girder bridge shown. Follow the A.R.E.A. instructions given in Table 6, § (g). The total weight of the 90-ft. span is 170,000 lb. Ans. 1.67.

174. Repeat Problem 173 for *DL, E-72 LL*, wind force from (a) and (b) of Table 6, nosing force from (c), but without allowance for impact. Use E.U.L. for moment at midspan to replace *E-72*.

175. Lay out a lateral system for the girder of Problem 173. Compute maximum stresses in all diagonals due to wind and nosing or sway of the *E-72* locomotive loading. For safety consider upper lateral forces transferred 67% by the intermediate cross frames and lower laterals, and also 67% by the upper laterals and end cross frames.

STRESSES PRODUCED BY LONGITUDINAL FORCES

217. Traction Stresses in Railway Bridge Trusses. Traction stresses are not important in highway-bridge design, but they must be considered in the design of railway bridges. Traction stresses are caused by the effect of the acceleration or braking of the train. This *inertia force*, acting in the direction of the track, is applied at the center of gravity of the train, or about 6 ft. above the top of the rail. In the past its value has been taken as 20% of the weight of the live load, but more recently the specification has been 15% of the entire live load due to braking or 25% of the load on the drivers for traction. This force is placed 6 ft. above the top of the rail. For highway bridges we may use 5% of the live load at 4 ft. above the roadway. The traction forces are brought to the panel points either by the floor beams acting in flexure or else by the lower laterals when they are connected to the stringers at the points of crossing. The second arrangement is to be preferred, at least for open-floor bridges. For highway bridges the low figure of 5% for traction is offset by the stringent requirement that all lanes are considered to be live loaded with traffic moving in the same direction.

Wheel Reactions. In the study of traction stresses, it is permissible to replace the traction force at the center of gravity of the train by an equal horizontal force at the top of the rail and by vertical forces at the trucks of each car caused by the overturning moment. It will be noticed in Fig. 186 that the vertical forces at the front wheel of one car and at the rear wheel of the preceding car act in opposite directions.

218. Example of the Calculation of Traction Stresses. The horizontal reaction caused by traction is resisted entirely at the fixed end of the truss, which is the right-hand end in Fig. 186.

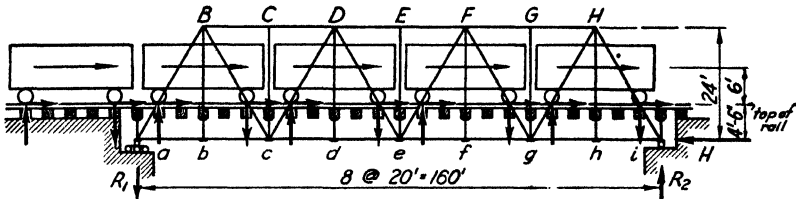


FIG. 186. TRACTION LOADS ACTING ON A TRUSS.

Place the traction force 6 ft. above the top of the rail. See Table 6. Wheel reactions on the rail due to traction alone are shown; when reversed they represent rail reactions upon the cars. The wheel reactions shown replace the traction forces and may be used to study their effect upon the floor system.

CALCULATIONS. Traction Reaction. If the loading is taken to be the equivalent uniform load for moment at the center of a 160-ft. span due to *E-72* loading, it is found to be 4500 lb. per ft. of truss. The total live load is $4500 \times 160 = 720,000$ lb. The traction force will be taken as $0.20 \times 720,000 = 144,000$ lb. The value of R_1 or R_2 is seen to be $144,000 \times (6 + 4.5) \div 160 = 9450$ lb. (Traction force applied at 6 ft. above the top of the rail.)

Web Stresses. Panel shears in the truss may all be considered to be equal, since the vertical wheel reactions in Fig. 186 tend to cancel each other. The stress in a diagonal then becomes the vertical reaction times the length of the diagonal divided by the height of the truss, or $9450 \times 31.2/24 = 12,300$ lb. This stress must be considered either as a tension or a compression because the traction force may reverse in direction.

Chord Stresses. The traction stress in a chord member may be found by taking moments about the proper panel point of the opposite chord. The diagrammatic picture of the application of the loads on brackets above the floor beams, Fig. 187, will help to clarify the

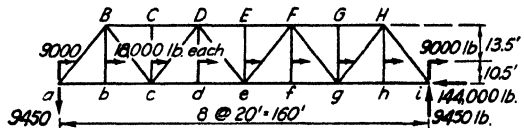


FIG. 187. CALCULATION OF TRACTION STRESSES.

equations of moments. In each case, all forces to the left (or right) of any section must be considered in computing the moment about the center of moments. The chord stress BC is found from an equation of moments about c . Thus we may write

- (a) Stress in BC : $(-9450 \times 40 + 27,000 \times 10.5) + 24 = 4,000$ lb. (T); $\Sigma M_c = 0$.
- (b) Stress in cd : $(-9450 \times 60 - 45,000 \times 13.5) + 24 = 48,900$ lb. (C); $\Sigma M_d = 0$.
- (c) Stress in hi : $(-9450 \times 140 - 135,000 \times 13.5) + 24 = 131,000$ lb. (C); $\Sigma M_h = 0$.

Incidentally, we can not compute the correct stress in the lower-chord member hi merely from the equation $\Sigma H = 0$ about the joint i . The horizontal force of 9000 lb. above the joint produces a moment about the joint and introduces shears into the members hi and Hi .

219. Use of Traction Stresses in Design. Since the traction stress in the top chord is a relatively small tension stress in a compression member, it is of no importance. The traction stresses in the web members may be large enough to affect their design. The tension stress produced in the lower chord may be of importance, but there is also the compression to be considered. This compression combined with the compression produced by wind forces (to be discussed in § 220) may reverse the stress in the end member of the bottom chord and make the use of a stiff compression member necessary. To check this possibility rather quickly it is common practice to use the full value of the traction force as the maximum stress in the lower chord rather than the smaller theoretically-correct value as determined above for the member hi of Fig. 187. Compare 131,000 with 144,000 lb. Note that $\Sigma H = 0$ applied to the joint i gives a stress of $144,000 - 9000 = 135,000$ lb. for hi . Although this equation neglects the small shears in hi and Hi mentioned above, it gives a reasonable result.

COMBINED EFFECTS OF VERTICAL AND HORIZONTAL FORCES

220. Stresses in the Main Trusses Caused by Lateral Forces. *The End Post.* This member of a through bridge receives considerable direct stress as a member of the portal. It is also heavily stressed in *flexure*. These two effects may be sufficiently important to require an increase in the size of the

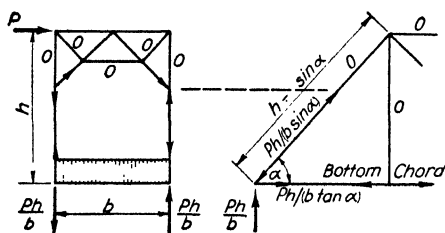


FIG. 188. PORTAL ACTION ON THE BOTTOM CHORD.

end post even though the working stress is allowed to be increased 25%, as is common for combinations of stress that include the effect of wind.

The Lower Chord. The direct stress in the inclined end post from portal action produces a direct tension or compression in the bottom chord of the truss. See Fig. 188. This stress is the *horizontal component* of the direct stress in the end post.

There is no similar stress set up in the top chord because the direct stress in the end post at its upper end (caused by portal action) is zero or is balanced by a stress in a diagonal member of the portal. See Fig. 188. Another chord stress that may reverse from tension to compression is the stress produced in the chord as a member of the lateral truss.

Overturning Moment. The overturning action that produces vertical loads on the trusses is illustrated by Fig. 189. The symbol P_1 represents the wind force on the train, which is taken as uniform; and P_2 , the force corre-

sponding to the sway of the engine or train, which was specified as a single moving concentrated load by the A.R.E.A.

In Fig. 189, the overturning moments are P_1a_1 and P_2a_2 and the truss reactions become P_1a_1/b and P_2a_2/b . These forces must be considered as moving uniform or concentrated loads on the trusses. The stresses obtained are quite as important in the design of the truss members as those caused by the *real vertical loads*. It should be remembered, however, that the stresses caused by the lateral and longitudinal forces are not increased for impact. In Fig. 189 the sway force is shown with a lever arm a_2 reaching only to the bottom of the rail. More recent specifications of A.R.E.A. specify that the nosing or sway force be applied at the top of the rail.

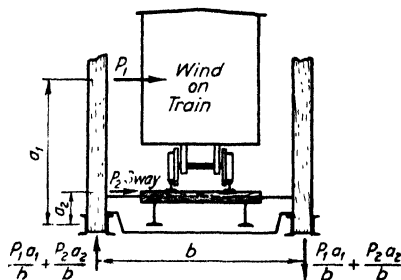


FIG. 189. LOADS ON TRUSS FROM THE OVERTURNING EFFECTS OF WIND AND SWAY.

221. Reversal of Stress in the Lower-Chord Member in the End Panel.

One of the most important effects to be investigated is the possibility of reversal of the stress in the end member of the bottom chord from tension to compression. With wind blowing from left to right, as shown in Fig. 189, the lower chord of the windward or left-hand truss is stressed as follows: (1) it receives compression as a member of the lower lateral truss from the action of wind on the truss, wind on the train, and sway; (2) the overturning effects of wind on the train and of sway produce upward loads on the windward truss and introduce compression into the lower chord although the vertical reactions due to the sway or nosing effect of the locomotive may be neglected according to A.R.E.A. specifications; (3) the direct stress in the left-hand post of the portal is a tension, and the horizontal component of this stress is resisted by a compression in the lower chord of the windward truss; (4) the traction stress in the lower chord may be a compression. Hence all of these effects may combine together to produce compression in the lower chord, or to cause reversal.

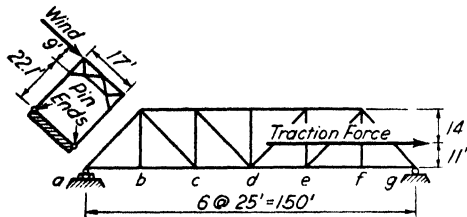
Actually, the lower-chord stress caused by dead load and live load is too high to be reversed in any panel except the *two end panels* of the bridge. Probably the reader has noticed the use of stiff lower-chord members in the end panels of a pin-connected railway bridge where the other tension members were eye-bars. Frequently this stiff member is used, even though reversal does not quite occur, in order to maintain the *same factor of safety* that exists in the other members of the bridge.

PROBLEMS

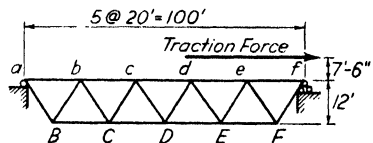
176. Lay out a complete lateral system for the truss of Problem 144. The trusses are spaced 30 ft. apart to accommodate a 27-ft. concrete slab. Determine the maximum unit stress in the lower chord as a member of the lower laterals and the maximum unit stress in the end post as a member of the portal. The lower chord has a net area of 75 sq. in. at the center. The end post has an equal area and a section modulus of 650 in.³ in the direction of bending. The lateral forces are 250 lb. per ft. at the upper chord and 575 lb. per ft. at the lower chord. Assume that one-third of the upper-chord load is transferred by the sway frames.

177. The maximum stress in the lower chord of a 150-ft. Pratt highway truss from dead load, live load, and impact is 290,000 lb. If the trusses are spaced 20 ft.-6 in. apart, find the lateral force necessary to increase this stress 25%. Assume that there are ten 15-ft. panels. If the area seen in elevation is equivalent to 40% of the outline of the truss which is 25 ft. deep, two-thirds of this area being tributary to the lower chord, apply the A.A.S.H.O. specifications of Table 6, §§ (a) and (b) and decide whether the chord of such a truss might need to be increased in size because of lateral forces.

Ans. Wind forces fail to produce reversal by 5.3 %.



PROBLEM 178.



PROBLEM 180.

178. The single-track Pratt railway bridge shown has a dead weight of 1500 lb. per ft. of truss. The live loading is *E-60*. Consider the traction force to be 20% of the equivalent uniform load for moment at the center of the span. Take the wind forces on the loaded chord at 400 lb. per lineal ft. and on the unloaded chord at 300 lb. per ft. Lay out a standard lateral system with tension diagonals for this bridge and determine whether the lateral and longitudinal forces will reverse the member *fg* to compression. Do not consider impact. Apply the 20,000-lb. sway force at 6½ ft. below the traction force and the wind on the train at 2 ft. above the traction force. Take the portal as pin-connected at the top of the 3-ft. floor beam.

Ans. No reversal; *DL + LL* stress = + 330,000 lb. while

- (a) Lower-lateral action = - 88,700
- (b) Overturning effects = - 18,700
- (c) Portal action = - 24,200
- (d) Traction stress = - 103,200

Total = - 234,800 lb.

179. Correct all data and calculations of Problem 178 to fit *E-72* loading and the A.R.E.A. specifications of Table 6. Assume that the floor is 4 ft. deep and that 44% of the outline of the truss is solid when the elevation of the floor is included.

180. The deck Warren bridge shown has a dead load of 1450 lb. per ft. of truss. The live loading is *M-60* (single track). Consider the traction force to be 15% of the equivalent uniform load for moment at the center of the span. Take the wind force on the bridge from the A.R.E.A. values of Table 6. The trusses are spaced 10 ft. apart. There are no

lower laterals, diagonally braced sway frames being used instead. Lay out a standard upper lateral system and determine the percentage increase in the *DL* and *LL* stresses caused by the effect of each lateral and longitudinal force for the members *cd*, *dD*, and *DE*.

181. Determine the possibility of reversal in the member *ab* of the Pettit railway truss of Fig. 182. Assume that this is a double-track structure designed for the *E-72* loading. Use your own judgment in the selection of all data.

222. Unified Specifications for Bridge Design. Since lateral forces are difficult to measure accurately and since the bracing systems of bridges are almost always statically indeterminate (more than one possible stress path), specifications play a very important part in the design of bracing systems. It is therefore confusing to find that the A.R.E.A. and A.A.S.H.O. specifications do not always agree. A difference would be expected in the specified traction force since the traction effect of a locomotive is far different from that of a line of motor trucks. But no difference is justified in the method of computing wind pressures on the structure although we find considerable variation between A.R.E.A. and A.A.S.H.O. specifications on this point in Table 6, § (*a*).

Another unjustified difference is found in the minimum permissible value of the slenderness ratio for bracing members that act in compression. The minimum l/r value for such members is 140 in the A.A.S.H.O. specifications and 120 in A.R.E.A. specifications. The safe value for slenderness ratio of a compression member can be established by tests in a laboratory. It does not depend upon the source of the loads. If some difference is necessary to take care of the severe vibration of railroad bridges, it could be placed in the specification of the loads and not made up by minor differences in design procedures. It is gratifying to note that the main working stresses for structural steel as specified by A.R.E.A. and A.A.S.H.O. (1949) do agree. A joint specification for highway and railway bridges could readily be prepared.

CHAPTER 9

THE PLATE GIRDER

223. Types of Plate Girders and Their Uses. The plate girder is simply an oversize built-up beam; as such, it may be looked upon as intermediate in carrying capacity between the rolled beam and the truss. Steel bridges of under 50-ft. span are usually of the beam type. Above a span of 50 ft. the plate girder may be found economical. The plate-girder bridge is used as a railway structure for spans up to 125 ft. or even more, although it is not as economical of material as the truss for spans near this upper limit of length. *Fabrication and erection costs* are less for the plate girder than for the truss. This accounts for exceptional girder spans of 200 to 300 ft.

The use of the girder bridge for long spans has often been dictated by the fact that it can be *rolled into place* in a few hours without serious delay of traffic, while the construction of a truss bridge requires several days unless elaborate preparations for its erection on falsework to one side of the old span are made. The plate girder is less widely used as a highway bridge. Lengths under 60 ft. can be spanned by the beam bridge of steel or of reinforced concrete. For spans above 60 ft. and below 140 ft., the low-truss bridge may be preferred.

Bridge Girders. Plate-girder bridges may be either of deck or of through construction. The floor may span laterally from girder to girder, or floor beams may be used, as in a truss bridge, with stringers to span from floor beam to floor beam. A layout of the floor system is necessary before an analysis of the girders can be made because the application of the live load to the girder occurs through the medium of the floor system. In a deck-girder bridge with ties resting directly upon the top flange, the live load is considered to be rolling along the top of the girder. If floor beams are used, as in a through-girder bridge, the live load (and the dead load of the floor) is concentrated at *panel points* just as in a truss bridge. The reader is referred to Fig. 111 for illustrations of several types of floor systems for railway plate-girder bridges.*

Building Girders. The use of the plate girder is not confined to bridge structures. It is commonly used in all types of buildings to span between

* Consideration is being given to aluminum alloy as a structural material for plate girders. R. L. Moore, *Observations on the behavior of aluminum alloy test girders*, Transactions, A.S.C.E., Vol. 112, 1947, pp. 901-920.



Courtesy American Bridge Co.

FIG. 190. BUILDING GIRDER ONE STORY DEEP.

Notice the horizontal web spline. Plates 12 ft. deep were not obtainable. Also notice the drift pins in use for making a temporary connection of the girder to the left hand column.

columns and to carry heavy concentrated loads. The analysis of a building girder such as Fig. 190 is relatively simple since there are no *moving* loads to be considered. The overhead crane is designed for a *moving* vertical load

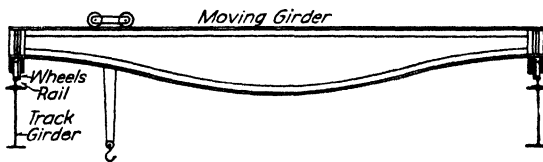


FIG. 191. OVERHEAD CRANE FOR A MILL BUILDING.

(as shown in Fig. 191) and also for a lateral traction force equal to 20% of the lifting capacity of the crane.

The Cross-Section. The typical cross-section of a riveted railway girder is similar to Fig. 192(a). Heavier sections are obtained by the use of heavier

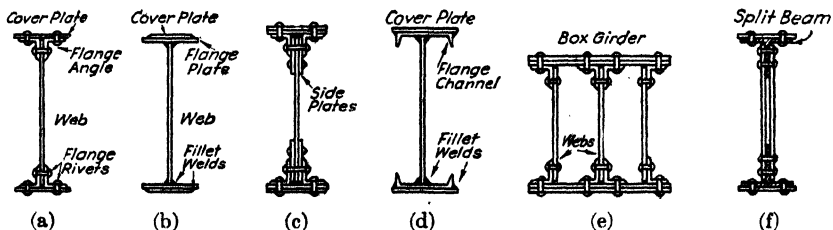


FIG. 192. PLATE-GIRDER SECTIONS.

angles with double rows of rivets and the addition of extra cover plates. A typical section for a welded building girder is shown in (b). Other sections are shown in (c) and (d). Figure 192(e) is a box-girder section that is used occasionally either in bridge construction or in building construction where great strength is required and the *depth is limited*. Type (f) may also be useful for a shallow section.*

224. Stresses to be Considered in Girders. A plate-girder section is made up of a web and two flanges. The web and the flanges are connected by means of rivets or by arc welding. The main function of the web is to resist the *vertical shear*. The flanges, which are similar to the chords of a truss, produce the major resistance of the section to *moment*. The connection between the web and the flange must resist the *horizontal shear* existing along this line.

Web Stresses. The web of a girder, being relatively tall and thin, is a poor compression member. Wherever a load is concentrated upon the upper flange or a reaction acts upon the lower flange, the web must be stiffened to prevent it from *buckling* or *wrinkling*; these types of failure are illustrated by Fig. 193. Stiffener angles may be riveted to the web or stiffener plates may be welded to the web to prevent such failure.

An analysis shows that diagonal compression always accompanies shear. See Fig. 193(c). A thin web must be stiffened at intervals to prevent buckling from the effect of this diagonal compression. Experience has shown that a web which is at least $\frac{1}{80}$ of the *unsupported depth* in thickness is safe from buckling by diagonal compression, but a web that is thinner than $\frac{1}{80}$ of the unsupported depth must be stiffened at intervals not exceeding the *clear depth* of the web.† Stiffeners more widely spaced would permit one complete diagonal wrinkle between stiffeners. The minimum web thicknesses are $\frac{5}{16}$ in. for building girders, $\frac{3}{8}$ in. for bridge girders, and $\frac{1}{16}$ of the *unsupported depth* or the *clear depth* between the flanges.

The purpose of this chapter is to analyze the plate girder for all of the stresses mentioned above, and to determine in general how to apply these methods of analysis to the design of plate girders. It will not be possible to go into the subject of standard specifications and their application to design.

Tension Field Girders. In airplane design it is necessary to reduce weight to the minimum. Hence webs may have to be much less than $\frac{1}{80}$ times the depth in thickness. Such thin webs will wrinkle at 90 degrees to the direction of the diagonal compression shown in Fig. 193(c). Collapse would occur if the vertical stiffeners were to be omitted. However, with vertical stiffeners designed to resist compression, the thin web can reproduce the effect of the tension diagonals of a Pratt truss, and the resulting structure will be stable.

* For a description, see Engineering News-Record, October 29, 1931.

† This factor is set at $\frac{1}{80}$ for building girders.

The stress analysis of the chords and verticals would be identical with the analysis of a corresponding Pratt truss. The total stress calculated for a diagonal of the truss might be considered to be resisted by a diagonal strip of web equal to $\frac{1}{4}$ to $\frac{1}{3}$ of the depth. A width of web equal to $24t$ on each side

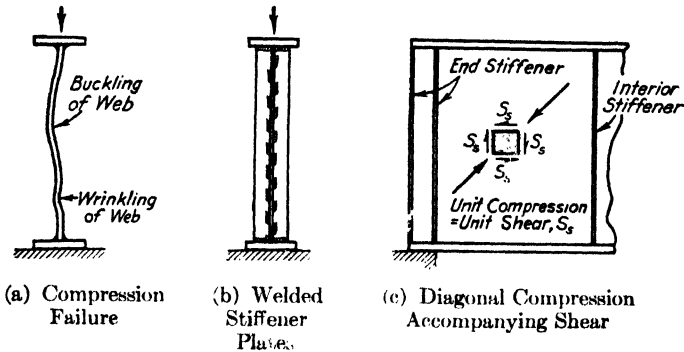


FIG. 193. PURPOSE OF STIFFENERS.

of the stiffener adds to the stiffener area resisting compression. Such girders of steel might prove useful for deep building girders where a web thickness of $\frac{1}{10}$ times the depth would prove uneconomical.

225. Loads, Shears, and Bending Moments. The loads acting on a railway plate girder may be applied directly to its upper flange, or the loads may be concentrated at panel points as floor-beam reactions. A building girder may have heavy concentrations of live load and dead load. A crane girder is subjected to a moving pair of wheel loads and to a lateral force caused by inertia. In each case, the calculation of the maximum shear and the maximum moment at any section follows directly from the methods developed in the preceding chapters. These methods need not be reviewed here.

APPROXIMATE ANALYSIS AND THE BEAM THEORY

226. Beam Formulas Applied to a Riveted Girder. The treatment of a plate girder as a solid beam for the calculation of stresses is the standard practice. This procedure needs no justification for a welded girder. However, an investigation of the preceding assumption becomes necessary for the riveted girder, since we know that there is slip of the rivets at a stress slightly above their normal working stress. The reader will notice that all flange rivets in the plate girder occur in long horizontal lines. See Fig. 195. Such a row of rivets will contain a hundred or more. We can normally expect better frictional resistance between the connected parts than would exist between the parts of a *small* riveted connection. Moreover, since the horizontal shear on

all rivets of a row will not be a maximum at the same time, any tendency toward localized slip will be resisted by adjacent rivets. This argument seems to justify the assumptions that the riveted girder acts as a solid beam and that its fiber stress and its shearing stress at any point may be computed from the *beam formulas*:

$$(1) \quad f = \frac{Mc}{I},$$

where f = fiber stress (lb. per sq. in.), M = bending moment at the section (in.-lb.), I = moment of inertia of the cross-section about the neutral axis with an appropriate deduction for rivet holes (in.⁴), c = extreme fiber distance measured from the neutral axis (in.); and

$$(2) \quad v = \frac{VQ}{I},$$

where v = horizontal shear per lineal inch of girder (lb. per in.), V = vertical shear acting at the section (lb.), I = moment of inertia of the cross-section about the neutral axis, gross moment of inertia in this case (in.⁴), Q = statical moment about the neutral axis of that part of the cross-sectional area *outside* of the line on which shear is to be calculated (in.² × in. = in.³)

227. Maximum Unit Shear in the Web of a Plate Girder. Since the horizontal and the vertical unit shears are equal at any point in the web of a girder, the unit vertical shear might be calculated by use of equation (2) above. The value of v in pounds per lineal inch of girder must be divided

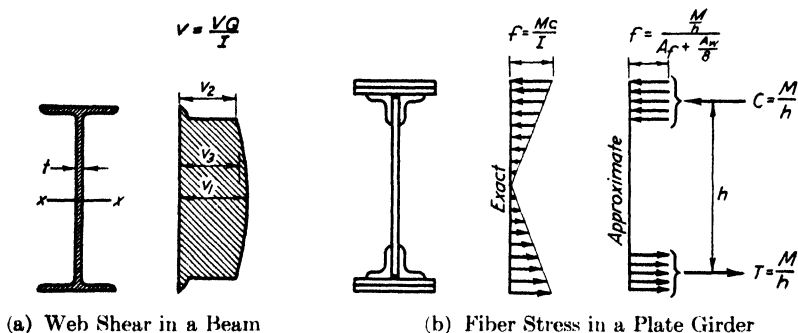


FIG. 194. STRESS VARIATION IN BEAMS AND GIRDERS.

by the thickness of the web to obtain the unit shear s_v in pounds per square inch. The variation of shear will correspond closely to the variation for an I-beam, as shown in Fig. 194(a) where it is noticed that the average shear v_3 is but little smaller than the maximum shear v_1 at the neutral axis. For this reason, an exact computation of the maximum shear by means of the shear formula is seldom made. Instead, it is more common to assume that the

vertical unit shear is *uniform over the entire depth of the web* and to select the web of such thickness that the average unit shear on its *gross area* will not exceed the allowable value of 11,000, 12,000 or 13,000 lb. per sq. in. according to the requirement of the specifications. It is assumed that the web carries the *entire shear*. The allowable unit shear is specified on the *gross area* of the web.

228. Fiber Stresses by the Exact Method. *Welded Girder.* The flange stresses in a plate girder can be calculated exactly by use of the beam flexure formula. See equation (1) above. When we use the term "exact calculation," as above, it is naturally with the mental reservations that the assumptions involved in the flexure formula require. The use of the flexure formula for checking the fiber stress in a welded plate girder is extremely simple. The *gross* and *net* sections are identical; hence the neutral axis must lie at the center of gravity of the gross cross-section or at the mid-height of a symmetrical section. The gross moment of inertia about the neutral axis is used and the fiber stresses are calculated as for a rolled beam. Specifications of the American Institute of Steel Construction (A.I.S.C.) permit the neglect of rivet holes up to 15% of the section and the use of the full gross cross-section of a riveted building girder just as for the case of a welded girder. However, bridge specifications are more conservative and require the consideration of the net section.

Riveted Girder. The calculation of the fiber stresses for a riveted plate girder is complicated by the fact that the *net* and *gross* sections have considerably different areas. The net area is considered effective on the tension side of the girder, while the gross area is effective on the compression side. Since the neutral axis lies at the center of gravity of the effective section, it is *above* the mid-height of the normal symmetrical girder. When the neutral axis has been located, the moment of inertia of the effective section about this axis can be determined, and then the fiber stresses may be obtained by use of the flexure formula.

Unknown Factors Involved. Many engineers have felt that the determination of the exact location of the neutral axis as the center of gravity of the effective section is an unjustifiable procedure.* This seems particularly true when we consider the fact that the *fully-reduced* net section used occurs only at stiffeners and at web splices where vertical rows of rivets are driven through the web. The neutral axis can not be a straight horizontal line, but it must shift upward at each vertical row of rivets, as shown in Fig. 195. The neutral axis between stiffeners would shift slightly at each rivet in the tension flange. It seems impossible that there can be a sudden break in the neutral axis, for this break would produce a *sudden change in fiber stress*, which is

* S. B. Lilly and S. T. Carpenter, *Effective Moment of Inertia of a Plate Girder*, Transactions, A.S.C.E., Vol. 105, 1940, pp. 1462-1518.

quite obviously impossible. Accordingly, the neutral axis is shown as a smooth curve, although the shape of the curve is admittedly unknown.

Simplified Semi-Exact Procedure. Because of these unknown factors, which make the use of the effective section rather questionable, a simpler method has been recommended in A.R.E.A. and A.A.S.H.O. specifications. By this simplified method, we consider the full gross area effective when we compute the fiber stress in the compression flange, but we deduct all rivet holes for the full depth of the girder to obtain the net section for calculating the stress in the tension flange. In both cases, the neutral axis lies at the mid-height if the girder is of the usual symmetrical type. Both net and gross moments of inertia are easily computed.

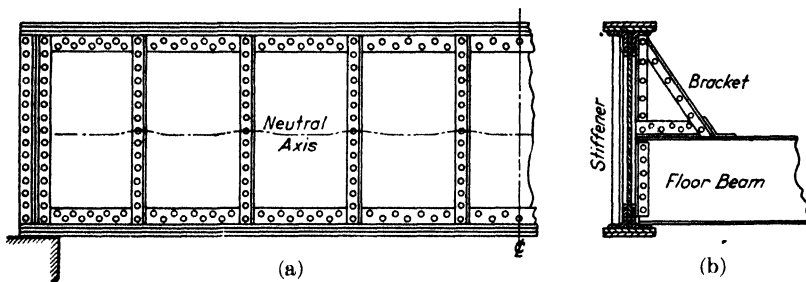


FIG. 195. VARIATION OF THE NEUTRAL AXIS WITH NET SECTION.

Comparison of Methods. At first thought, we probably would expect this second method to show a much higher tension stress and a correspondingly lower compression stress than the first method because of the changed moments of inertia. Note, however, that the neutral axis has been lowered, and the extreme fiber distance to the tension side has been decreased, while the distance to the compression side has been increased. This *shift in the neutral axis* may amount to 2 in. or more. The corresponding change in the extreme fiber distances has an important effect upon the calculated fiber stresses. The result is that the fiber stresses determined by the two methods will not differ by more than about *two per cent*.

229. Approximate Method of Stress Calculation for Use in Design. In the design of a plate girder, we must obtain a reasonable cross-section before the moment of inertia can be calculated and the stresses checked. An approximate method is widely used for this purpose, and, in fact, frequently it is the only method used for the design of a deep girder. The stresses calculated by the approximate method for a shallow girder should be checked by the exact method or the semi-exact procedure just described.

The reader undoubtedly has observed that the flange material in a plate girder is grouped quite close together and that the moment of inertia of the section is furnished largely (about 85%) by the flanges rather than by the

web. Therefore the assumption that the flange stress is nearly constant over the entire area of the flange can not be seriously in error. See Fig. 194(b). If the distance between centers of gravity of the flanges could be known, we might divide the bending moment by this effective depth, shown as h in Fig. 194(b), and obtain the total flange stress C or T . Since the resistance of the web even then would have to be introduced by approximation, it is common practice to estimate the *effective depth* as the depth of the web or the distance back to back of flange angles for a deep girder with more than three cover plates, and from two to four inches less than the depth of the web for a girder of average proportions.

Moment Resistance of the Web. Some of the older specifications did not allow the web to be considered as furnishing moment resistance in a plate girder, but present-day design allows for the resistance of the web. It is necessary to determine how much the *net area* of the tension flange may be assumed to be increased by the effect of the web. The moment of inertia of the gross web is $\frac{1}{12}th^3$ or $\frac{1}{12}Ah^2$, where t is the web thickness, h is the web depth, and A is the *gross web area*. An area equal to $A/6$, if placed in each flange, would have an approximate moment of inertia of

$$2 \left(\frac{A}{6} \times \frac{h^2}{4} \right) = \frac{1}{12} Ah^2.$$

We conclude that $\frac{1}{6}$ of the gross area of the web may be considered to form a part of the gross area of each flange when we are calculating the stress in a welded girder or when we are investigating the compression flange of a riveted girder.

Net Section. At the net section of a riveted girder, we allow for a vertical row of 1-in. holes in the web, spaced 4 in. apart. The cross-sectional area and the moment of inertia will be reduced 25% when the holes are deducted. This reduces the factor $\frac{1}{6}$ down to $\frac{1}{8}$ and shows that but $\frac{1}{8}$ of the gross area of the web should be considered as *net area of the tension flange*. It would be permissible to use $\frac{1}{6}$ of the net web area, but the use of $\frac{1}{8}$ of the gross area is more common.

Girder Design. The reader will find that this method of analysis lends itself unusually well to design. The maximum bending moment is divided by the assumed effective depth, taken as the depth of the web, or somewhat less, to obtain the total flange stress. This total stress is divided by the allowable fiber stress in tension to obtain the required *effective net flange area*. From this area, $\frac{1}{8}$ of the gross web area is deducted to arrive at the *net flange area*, which is made up of *two angles and one or more cover plates*. A comparison of the fiber stresses as computed for a standard riveted girder by the exact method, by the simplified semi-exact method, and by the approximate method, is given on the computation sheet, Fig. 196.

CALCULATION SHEET — APPROXIMATE AND EXACT CALCULATION OF GIRDER STRESSES

Area of Cross-Section —

Gross = 105.3 sq. in.
 Net = 86.7 (2 holes out of each L and cov.)
 Effective = 96.0 (holes out of tens. flange only)

Effective Area of Flange —

Gross = 45.1 sq. in. (incl. 1/8 A_{web})
 Net = 37.7 (incl. 1/8 A_{web})

Gross Moment of Inertia —

Web $\frac{1}{12} \times 0.375 \times 60^3 = 6,750$
 Angles $I_1 = 4 \times 28.2 = 110$
 $Ad^2 = 4 \times 8.44 \times 28.5^2 = 27,700$
 Covers $I_0 = 2 \times \frac{1}{12} \times 14 \times 1.75^3 = 10$
 $Ad^2 = 2 \times 14 \times 1.75 \times 31.1^2 = 47,400$
 Gross $I = 81,770 \text{ in.}^4$

Deduction for 1-in. Holes —

Web 25% of Gross $I = 1,690$
 Angles $8 \times 1 \times 0.75 \times 27.7^2 = 4,600$
 Covers $4 \times 1 \times 1.75 \times 31.1^2 = 6,770$
 Deduction = 13,060
 Net $I = 81,770 - 13,060 = 68,710 \text{ in.}^4$

Location of the Neutral Axis of the Effective Section — (Area = 96.0)

(The statical moment about the center line is equal to the statical moment of the holes below the mid-height.)

Web 25% $(30 \times 0.375 \times 15) = 42$
 Angles $4 \times 1 \times 0.75 \times 27.7 = 83$
 Covers $2 \times 1 \times 1.75 \times 31.1 = 109$

Statistical moment about the center line = 234 in.³

Eccentricity of the Neutral Axis above the Mid-Height = $234/96.0 = 2.44 \text{ in.}$

Gross I about the Neutral Axis = $81,770 + 105.3 \times 2.44^2 = 82,400 \text{ in.}^4$

Deduction for Holes below the Neutral Axis

Web 25% $(11/3 \times 0.375 \times 32.5^3) = 1,080$
 Lower Angles: $4 \times 1 \times 0.75 \times 30.1^2 = 2,720$
 Lower Covers $2 \times 1 \times 1.75 \times 33.5^2 = 3,930$
 Total Deduction = 7,730 in.⁴

Effective I about the Neutral Axis = $82,400 - 7,730 = 74,670 \text{ in.}^4$

Stresses Calculated for a Bending Moment of 34,300,000 in.-lb.

From Net I : $f_t = \frac{Mc}{I} = \frac{34,300,000 \times 32}{68,710} = 16,000 \text{ lb. per sq. in.}$

From Gross I : $f_c = \frac{Mc}{I} = \frac{34,300,000 \times 32}{81,770} = 13,500$

From Effective I : $f_t = \frac{Mc}{I} = \frac{34,300,000 \times 34.45}{74,670} = 15,900$

From Effective I : $f_c = \frac{Mc}{I} = \frac{34,300,000 \times 29.55}{74,670} = 13,600$

Assumption that Effective Depth is 3 in. less than Depth of Web, or 57 in.

Approximate $f_t = \frac{M}{hA_{\text{net}}} = \frac{34,300,000}{57 \times 37.7} = 15,900 \text{ lb. per sq. in.}$

Approximate $f_c = \frac{M}{hA_{\text{gross}}} = \frac{34,300,000}{57 \times 45.1} = 13,400 \text{ lb. per sq. in.}$

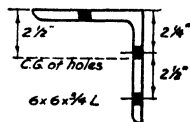
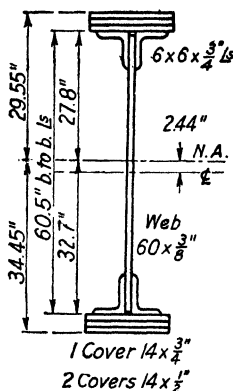


FIG. 196

If the girder is to be welded, the procedure is varied by deducting $\frac{1}{8}$ of the gross web area from the required gross flange area. Then the flange plates or channel and cover (see Fig. 192) are selected to furnish the required flange area. The compression flange commonly is identical with the tension flange. The method of checking the compression flange for safety against buckling is given below.

PROBLEMS

182. A light plate-girder section is composed of $6 \times 6 \times \frac{1}{2}$ -in. angles placed 40.5 in. back to back, a $14 \times \frac{1}{2}$ -in. cover plate in each flange, and a $40 \times \frac{3}{8}$ -in. web plate arranged as in Fig. 192(a). The total vertical shear is 150,000 lb. Compute the unit shear at the neutral axis and at the connection between web and flange by use of the shear formula. Compare the two computed values of the unit shear with the average unit shear in the web.

Ans. Maximum unit shear = 10,900 lb. per sq. in.

183. Same as Problem 182 except the girder is of welded construction, having a web plate $42 \times \frac{3}{8}$ in., a flange plate $12 \times \frac{5}{8}$ in., and a cover plate $10 \times \frac{5}{8}$ in. The section is made up as shown in Fig. 192(b). *Ans.* Maximum unit shear = 10,000 lb. per sq. in.

184. Locate the neutral axis and compute the fiber stresses caused by a moment of 750,000 ft.-lb. in the girder of Problem 182. Deduct holes as follows from the tension side of the girder: two 1-in. holes from each tension angle, two 1-in. holes from the cover plate, a vertical row of 1-in. holes placed 4 in. apart from the tension side of the web.

Ans. 12,000 and 14,000 lb. per sq. in.

185. Determine the fiber stresses for the girder of Problem 184 by using the full gross moment of inertia and the fully reduced net moment of inertia with the neutral axis at the mid-height.

Ans. 11,800 and 14,200 lb. per sq. in.

186. Compute the fiber stresses for the girder of Problem 184 by use of the approximate method, using a lever arm first of the depth of the web and then of the distance back to back of angles. Compare these stresses with those of Problems 184 and 185. The discrepancy should be greater for this shallow girder than for a deeper one. Find the effective depth.

Ans. 36.3 in.

187. A welded girder is composed of a $48 \times \frac{1}{2}$ -in. web, two 12-in. $\times$ 25-lb. flange channels, and two $14 \times \frac{1}{2}$ -in. covers as shown in Fig. 192(d). The bending moment is 1,200,000 ft.-lb. Compare the fiber stress computed by the beam formula with the fiber stress determined from an estimated effective depth of 46 in.

188. A riveted girder has a web plate $80 \times \frac{7}{16}$ in., flange angles are $6 \times 6 \times \frac{3}{4}$ in., and there are three $14 \times \frac{3}{4}$ -in. covers on each flange. The angles are placed $80\frac{1}{2}$ in. back to back. When determining net section, allow two holes out of each angle and each cover (1-in. holes for $\frac{7}{8}$ -in. rivets). The holes in the stiffener angles are at 5-in. spacing. Compare the resisting moment of the section as determined by the three different methods for a maximum fiber stress in tension of 18,000 lb. per sq. in. Repeat for a compression fiber stress of 16,000 lb. per sq. in. Use the gross section. Use the depth of the web as the effective depth.

189. Determine the resisting moment of the welded girder of Problem 183 by the exact method and by the approximate method. Assume that the fiber stress in tension is 20,000 lb. per sq. in.

Ans. 1130 kip-ft. by the exact method.

230. Allowable Stress in the Compression Flange. The compression flange of a girder acts essentially as a *column*. Failure of this column may

occur by local wrinkling of a plate or by sidewise buckling of the flange over a considerable length that is unsupported laterally. Specifications usually require that the outstanding or *unsupported width* of a plate which forms part of a compression member shall not exceed 12 (A.I.S.C. permits 16) times its thickness. An analysis shows that this specification is highly conservative and that local wrinkling of such a plate will not occur.

Column Action. The compression flange of a through railway girder can be supported laterally by brackets to the floor beams, as shown in Fig. 195. The flange must act as an unsupported column *between* these brackets. If the length of span between lateral supports exceeds 15 times the flange width, there is danger that failure may occur from lateral buckling at a reduced stress. In order to compensate for this weakness, it is necessary to reduce the allowable flange stress in compression in the same manner that the allowable stress on a long column is reduced.

SPECIFICATIONS. All of the standard types of column formulas have been used to limit the allowable compressive stress in a girder flange which must be reduced with increase in the unsupported length L and with decrease in width b of the compressive flange. For example:

$$(3) \quad \frac{P}{A} = 16,000 - 150 L/b \text{ (old A.R.E.A. formula)}$$

$$(4) \quad \frac{P}{A} = \frac{20,000}{1 + \frac{L^2}{2000b^2}} \text{ (old A.I.S.C. formula)}$$

$$(5) \quad \frac{P}{A} = 18,000 - 5 \frac{L^2}{b^2} \text{ (1949 A.R.E.A. and A.A.S.H.O.)}$$

These formulas should be compared with the current A.I.S.C. formula which involves more variables.

231. Reduction of Flange Area. The maximum bending moment in a girder occurs near the center of the span. Irrespective of the type of loading, the variation in maximum moment for all points along the span is *nearly parabolic*. In other words, the maximum moment at the quarter point is approximately 75% of the maximum moment at the center of the span, etc. Note that the reduction in maximum moment varies as the *square* of the distance out from the center. At one-half of the distance from the center to the end, the reduction in moment will be one-quarter, at three-quarters of the distance from the center to the end the reduction will be nine-sixteenths, etc.

Point of Cut-Off for the Cover Plate. Since the entire cover plate must be cut off at one point, this point of cut-off is determined by the resisting moment of the *remainder* of the section. Either by plotting a true curve of maximum moments, or by a simple computation based upon an assumed parabolic variation of maximum moment, the point is located at which the bending moment just equals the resisting moment of the section with one cover plate removed. It is then common practice to cut off the cover plate not less than

one foot nearer the end of the beam than this *theoretical point of cut-off*. The extra length provides an added assurance of safety because it allows the cover plate to be partially developed by the rivets at the point where it theoretically becomes partly needed. A cross end-weld might be used to develop the full value of a cover in welded construction, but this is not possible with rivets.

Design Practice. The procedure explained above is repeated in determining the points of cut-off for successive cover plates. All covers may be cut off of the lower flange, leaving only the flange angles, but one cover must be run for the full length of the upper flange to act as protection from the weather unless the girder is encased in concrete. Otherwise, water will work down between the angle legs and excessive corrosion may occur.

Fatigue Fracture at End of Cover Plate. Experiments by W. M. Wilson have established the anticipated result that the cut-off point of a cover plate is a point of high stress concentration at which a fatigue crack may develop at a relatively low load. Bridge girders, crane girders and girders supporting moving industrial equipment are subjected to a sufficient number of repetitions of the design loading to make fatigue fracture an important consideration.

Fracture at 2,000,000 Cycles.

For fracture at 2,000,000 repetitions of a flexural load producing a stress cycle from zero to a maximum stress, the fracture stress was found to be:

- (a) 32,700 lb. per sq. in. for 16-in., 36-lb. *WF* beams without holes or attachments on the tension side of the beam.
- (b) 8000 to 11,000 lb. per sq. in. for similar beams with partial-length cover plates attached with longitudinal welds.
- (c) 22,800 lb. per sq. in. for similar beams with full-length cover plates attached with continuous fillet welds.
- (d) 16,500 lb. per sq. in. for similar beams with full-length cover plates attached with intermittent fillet welds.
- (e) 16,900 lb. per sq. in. for a beam fabricated by welding three plates together with continuous fillet welds.

Effect of Welded Intermediate Stiffeners:

- (a) 18,800 lb. per sq. in. with stiffeners welded to the web and to both flanges.
- (b) 26,600 lb. per sq. in. with stiffeners welded to the compression flange and the full depth of web.
- (c) 32,700 lb. per sq. in. with stiffeners welded to the compression flange and the compression side of the web.

Effect of a Rivet Hole:

- (a) 26,000 lb. per sq. in. is the established fatigue resistance of a plate with a hole acting in tension for 2,000,000 applications of load increasing from zero to a maximum. A hole in the tension flange of a beam would be expected to have a similar effect.
- (b) This effect seems to be independent of the ultimate strength or the alloy composition of the steel.

Fracture at 100,000 Cycles.

- (a) 40,000 lb. per sq. in. for welded full-length cover plates.
- (b) 20,000 lb. per sq. in. for welded partial-length cover plates.

CONNECTIONS AND DETAILS

232. Shear between Web and Flange — *Exact Method.* The connection between the web and the flange is made either by riveting or by welding the two parts together. It will be observed that flange rivets, which pass through two angles and the web, are in double shear or in bearing on the web, while flange welds are simply stressed in longitudinal shear. It is sufficient that methods be developed for finding the longitudinal shear per lineal inch of girder. With this information, we can readily determine the shear per rivet and the required rivet spacing or the size of fillet for a longitudinal weld.* The exact calculation of the horizontal shear per lineal inch between the web and the flange can be made by use of the *shear formula*

$$(6) \quad v = \frac{VQ}{I}.$$

In this formula, Q is the statical moment about the *neutral axis* of the cross-sectional area of all metal *outside* of the section on which shear is to be calculated. In Fig. 197(a), this area includes two angles and a cover plate, while in (b) it includes the channel and the cover plate.

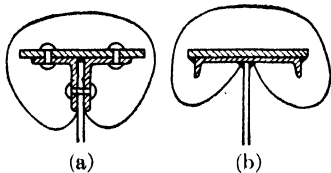


FIG. 197. CONNECTION BETWEEN WEB AND FLANGE.

If the shear acting between the angles and the cover plate of Fig. 197(a), or between the cover plate and the channel in (b), is desired, the value of Q is determined from the area of the cover plate only. In all cases I represents the moment of inertia of the *gross cross-section*.

233. Shear between Web and Flange —

Approximate Method. The approximate method of determining shear between the web and the flange will be made dependent upon the assumption that the maximum web shear is the same as the average shear on the web. Clearly, the horizontal shear between the web and the flange per lineal inch of girder must be the same as the horizontal shear per lineal inch in the web itself near this connection line. Therefore, since the horizontal and vertical unit shears in the web are equal at any point, the horizontal shear per lineal inch between the web and the flange is equal approximately to the *average* vertical shear, or

$$(7) \quad v = \frac{V}{h}.$$

OTHER DERIVATIONS. This simple rule can be derived in several other ways. For instance, let us assume that the moment of inertia of the section is equal to the moment of

* Working stresses for welds are not available in all structural handbooks. The common value in shear is 1200 lb. per lineal inch per $\frac{1}{8}$ in. of fillet leg, or 3600 lb. per lineal inch for a standard $\frac{3}{8}$ -in. fillet weld. Higher stresses have been permitted but exceptional control of workmanship is required.

inertia of the flanges alone. Then, if the effective depth is taken as h , the depth of the web, we have

$$(8) \quad v = \frac{VQ}{I} = \frac{V(A_f \times h/2)}{2(A_f \times h^2/4)} = \frac{V}{h}.$$

The same relation may be derived from an ordinary equilibrium study of the section of web of unit length shown in Fig. 198. If all the horizontal stress is considered to be resisted by the flanges, the only forces acting on this piece of web are the total vertical shears V and the horizontal shears v between the web and the flange. Each of these pairs of forces forms a couple. For equilibrium these couples must have equal values and they must act in opposite directions. Therefore, if h is the vertical distance between the flange welds, we may write

$$(9) \quad v \times h = V \times 1, \quad \text{or} \quad v = \frac{V}{h}.$$

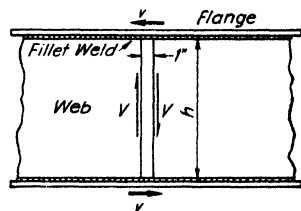


FIG. 198. SHEAR BETWEEN WEB AND FLANGE OF A GIRDER.

Shear between Parts of the Flange. The determination of the shear between the angles and the cover plates by means of the approximate method is also quite simple. A study of the exact formula, $v = VQ/I$, shows that Q varies almost directly with the flange area outside of any given section, while V and I remain unchanged. Hence the shear per lineal inch between the cover plate and the angles of Fig. 197(a) can be found approximately by multiplying the shear per lineal inch between the web and flange by the ratio of the area of the cover plate to the total area of the flange.

234. Resultant Stress on Flange Rivets. In railway plate girders of deck construction, there is a vertical shear on the rivets connecting the flange to the web caused by the direct vertical load on the top flange of the girder. In addition, there is the horizontal shear discussed above. The maximum shear on a rivet is the *resultant* of these horizontal and vertical shears.

The Vertical Shear per Rivet.

This shear for a deck railway girder is dependent upon the longitudinal distribution of a wheel load. The common assumption is that the stiffness of the rail is sufficient to distribute the wheel load over three ties. Accordingly, the vertical shear per rivet is

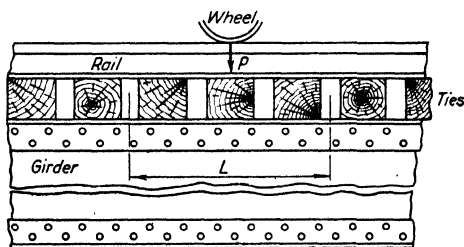


FIG. 199. DISTRIBUTION OF WHEEL LOAD TO TIES.

the wheel load divided by the number of rivets covered by three ties, as shown in Fig. 199, or the vertical shear per lineal inch of web is P/L . The value of L for 10-in. ties spaced 4 in. in the clear, as is usual, is 42 in. The A.R.E.A. specification limits this length to 3 ft.

Rivet Pitch. If the horizontal shear per lineal inch between the web and the flange is called v and the vertical shear is P/L , the resultant shear per lineal inch is R , as shown in Fig. 200. The *resultant shear per rivet* is R times the spacing or the pitch of the rivets. In other words, the required spacing is the rivet value divided by the resultant shear R . The rivet pitch under concentrated loads on building girders can be determined in a similar manner.

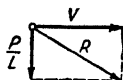


FIG. 200.
RESULTANT
RIVET
SHEAR.

235. Web Stiffeners. The web of a plate girder is a relatively slender compression member. Either the compressive stress in the web must be kept low enough to prevent buckling* or else the web must be stiffened by the use of riveted *stiffener angles* or by welded *stiffening plates*. There are two kinds of stiffeners used on plate-girder webs which serve distinctly different purposes. End stiffeners, as shown in Fig. 193, are used to prevent the web from buckling or wrinkling under the effect of the concentrated end reaction. Similarly, stiffeners are used in building girders under any large concentrated load that is placed on the top flange. Interior stiffeners (Fig. 202) are used to prevent the formation of *diagonal wrinkles* in the web perpendicular to the direction of the diagonal compression. Such wrinkles must not develop except in the special case of tension-field girders discussed in § 224.

Diagonal Stiffeners. It might seem that diagonal stiffeners (Fig. 201) rather than vertical stiffeners would be most effective in resisting the diagonal compression. This is probably true, but another point of view is of importance. The web itself will properly resist the diagonal compression if it is prevented from buckling by stiffeners. Vertical stiffeners as shown in Fig. 202 can prevent buckling as effectively as diagonal stiffeners, and they simplify the fabrication of the girder. *Horizontal stiffeners* have been used on very deep girders. To be effective they are placed well above the midheight of the web in the compression zone. A single longitudinal stiffener running the full length of the girder above the midheight may prove adequate as a replacement for closely spaced vertical stiffeners. However, the longitudinal stiffener itself must be braced to prevent buckling.

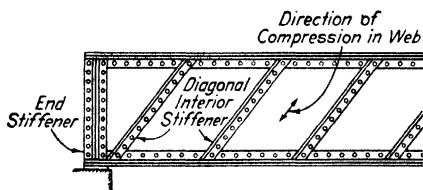
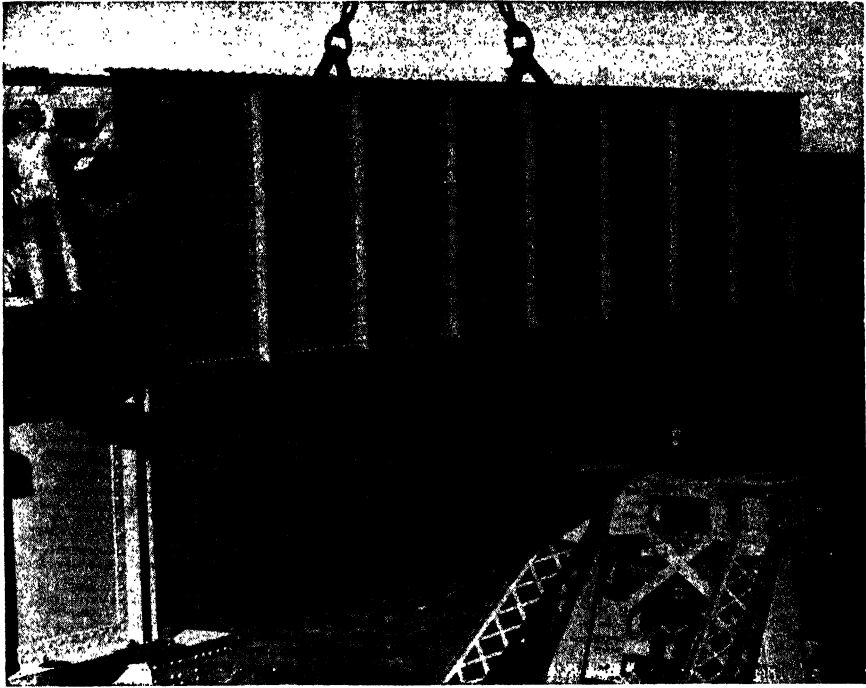


FIG. 201. DIAGONAL STIFFENERS ON A PLATE-GIRDER WEB.

236. Stresses in End Stiffeners. The load on the end stiffener is equal to the maximum end reaction of the girder. The usual assumption is that the *entire* load is carried by the stiffener without aid from the web. The outstanding legs of the stiffeners are milled to bear against the legs of the flange

* See *Theory of Plates and Shells*, S. Timoshenko, McGraw-Hill, 1940, pp. 311-328.



Courtesy Illinois Central R.R.

FIG. 202. SWINGING A PLATE GIRDER INTO PLACE.

The crane is lowering the girder into place. The workmen at the left are guiding it downward while those at the right are ready to seat it on the viaduct tower.

angles, or else the two are welded together. The area in bearing between the flange and the stiffener must be sufficient so that the allowable stress in bearing (27,000 lb. per sq. in. by the A.R.E.A. specifications) will not be exceeded.

Column Action. The end stiffener for a deep girder may be sufficiently tall to act as a slender column. More commonly, the slenderness ratio is small enough so that the allowable compressive stress is limited by the maximum value for compression members, 15,000 lb. per sq. in. for railway girders and 17,000 lb. per sq. in. for building girders. In calculating the value of L/r for an end stiffener, the value of L may be taken as *one-half* of the depth of the web ($0.75d$ by A.I.S.C. spec.) to allow for the fact that the compressive stress is entirely transferred out of the stiffener within the depth of the web. The value of r to be calculated is a horizontal radius perpendicular to the plane of the girder web. Buckling of the end stiffener in the plane of the girder web is obviously impossible.

Connection to the Web. The end stiffeners should be riveted or welded to the web with a sufficient number of rivets or length of weld to transfer the

entire value of the end reaction from the stiffener into the web. In riveted construction, the rivets are either in double shear or in bearing on the web because web stiffeners are always used in *pairs* on opposite sides of the web.

237. Need for Interior Stiffeners. Railway plate girders must have interior stiffeners if the unsupported depth of the web exceeds 60 times the web thickness. The unit shear as then allowed by the A.R.E.A. specifications is 11,000 lb. per sq. in. on the gross area of the web. The specifications of the A.I.S.C. for building girders are more lenient; they permit a web thickness of $\frac{1}{70}$ of the unsupported depth of the web without requiring stiffeners. The allowable shear on a web of this thickness, or on a thinner web that is stiffened at proper intervals, is 13,000 lb. per sq. in.

There is also a specification in the A.I.S.C. code to the effect that a web without stiffeners and with an unsupported depth of *more* than 70 times the thickness may be used provided that the unit shear does not exceed

$$(10) \quad s_s = \frac{64,000,000}{\left(\frac{h}{t}\right)^2},$$

where h and t are the unsupported depth and the thickness of the web, respectively. It is noticed at once that this expression is similar to a column formula in that the allowable stress is reduced with increase of $\frac{L}{r}$ or $\frac{h}{t}$. If we reduce the allowable shear on the web by a column formula, we are in effect designing the web as a column to resist the diagonal compression. This procedure seems justified, for, if the web is itself amply strong as a column, interior stiffeners are not needed.

238. Size and Spacing of Interior Stiffeners. There is seldom any attempt made to determine the stress in an interior stiffener. Instead, they are selected to provide the necessary lateral stiffness. The outstanding leg of the stiffener should be not less than $\frac{1}{30}$ of the depth of the girder plus 2 in. Its thickness is at least $\frac{1}{16}$ of the width of the outstanding leg. For railway plate girders, the interior stiffeners are usually $5 \times 3 \times \frac{3}{8}$ -in. angles where 6-in. flange angles are used. Stiffeners of $6 \times 3\frac{1}{2} \times \frac{3}{8}$ -in. angles are used with 8-in. flange angles. The stiffener angles are riveted to the web by a row of rivets spaced not to exceed 8 diameters or 6 in. apart. Welded stiffener plates may be made to correspond to the sizes of the outstanding angle legs suggested above. The stiffness of the welded plate will be nearly the same as the stiffness of the angle.

The proper spacing of interior stiffeners has been a much debated question. It is common practice to place an upper limit of 6 ft. on the stiffener spacing, irrespective of the depth of the girder, (7 ft. by A.I.S.C.). The stiffener spacing should not exceed the *clear depth* of the web in order to prevent a buckling failure between stiffeners on a line at 45 degrees. Moreover, it is obvious that

the stiffener spacing should be related to the column formula, since the critical stress in the web is compression.

Formulas for Spacing Stiffeners. The value of the allowable shear on the web is controlled in the A.I.S.C. specifications by equation (10). Hence, if the value of the actual average shear in the web is known, we should be able to set it equal to this expression and solve for the value of h , which will be the limiting *vertical or horizontal* dimension of an unsupported plate or the stiffener spacing. In this way we obtain the following formula for stiffener spacing,

$$(11) \quad h = d = \frac{8000t}{\sqrt{s_s}},$$

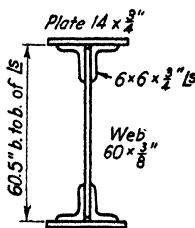
in which s_s is the value of the *average unit vertical shear* (lb. per sq. in.) in the web at the section under consideration. Formula (11) is typical of the stiffener spacing formulas generally found in present specifications. However, the numerator term is changed to 11,000 by A.I.S.C., 10,500 by A.R.E.A. and 9000 by A.A.S.H.O. specifications. Greater uniformity will no doubt develop in the future.

Note that because of the variation of the unit shear s_s , the stiffener spacing will increase from the end toward the center of the girder.

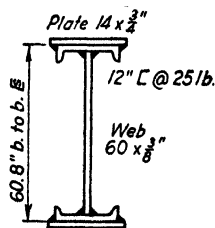
Longitudinal Stiffener. By A.A.S.H.O. specifications a longitudinal stiffener may be placed at $\frac{1}{3}$ of the depth from the toe of the compression flange and then the thickness of the web may be reduced to $\frac{1}{2}\frac{1}{6}$ of the depth between flanges instead of $\frac{1}{16}$ D . Sufficient tests are not available as yet to establish the correctness of the A.A.S.H.O. formula for proportioning the size of the longitudinal stiffener, nor is it clear when a longitudinal stiffener may be used alone without the support of vertical interior stiffeners. However, the longitudinal stiffener seems to be a useful contribution to the strength of a plate-girder web. When its action is fully explained by tests it will undoubtedly find its place in plate-girder design.

PROBLEMS

190. Compute the maximum fiber stresses for this riveted girder by the exact and semi-exact methods. The maximum moment is 2,500,000 ft.-lb. Deduct 1-in. holes for $\frac{7}{8}$ -in. rivets as follows: two holes from each cover plate and from each angle, a vertical row of holes at 4-in. spacing from the web. Then determine the effective depth to give the same tensile fiber stress as the exact method.



PROBLEM 190.



PROBLEM 191.

191. Compute the maximum fiber stress in this welded girder by the exact method. The bending moment is 2,500,000 ft.-lb. Then determine the effective depth to give the same tensile fiber stress as the exact method.

192. Compare the maximum shear at the neutral axis with the average shear in the web for the two girders of Problems 190 and 191. The shear is 250 kips.

193. The compression flange of a girder of 7-ft. depth is unsupported laterally for a distance of 30 ft. Its width is 14 in. Compare the allowable compressive stress obtained from equations (3), (4) and (5) with current A.I.S.C. specifications.

194. The plate girder of Problem 191 has two extra cover plates added. These plates are $14 \times \frac{1}{2}$ in. If the length of the girder is 60 ft., find the theoretical point of cut-off for each of these plates. Assume that the section is fully stressed at the center of the span and that the curve of maximum moments is a parabola. Base your calculations on gross area.

Ans. 11.8 and 16.7 ft. from the center.

195. Same as Problem 194, except work on the assumption that the section is stressed to 95% of its capacity at the center.

Ans. 10.0 and 15.7 ft. from the center.

196. Determine the required spacing of $\frac{3}{8}$ -in. rivets connecting the web and flange of the girder of Problem 190, where the vertical shear is 170 kips. Allow 15,000 lb. per sq. in. for single shear and 40,000 lb. per sq. in. for bearing on the web. Use the exact method and check by the approximate method.

Ans. Approximately $3\frac{1}{2}$ in.

197. Compute the spacing of rivets between the cover plate and the flange angles for the girder of Problem 196. Again use the exact method and check by the approximate method.

198. If the welded girder of Problem 191 is subjected to a vertical shear of 150 kips, compute the horizontal shear per lineal inch between the cover plate and the channel and between the channel and the web. Check by the approximate method.

199. Determine the reduction in the required spacing of rivets in Problem 196 if there is a vertical wheel load of 36,000 lb. distributed over 36 in. (three ties) to be considered as well as the shear of 170 kips. Increase the wheel load for 100% impact.

Ans. Reduced approximately $\frac{1}{8}$ in.

200. Select end stiffeners for the girder of Problem 190 for a maximum end reaction of 100,000 lb. Follow current A.I.S.C. or A.A.S.H.O. specifications.

201. What would be the total allowable shear by the A.I.S.C. formula for the girder of Problem 190 if there are no interior stiffeners?

Ans. 86,300 lb.

202. Determine the required stiffener spacing for the girder of Problem 190 where the shear is 170 kips. Use the formula $d = \frac{11,000t}{\sqrt{s_s}}$.

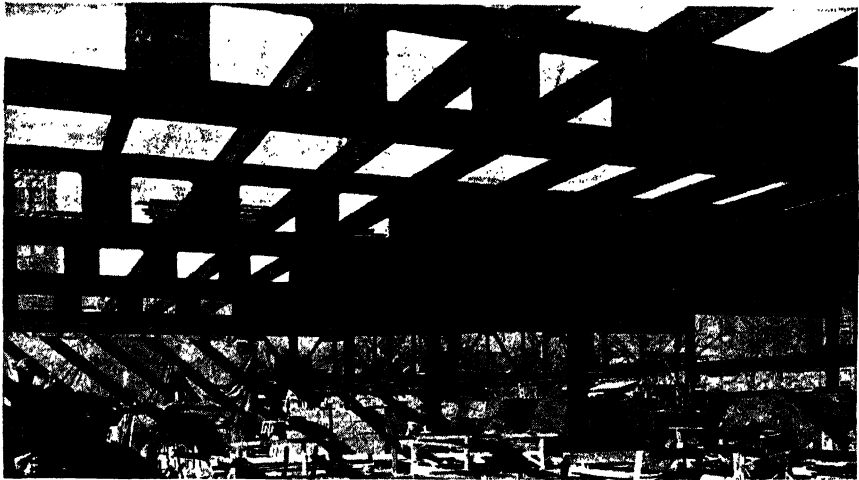
Ans. $47\frac{1}{2}$ in.

WEB AND FLANGE SPLICES

239. The Web Splice. The use of a web splice becomes necessary where the length of the girder exceeds the length of plate that is stocked. Splices usually occur in pairs because it is not considered good design to place a web splice at the center of the girder where the moment is a maximum. The splice *must* be able to replace the resistance of the web in so far as this resistance is needed at the spliced section. The common procedure is to make the web splice capable of replacing the *full moment resistance* of the web, as nearly as possible, and of resisting the *actual shear* at the section. However, the conservative A.R.E.A. specifications require the splice to replace the *full value* of the web both for shear and for moment. A.A.S.H.O. specifications are less conservative.

Welded Web Splice. It is very easy to splice a welded girder because the full depth of the web is exposed. Splice plates may be placed across the joint in the web and welded to the web to replace its full value of moment and shear resistance. In fact, splice plates are usually dispensed with entirely. The abutting edges of the web may be V-ee'd with a torch and butt-welded together to replace the full value of the web. A proper butt weld will reproduce the full strength of the web plate.

Riveted Web Splice. A riveted web splice is complicated by the fact that the entire depth of the web is not exposed.* A part of it is covered by the



Courtesy Werskopf and Peckworth

FIG. 203. BUILDING ROOF GIRDER WITH DUCT OPENINGS.

The solid web girder at the right turns into an open-web or Vierendeel girder at the left to provide openings for the passage of ducts. The analysis of open-web girders is treated in Vol. 2. They are very useful in building construction but are uneconomical when compared with solid web girders or trusses. Note the web splice at the right between the solid web and the open-web girder.

flange angles. Probably the most satisfactory type of web splice for use in a riveted girder is one that consists of riveted splice plates between the flange angles plus narrow splice plates riveted over the vertical legs of the flange angles to splice that part of the web *underneath* these angles. Sometimes the large splice plates can be made sufficiently strong so that the narrow plates can be omitted. The crude procedure of using a narrow vertical splice plate to replace the shearing resistance of the web, along with horizontal splice plates over the angle legs to develop the moment resistance of the web, is not

* J. M. Garrelts and I. E. Madsen, *An investigation of plate-girder web splices*, Transactions, A.S.C.E., Vol. 107, pp. 1303-1329.

considered satisfactory. The two parts of the splice should be designed to work together.

240. Stresses in Rivets Arranged in Long Vertical Rows. Before we can determine the stresses in rivets forming a web splice, it is necessary that we understand how shear and moment act in stressing a line of rivets. Since the shear acts vertically upward or downward upon the entire group of rivets to one side or the other of the splice, it is usually *assumed* that the vertical shear

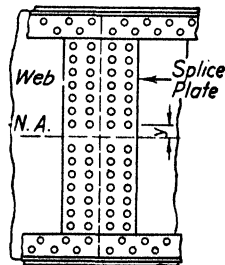


FIG. 204. A STANDARD WEB SPLICE.

per rivet is equal to the total shear divided by n , the number of rivets to one side of the splice. Moment tends to rotate the splice about its center. The stiff splice plates may be *assumed* to produce a *horizontal shear* on each rivet in proportion to its *vertical distance* from the mid-height of the girder. See Fig. 204.

Flexure Formula. Analysis by the flexure formula, $f = Mc/I$, follows the basic assumption above. This formula will be used for the calculation of the horizontal shears on the splice rivets. In order to make f equal to S_s , the horizontal shear per rivet, I must equal the moment of inertia of the rivet areas

about the center of gravity of the group *when each rivet is assumed to have a unit area*. Accordingly, if y represents the vertical distance from the center of the group to any rivet, I will be computed as Σy^2 , where the summation includes all the rivets *above and below* the neutral axis of the girder, but *on one side only* of the splice. Then S_s , the horizontal shear per rivet, is obtained from the expression

$$(12) \quad S_s = \frac{My}{\Sigma y^2}.$$

Maximum Rivet Shear. In calculating the maximum shear on a rivet that is a part of a web splice, we must obtain the resultant of the vertical shear per rivet caused by vertical shear at the splice and the horizontal shear caused by moment. Naturally, one of the rivets located *farthest* from the neutral axis of the girder will receive the greatest horizontal shear and therefore the greatest resultant shear. A rivet must be selected for which the two components of shear *add* rather than subtract. Since V/n is the average vertical shear per rivet where V is the total vertical shear at the splice and n is the number of rivets on one side of the splice, the maximum rivet shear will be $\sqrt{(V/n)^2 + (My'/\Sigma y^2)^2}$ where y' is the vertical distance from the neutral axis to the farthest rivet through the web splice plate.

241. Stresses in Rivet Groups. The assumption that the rivet shears produced by moment act in a horizontal direction would be seriously in error for a rivet such as (a) of the group shown in Fig. 205. Each rivet actually re-

ceives a stress from pure rotation which acts *perpendicularly* to a line joining that rivet to the C.G. of the group. In Fig. 205, the rivet (*a*) receives a shear acting downward to the right at an angle of 45 degrees.

Use of the Torsion Formula. The value of the shear on the rivet (*a*) of the group of Fig. 205 produced by the moment *M* can be computed with reasonable accuracy by the torsion formula,

$$(13) \quad S_s = \frac{Mr}{J} = \frac{Mr}{\sum r^2}.$$

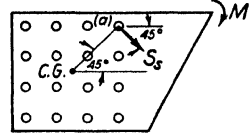


FIG. 205. ROTATION OF A RIVET GROUP.

In this formula, *M* is the twisting moment, *r* is the radius for any rivet, and *J* is the polar moment of inertia of the rivets about their centroid when each rivet is assumed to have a unit area. Then $J = \sum r^2$, and *S_s* is the shear per rivet at the distance *r* from the center of gravity of the group. The beam formula, when applied to the calculation of stresses in rivets, should be looked upon as a *special case* of the torsion formula. It should be used only when all rivets lie approximately in a single line. If we recall that the polar moment of inertia *J* is always equal to $I_{x-x} + I_{y-y}$, we see that equations (12) and (13) become identical when I_{y-y} is negligibly small.

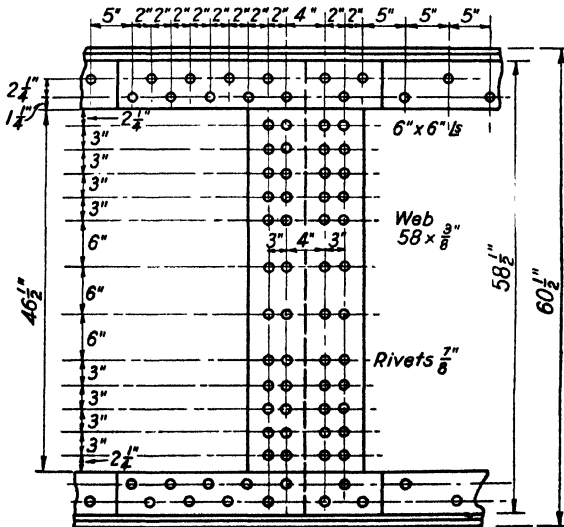


FIG. 206. WEB SPLICE TO LEFT OF CENTER LINE WITH PLATES OVER THE FLANGE ANGLES.

242. Analysis of a Standard Web Splice. The standard web splice to be investigated is shown in Fig. 206. The web of this girder is 58 in. deep and

$\frac{3}{8}$ in. thick. Splice plates of minimum thickness ($\frac{3}{8}$ in.) will be ample to splice the depth of web between the flange angles and to carry all of the vertical shear. Two additional splice plates $5 \times \frac{3}{8}$ in. placed over the vertical legs of each pair of flange angles will more than replace the depth of web underneath these angles.

EXAMPLE. Rivets through the Main Splice Plates. These rivets are assumed to resist the entire vertical shear. If the shear on the section is 112,500 lb., the shear per rivet is $112,500 \div 24 = 4690$ lb. Assuming that these rivets are $\frac{7}{8}$ in. in diameter and that the allowable stress in shear is 13,500 lb. per sq. in. and in bearing is 27,000 lb. per sq. in., we find the value of a rivet in double shear to be 16,200 lb. and its value in bearing to be 8870 lb. Bearing on the web will control; the allowable horizontal stress per rivet is

$$\sqrt{8870^2 - 4690^2} = 7500 \text{ lb.}$$

Note, however, that there is *another limitation* upon the stress in that the rivets through the flange also act as splice rivets for the web. The rivets farthest from the neutral axis can be stressed only to 8870 lb. The distance from the neutral axis out to the farthest rivet is 26.75 in. Based upon a straight-line variation of stress, the allowable *horizontal shear* for the farthest rivet in the main splice plate at 21.0 in. from the neutral axis will be 8870 $(21.0/26.75) = 6970$ lb. This allowable stress will control the design, for it is less than the value of 7500 lb. above. The allowable horizontal shear per rivet should be *further reduced* for a deck girder carrying a vertical load on the top flange.

The moment of inertia of the rivets to one side of the splice is

$$\Sigma y^2 = 4(3^2 + 9^2 + 12^2 + 15^2 + 18^2 + 21^2) = 4900.$$

The *resisting moment of the rivet group* is

$$M = \frac{S_r I}{c} = \frac{6970 \times 4900}{21} = 1,630,000 \text{ in-lb.}$$

Value of the Web between Flange Angles. If the flange stress computed from the net section is 18,000 lb. per sq. in., the stress is approximately 16,000 lb. per sq. in. when computed on the gross section. At the inside edge of the flange angles, or 23.25 in. from the neutral axis, the stress will be $16,000(23.25/30.25) = 12,300$ lb. per sq. in. The moment of inertia of the gross web between the flange angles is $\frac{1}{12} \times \frac{3}{8} \times 46.5^3 = 3140$ in.⁴ The *resisting moment of this part of the web* is

$$M = \frac{fI}{c} = \frac{12,300 \times 3140}{23.25} = 1,660,000 \text{ in-lb.}$$

The moment resistance of the rivet group is just about sufficient to develop the moment value of the depth of the web covered by the splice plates, or the part between the flange angles.

243. Horizontal Splice Plates. The stresses in the rivets of the splice plates placed over the vertical legs of the flange angles will now be considered. On account of the normal action of the horizontal shear between web and flange, there is a shear on each rivet which may be represented by an arrow

pointed toward the left provided that the rivet considered is through the upper flange to the left of the center, corresponding to the rivet (a) in Fig. 207. This arrow represents the pressure of the flange angles *against the rivet*. This is made clear in Fig. 207 by the enlarged cross-section at the rivet (a). This force normally is resisted by the pressure of the web acting to the right. However, the splice plates are in compression; hence they act to the left against rivets on the left of the break in the web and to the right against those

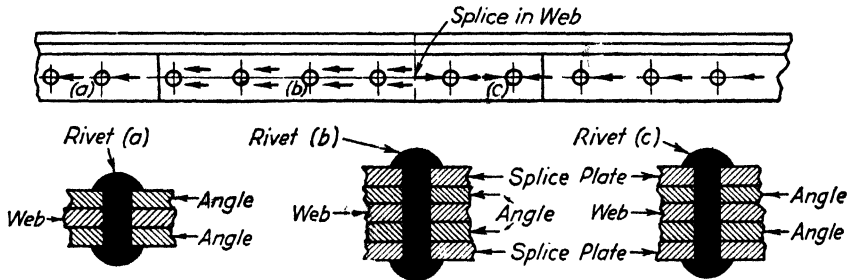


FIG. 207. SHEAR ON FLANGE RIVETS NEAR A SPLICE AT LEFT SIDE OF CENTER LINE.

rivets on the right or on the side nearer the center of the girder. It follows that the web must form the reaction for *both* of these forces for rivets such as (b) on the abutment side from the splice. A study of the enlarged diagram for the rivet (c) shows that the splice plates increase the bearing stress on the flange angles to the right of the splice, which usually is not serious, since it places the rivets in quadruple shear. The bearing stress on the web is decreased at rivet (c).

EXAMPLE. Rivets in the Horizontal Splice Plates. The correct procedure in riveting the splice plates over the vertical legs of the flange angles is to extend the plate far enough from the splice toward the near end of the girder to introduce *extra rivets* whose bearing resistance on the web is equal to the value of the web covered by the flange angles. This value may be found with reasonable accuracy by multiplying the gross area of the web covered by the flange angles by an *estimated average stress*. This stress for the example of § 243 would be between 12,300 and 16,000 lb. per sq. in. At 14,500 lb. per sq. in., the value of this part of the web is $14,500 \times 0.375 \times 5.75 = 31,200$ lb. Each rivet has a bearing value on the $\frac{3}{8}$ -in. web of 8870 lb. The required number of rivets to carry the stress is $31,200 \div 8870 = 3.5$. However, it is wise to provide a 33% increase to 5 rivets because the *splice is indirect*. The flange angles separate the splice plate from the web. Five extra rivets are shown to the left of the splice in Fig. 206.

No *extra rivets* are required on the side of the splice toward the center of the girder, but the splice plates must hold enough rivets to develop the section of the web underneath the angles. Since these rivets act in quadruple shear or in bearing on metal 1 in. thick, (for $\frac{1}{2}$ -in. *Ls*) the number required is $31,200 \div 23,600 = 2$ rivets. Again, this number is increased 33% because of the indirect splice. Three rivets are shown in the splice plate to the right of the splice in Fig. 206.

244. Analysis of a Special Web-Splice. A splice will now be investigated where the only splice plates are the ones placed between the flange angles as shown in Fig. 208.

CALCULATIONS. The moment of inertia of the gross web is $\frac{1}{12} \times \frac{3}{8} \times 58^3 = 6100 \text{ in.}^4$. The compressive fiber stress at 29.0 in. from the neutral axis will be taken as 16,000 (29.0/30.25) = 15,400 lb. per sq. in., based upon the gross section. The *moment resistance of the web* is

$$M = \frac{fI}{c} = \frac{(15,400 \times 6100)}{29} = 3,240,000 \text{ in-lb.}$$

EXAMPLE. The Splice Plates. Two $\frac{3}{8}$ -in. splice plates 46.5 in. deep are used between the flange angles, as shown in Fig. 208. These plates can be stressed by flexure to a maximum fiber stress which has been determined in the previous example to be 12,300 lb. per sq. in. Their moment of inertia is $\frac{1}{12} \times \frac{3}{4} \times 46.5^3 = 6280 \text{ in.}^4$. The *moment resistance of the splice plates* is

$$M = \frac{fI}{c} = \frac{12,300 \times 6280}{23.25} = 3,320,000 \text{ in-lb.}$$

The splice plates are amply strong.

The Splice Rivets. The rivet arrangement shown in Fig. 208 is about as strong as can be obtained in a girder of this depth. The stress per rivet caused by the vertical shear is $112,500 \div 36 = 3130 \text{ lb.}$ The value of a $\frac{7}{8}$ -in. rivet in bearing on a $\frac{3}{8}$ -in. web is 8870 lb. This value will not be reduced by the ratio 21.0/26.25 as in the previous example, because the flange rivets are not intended to function as an important part of the web splice. Instead, the

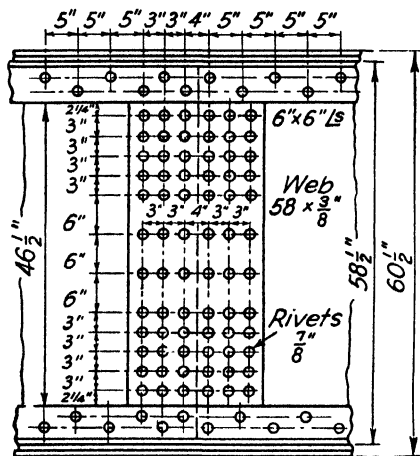


FIG. 208. SIMPLE WEB SPLICE.

horizontal shear must be reduced so that the *resultant* of the horizontal and vertical shears will not exceed 8870 lb. Hence the maximum horizontal stress per rivet is $\sqrt{8870^2 - 3130^2} = 8300 \text{ lb.}$ The moment of inertia of the rivet group is $6(3^2 + 9^2 + 12^2 + 15^2 + 18^2 + 21^2) = 7350$. The *moment resistance of the rivets* is

$$M = \frac{S_r I}{c} = \frac{8300 \times 7350}{21.0} = 2,910,000 \text{ in-lb.}$$

Transfer of Stress from Web to Flange. The moment resistance of the web is not replaced fully by the rivets, but the difference only amounts to $3,240,000 - 2,910,000 = 330,000 \text{ in-lb.}$ or 10 per cent. If there is any excess area available in the flanges, the difference can be made up by the *transfer* of a small amount of stress *from* the web *into* the flange. The excess gross area required in the flange to compensate for this deficiency in moment resistance of the web is $330,000 \div (14,500 \times 58) = 0.39 \text{ sq. in.,}$ approximately.

Extra Flange Rivets. The flange rivets transfer stress into the flange as the moment increases toward the center of the span. Thus it becomes clear that the transfer of stress

considered above throws an *extra duty* upon the flange rivets to the *left* of the splice since they must introduce more stress into the flange in the length between the left-hand reaction and the splice than would be required without a splice. Since the extra stress to be transferred is only $330,000 \div 58$ or 5700 lb., one extra rivet should be provided on the left-hand side of the splice.

It is recommended that splice plates should be used over the vertical legs of the flange angles whenever their omission makes necessary a heavy transfer of stress from web to flange, and, of course, wherever excess flange area is not available to take care of the transferred stress. However, in the example above the transfer of stress is small enough so that it can not be considered objectionable.

245. Flange Splices. Flange splices are not used except for a girder of unusual length. If it is available, flange material of full length should be specified by the designer. The flange splice may consist of the splice of a flange angle, of a cover plate, or of both. Not more than one element of the flange should be spliced at a single point. The splices are placed preferably at points where *excess flange material* exists. Frequently, a flange plate can be extended a short distance past its normal point of cut-off to provide such excess flange area.

246. Field Splice in Web and Flange. A field splice in a plate girder is seldom used, since it would greatly increase the cost of the structure. If the girder is so long that a field splice is necessary to permit transportation, the use of a truss should be given serious consideration.

Erection. To provide for easy erection of a girder that has a field splice, the flange elements may be *fanned out* as shown in Fig. 209. This arrange-

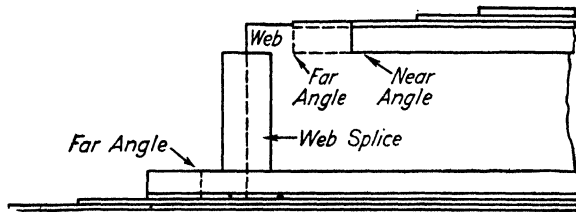


FIG. 209. FIELD SPLICE IN FLANGES AND WEB OF A LARGE GIRDER.

ment provides the possibility of erecting the right-hand portion of the girder first. Then the left-hand part can be lowered directly into place without *horizontal insertion*. A temporary falsework pier is required near the splice. The obvious objection to this type of field splice is that it can not be made as compact as the type which is locked together by a horizontal movement.

Design. In any field splice, the separate elements of the flange, such as adjacent cover plates, should be cut far enough apart so that the rivets between the two sections will offer sufficient resistance in single shear to develop the inside plate or angle. When arranged thus, no section will be reduced in

strength by more than the resistance of one flange element. An extra cover plate or splice plate can be made to overlap all breaks to replace the moment resistance of the section. However, if a flange angle is cut, the addition of a splice angle is desirable.

247. Splice of a Flange Angle and Cover Plate. A flange angle should always be spliced by another angle which is ground to fit the inside of the flange angle as shown by Fig. 210(a) and (b). Either the splice angle must be selected of thicker section than the flange

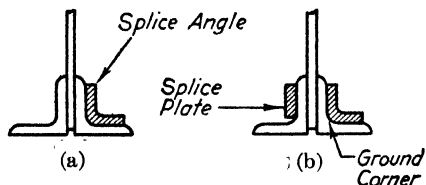


FIG. 210. SPLICE OF A FLANGE ANGLE.

angle (to reproduce its area) or else an extra splice plate should be used, as shown in (b). The use of a single angle seems desirable because the entire splice then becomes direct. The use of an extra plate changes the splice to partially indirect, and the number of rivets must be increased for this reason. The

number of rivets to each side of the splice should have sufficient resistance in single shear to develop the strength of the angle that is being spliced. These rivets may be at the normal spacing, or extra rivets may be added in order to shorten the splice angles.

Excess Flange Area. Some engineers prefer to place the splice in a flange angle near the end of a cover plate where excess flange area is available. Then splice angles of the same thickness as the flange angles are used. The excess stress must be transferred out of the angle and into the cover plate. An extra duty is thrown upon the rivets between the angles and cover plates. Unless these rivets are already in excess of the normal requirement, additional rivets must be used. *The extra rivets are needed toward the nearest abutment from the splice.* Another reason why one or more extra rivets should be added is that the outer cover plate furnishes the excess area by acting as an indirect splice for a part of the flange angle.

Splice of a Cover Plate. If the cover plate to be spliced is an outside plate, the splice is direct and the splice plate must simply develop the value of the cover by rivets in single shear. An inside cover is also spliced by a plate placed over the *outside* cover. This forms an indirect splice. The number of rivets must be increased at least 33% if there is one intermediate cover, 67% if there are two intermediate covers, etc. A splice plate usually carries several lines of rivets at 90° to the direction of stress. A.A.S.H.O. specifications require the addition of two such lines of rivets on each side of the splice if there is one intermediate plate or four additional transverse lines of rivets if there are two intermediate plates, etc. Covers must not be cut closer together than the minimum distance required to introduce enough rivets to develop the value of the inner plate by single shear.

248. Girders with Sloping Flanges. Crane girders, haunched girders for buildings, and deepened girders used as floor beams for wide bridges have sloping flanges. These three types are illustrated by Fig. 211. There is about the same correspondence between the analysis of these girders and ordinary girders with horizontal flanges as was found between the analysis of curved-chord trusses and parallel-chord trusses. The stress in the sloping flange, as shown by Fig. 212, has a horizontal component equal to the bending moment divided by the effective depth between flanges. The stress in the sloping flange then becomes

$$(14) \quad T = \frac{M}{h \cos \theta},$$

where h is the effective depth and θ is the inclination of the flange to the horizontal.

Modified Shear. The stress in the sloping flange has a vertical component of $(M/h) \tan \theta$. This vertical force may act to reduce or to increase the effective shear on the web. In Fig. 211(c) the modified shear over the length k is less than the true vertical shear because of the tension in the sloping lower flange. On the other hand, the modified shear will be greater than the true vertical shear over the length k in Fig. 211(b). In both cases the average unit shear on the web is to be taken as the modified shear divided by the cross sectional area of the web at the section under consideration.

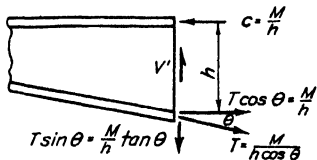
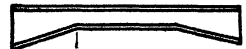


FIG. 212. STRESS IN THE SLOPING FLANGE.



(a) Crane Girder



(b) Haunched Girder



(c) Deepened Floor Beam

FIG. 211. GIRDERS WITH SLOPING FLANGES.

Rivet Spacing. The approximate method of determining rivet spacing will be extended to take care of a sloping flange. If V' represents the modified shear, then V'/h represents the approximate horizontal shear between the web and the flange per unit length of the girder. The horizontal shear over a short length x will be $V'x/h$ which determines the rivet spacing in the horizontal flange. However, this shear is only the horizontal component of the shear in the rivets or in the weld connecting the web and the sloping flange. The shear along the slope is $(V'x/h) \div \cos \theta$. But $x/\cos \theta$ is the distance s measured along the slope (Fig. 213). Accordingly, the true shear over the sloping length s is $V's/h$, which then represents the shear per rivet if s is the actual rivet spacing. Hence we may write

$$(15) \quad s = \frac{Rh}{V'},$$

where R is the limiting value of a rivet. When s is taken as unity, V'/h is the shear per lineal inch irrespective of the slope to be resisted by the flange welds. Remember that V' is the *modified shear* and that it varies continuously along the girder.

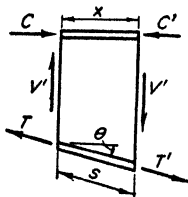


FIG. 213. RIVET OR WELD STRESS WITH SLOPING FLANGE.

PROBLEMS

203. Determine the moment resistance of the group of rivets forming the web splice of this girder. Compare the result with the moment resistance of the part of the web that lies between the flange angles. Allow 12,000 lb. per sq. in. for shear and 24,000 for bearing. The maximum fiber stress in tension is 16,000 lb. per sq. in. Take account of a vertical shear of 150,000 lb. at the splice.

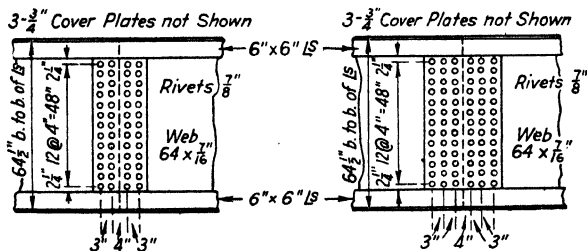
Ans. Deficient by 410,000 in.-lb.

204. Repeat Problem 203 for current A.I.S.C. working stresses.

205. Determine the size of splice plates to be placed over the vertical legs of the flange angles of the girder in Problem 203 to replace the value of the web underneath which compensates for the weakness of the main splice as found in Problem 203. Use $\frac{7}{8}$ -in. rivets. Calculate the number of rivets needed in these small splice plates to each side of the splice.

206. This web splice is used without splice plates over the vertical legs of the flange angles. The shear at the section is 150,000 lb. Find the required thickness of splice plates to replace the moment resistance of the web. Compute the amount of excess area needed in the flange to make the use of this splice permissible. Working stresses are the same as for Problem 203.

Ans. 1.4 sq. in.



PROBLEM 203.

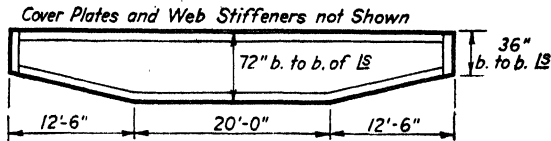
PROBLEM 206.

207. Repeat Problem 206 for current A.I.S.C. working stresses.

208. Select a splice angle to replace the tension value of a $6 \times 6 \times \frac{3}{4}$ -in. flange angle with two holes out. Determine the number of $\frac{7}{8}$ -in. rivets required in the splice. Follow current A.I.S.C. working stresses.

209. Find the number of $\frac{3}{4}$ -in. rivets to splice a $12 \times \frac{5}{8}$ -in. cover with two holes out. There is one plate between the cover and its splice plate. Use current A.I.S.C. working stresses.

210. The web plate of this deepened girder is $\frac{3}{8}$ in. thick. Each flange is composed of two $6 \times 6 \times \frac{1}{2}$ -in. angles and two $14 \times \frac{1}{2}$ -in. covers. Compute the fiber stress in each flange at the quarter point of the span by the approximate method. The bending moment at the quarter point is 1,600,000 ft.-lb. Deduct two $\frac{7}{8}$ -in. holes for $\frac{3}{4}$ -in. rivets from each element of the tension flange and 25% from the web area.



PROBLEM 210.

211. The shear at the quarter point of the deepened beam of Problem 210 is 120,000 lb. The depth of the web is $\frac{1}{2}$ in. less than the distance back to back of angles. Find the required spacing of $\frac{3}{4}$ -in. flange rivets in the sloping flange when h is taken as the depth of the web at the quarter point. Working stresses are the same as in Problem 203.

Ans. 6 in. on slope.

249. Plate-Girder Design. It is nearly impossible to separate the two processes of analysis and design in the discussion of built-up girders. The reason is that many features of the structure are governed by practical rules of design rather than by theoretical analysis. The selection of an interior stiffener is a case in point. An extension of our information regarding problems in the buckling of plates* has made a theoretical analysis and scientific design of such members possible.

The usefulness of the approximate methods of analysis that have been given is due to their direct applicability to design. They make possible the calculation of the total stresses in the web and flange before the cross-section has been selected. In fact, the cross-section is selected to resist these approximate stresses and is then checked by the exact methods if this refinement is considered necessary.

250. Survey of the Theory of Beams. The structural engineer is tempted constantly to extend the theory of the flexure of beams to the solution of a wide variety of problems. The flexure formula has been used in Chapter 4 of this text to calculate the column stresses in complex building bents. In this chapter, the same formula has been used to calculate the stresses in the rivets of web splices. Other extensions are quite as common; for instance, the determination of stresses in groups of piles, shears in welds, and pressures under foundations. When such applications are contemplated, it is expedient to review the basic assumptions of the flexure formula in order to determine to what extent these assumptions are fulfilled.

Evidently the basic assumption of the flexure formula is that there is a planar distribution of normal stress across the section. The stress will have a planar distribution if the strain is planar and if all stresses are below the elastic limit. It seems probable† that plane sections before bending *must*

* S. Timoshenko, *Problems concerning elastic stability in structures*, Transactions, A.S.C.E., 1930, p. 1014, also *Theory of Plates and Shells*, S. Timoshenko, McGraw-Hill, 1940, pp. 311-328.

† George F. Swain, *Structural Engineering-Strength of Materials*, McGraw-Hill, p. 233.

remain plane after bending in so far as the effect of the normal stresses are concerned, but a different condition obtains when heavy shearing stresses exist. As shown in Fig. 214(b), the effect of the parabolic distribution of shear on a rectangular section is to change a plane section into one of *S-shape*.

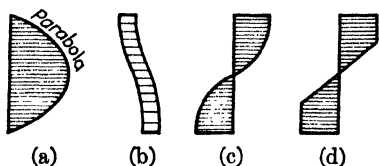


FIG. 214. BEAM ANALYSIS.

Since sections at points of zero shear are plane, while elsewhere all sections must be warped, we conclude that the beam formula is probably unsatisfactory when applied to short deep members where serious shear distortion occurs.

The beam formula will also give incorrect stresses when the fiber stresses exceed the elastic limit. Plane sections may remain plane and still the stress variation will not be linear if the elastic limit is passed. The action is apparently as follows: The extreme fiber passes the elastic limit and will then deform considerably *without* increase of stress. But its deformation is restrained by the adjacent fibers which build up stress until they too pass the elastic limit. The stress variation then probably corresponds to that shown by Fig. 214(c) or (d). This distribution might also represent the shear variation for a group of rivets forming a web splice. At a shear slightly above the normal working stress, the rivets show a considerable *slip*, which would be almost identical to the effect of passing the elastic limit described above. Incidentally, if the diagram (c) is *assumed* to be made up of two parabolas, we find that the moment resistance for a given fiber stress has been increased 25% over the moment resistance for planar stress distribution across a rectangular cross-section. This is interpreted as an added factor of safety against ultimate failure.

To the extent that the use of the flexure formula tends to overestimate fiber stresses it should promote conservative design. On the other hand, we can easily overestimate the effective section in complicated structures and thus err on the side of danger. Particularly where the cross-section is discontinuous, shear distortion may be large enough to make the outer parts of the cross-section ineffective in resisting moment. The structure must be designed and built adequately stiff to maintain plane sections before the flexure formula can be used. Here is another use for engineering judgment.

CHAPTER 10

OFFICE AND COMMERCIAL BUILDINGS

251. Types and Uses. The group of structures to be discussed in this chapter includes the warehouse, the office building of moderate height, and the store or apartment building with elevator service. Such buildings may be from four to ten stories high without being subjected to serious wind stresses since the walls and partitions will usually provide sufficient resistance to wind in structures below 100 ft. high. Actually, the height is not the determining factor, but it is the height-width ratio instead. Some building codes have permitted the omission of special wind bracing when the height of the building does not exceed twice its least width. A warehouse with brick walls covering half a square block might be twenty stories high without special consideration being given to wind stress calculations. If the building has only glass enclosure, the wind-stress problem will become more serious. This chapter will be devoted mainly to the problems found in buildings in which the wind stresses need not be calculated. Chapter 11 will cover the analysis of wind stresses.

There is little difference between the steel frame of an office building and that of a warehouse. Naturally, the latter carries greater loads and is a heavier structure. Many details of the steel frame are shown in Fig. 215. Today, buildings of each of these types are of skeleton construction, that is, the exterior walls above the basement level are mere enclosures. Such curtain walls are supported entirely by the steel frame. Wall-bearing construction, in which the massive exterior walls not only were self-supporting but actually supported a part of the floor loads, has been superseded by steel-frame or skeleton construction. Modern buildings have such a large part of the walls glazed that wall-bearing construction would be quite impractical today even if its greater cost had not caused it to be discarded years ago. It has been well established that the Home Insurance Building erected in Chicago in 1885 was actually the world's first building of skeleton construction.*

252. The Steel Structure. The steel frame of the building consists of vertical steel columns, usually spaced from 16 to 24 ft. apart, which are connected together in both directions by beams and girders at each floor level. The arrangement of beams and girders depends upon the type of floor construction to be used and upon the design of the wind bracing. A common

* Robins Fleming, *Whence the skyscraper*, Civil Engineering, October, 1934, pp. 505-509.

arrangement is to frame girders between the columns in the longer direction of the floor panel and to frame beams or joists between the girders in the short

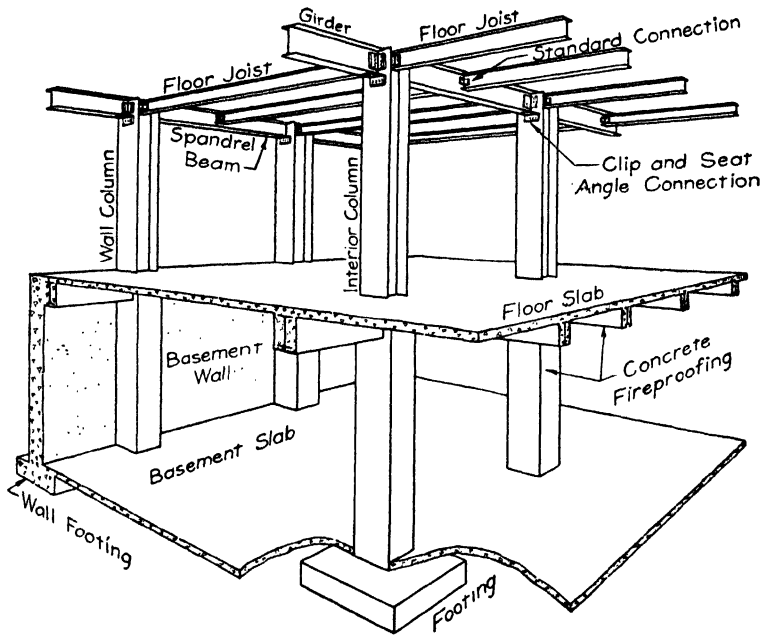


FIG. 215. THE FIREPROOFED STRUCTURE OF A STEEL-FRAME BUILDING.

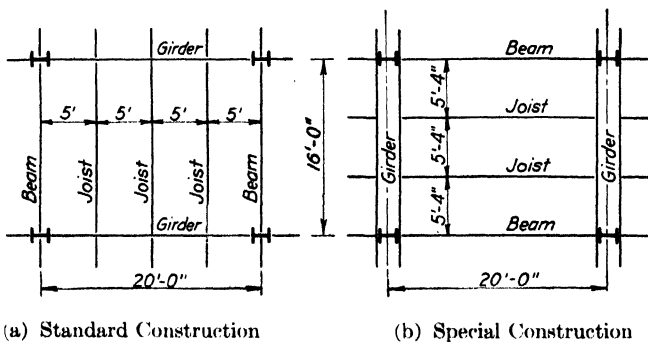


FIG. 216. TYPICAL FRAMING OF A FLOOR PANEL.

direction. See Fig. 216(a). The distance between joists is limited by the allowable clear span of the floor. Where headroom is limited, the long deep girders required by the arrangement of Fig. 216(a) can be reduced in depth by

the use of double channels, double I-beams, or a single wide flange beam spanning in the opposite direction, as shown in (b). Naturally, heavier joists are required and the total weight of steel will be increased, but several inches of headroom can be saved. Six inches of headroom saved in each story will reduce the required height of a 20-story building by *one entire story*. This saving represents an important economy.

253. Riveted Joints. The connections between beams, girders, and columns form important parts of the structural frame. For low buildings (under ten stories), the connections between girders and columns are not ordinarily expected to resist moment. The simple clip-angle connection of Fig. 217(a)

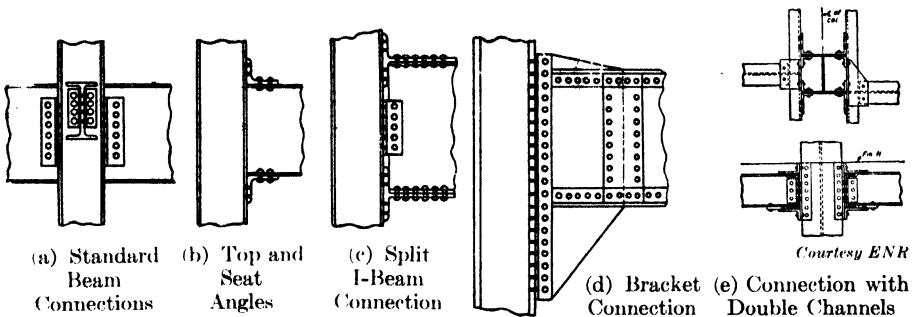


FIG. 217. RIVETED JOINTS USED IN BUILDING CONSTRUCTION.

is employed. Taller buildings constructed without diagonal bracing require the use of joints that will resist wind moments. Moment-resistant connections may also be used to reduce the simple-span bending moments in beams or girders and produce a shallower floor construction. Brackets on columns to support eccentric loads must also be moment resistant. Connections to the *flanges* of the girders become necessary to provide moment resistance. The top-angle and seat-angle connection shown in Fig. 217(b) and in Fig. 215 may be used where only a small moment resistance is required. This joint has been shown by tests* to be inadequate to resist heavy moments because distortion of the connection angles permits considerable rotation to occur between the column and the girder. The moment resistance of this connection is not much improved by the addition of clip angles to the girder web.

Split-Beam Connection. The most common type of moment-resistant riveted connection in use today is shown in Fig. 217(c). In addition to the clip angles to the web of the beam, whose purpose is to resist the vertical shear, there are heavy connections to the flanges. These connections are made from Tees obtained by splitting or by burning one flange away from a short piece of I-beam. Since rolled beams are available up to 36 in. in depth and up to

* W. M. Wilson and H. F. Moore, *Tests to Determine Rigidity of Riveted Joints of Steel Structures*, Bulletin No. 104, University of Illinois Engineering Experiment Station.

300 lb. per ft. in weight, the moment resistance of the split-beam connection can be comparatively high. Before the larger beam sizes became available, considerable use was made of the *bracket* or *gusset* connection shown in Fig. 217(d) and also in Fig. 227. This joint still must be used to resist extremely

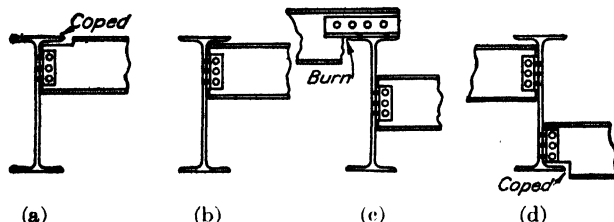


FIG. 218. SPECIAL BEAM AND GIRDER CONNECTIONS.

heavy joint moments unless welding is adopted. The advantage of the split-beam connection is that there are no projecting gusset plates to interfere with the architectural treatment of the interior of the building.

Connections between Joists and Girders. These joints preferably are made by means of the standard clip-angle connections which are given in all steel

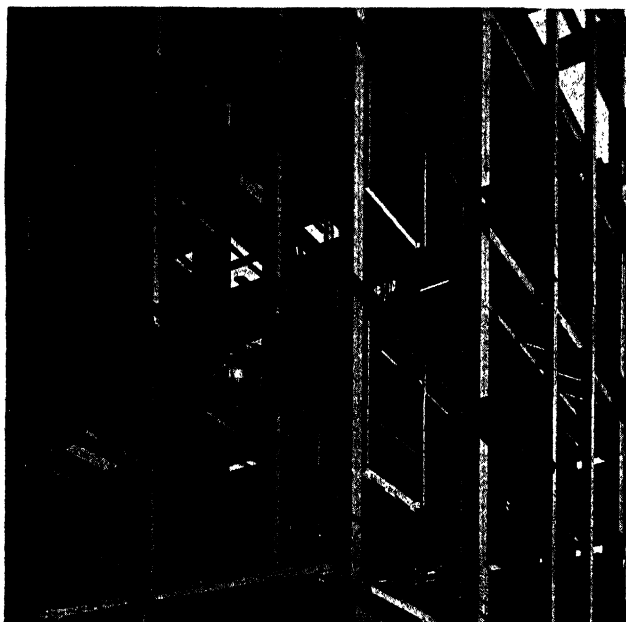


FIG. 219. BEAM-AND-COLUMN CONSTRUCTION FOR A LOW BUILDING.

This framework is for a powerhouse. It is of simple beam-and-column construction without wind connections. Notice the spandrel angles projecting beyond the face of the steel structure to pick up and support the brick walls.

handbooks. Illustrations of standard connections, coped and uncoped, are shown in Fig. 218 (a) and (b). *Special connections*, such as those shown in (c) and (d), may become necessary where members meet at different levels. Such connections usually are found in the framing of stairways, landings, and ramps. Beams are seldom allowed to rest on top of the girders, for this arrangement is wasteful of headroom.

The illustration Fig. 219 shows typical beam and column construction for a low building. Shelf angles may be seen attached to the outside face of the structure to support brick walls. It will be noticed that no heavy connections appear. Hence it is clear that wind stresses are unimportant because of the low height of the structure.

254. Welded Joints. The connections discussed above are all of the riveted type. The welded connection has found considerable favor in building construction. The simple welded connections shown in Fig. 220(a) are intended to be sufficiently flexible to permit the beam to rotate at the end and relieve almost all moment restraint. Bolt holes are provided for erection.

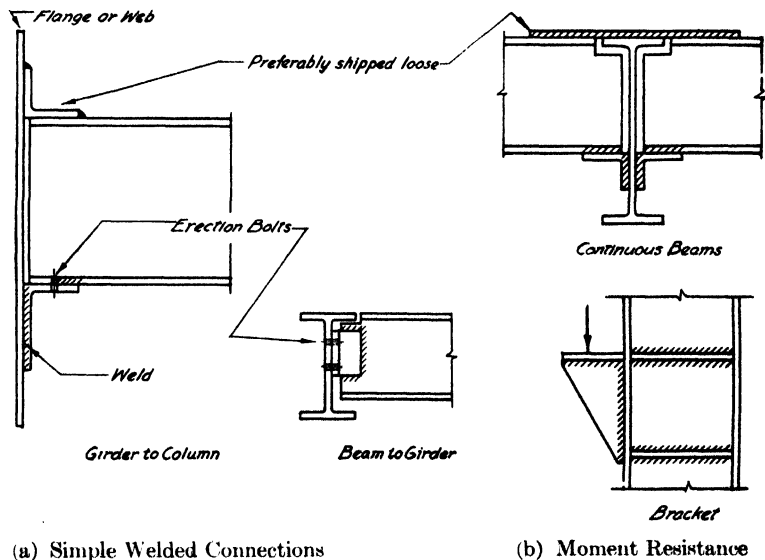


FIG. 220. WELDED CONNECTIONS FOR ORDINARY BUILDINGS

Some moment resistance can be developed in the top and seat angle connection by extending the welds along the sides of the top angle, but a better arrangement for this purpose is shown in Fig. 221(b). Continuous beams and brackets must develop the full value of the calculated moment. The connections shown in Fig. 220(b) are recommended for this purpose. The partially welded

connection shown in Fig. 221(a) is a satisfactory moment-resistant type for use in exterior walls and partitions where the knee brace will not project above the floor. Its main advantage is that the welding may be done after the frame has been erected by the ordinary methods. However, since rivets slip before they develop their full resistance and thereby transfer their loads to the welds, the use of rivets and welds in the same connection is not recommended. Bolts are used for the erection of welded structures. An all welded

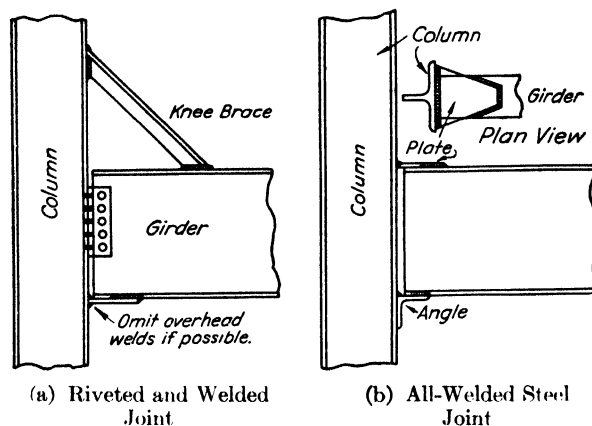


FIG. 221. WELDED COLUMN AND GIRDER CONNECTIONS.

connection is shown in (b). This joint can be made to develop the *full moment resistance* of the girder, although the welds may become objectionably thick. In addition to serving as wind bracing, welded joints of the moment-resistant type force the structure to act as a *rigid frame*.

FLOOR AND WALL CONSTRUCTION

255. Steel-Joist Floors. The simplest floor construction consists of a reinforced concrete slab resting on steel I-beam joists. The steel joists are encased in concrete for fire protection. Steel lath usually is suspended from the bottom flanges of the beams to provide support for a plaster ceiling. The slab may be varied considerably in thickness to correspond to the span between joists without serious effect upon the economy of the construction. A span of about 6 ft. between joists represents average conditions.

Precast Roof and Floor Slabs. Precast, light-weight ribbed slabs are available in many cities. Such slabs are usually of short span although some have been patented for use up to 20 ft. Precast slabs are not widely used for floors, but they have become quite popular for roof construction where the loads are

much lighter. They may be supported upon standard steel beams or on light trussed joists.

Slab Reinforcing. Slabs are reinforced with bars or wire mesh, the latter being preferred for slabs 4 in. or less in thickness. A small amount of *temperature steel*, about 0.1%, is placed in the slab parallel to the joists. Its purpose is simply to prevent contraction cracks since the load is carried in the direction of the main steel. A cross-section through a slab floor is shown in Fig. 222. The fireproofing is sometimes omitted for very light floor construction or roof construction. The steel beam joists are then replaced by Junior beam sections or by metal lumber, bar joists, or expanded-metal joists.

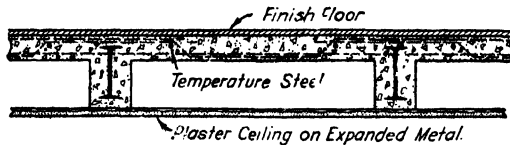


FIG. 222. CONCRETE SLAB AND STEEL-JOIST FLOOR CONSTRUCTION.

Omission of Steel Joists. It will be noticed that the fireproofing around each steel joist in Fig. 222 could readily be used to form the stem of a reinforced concrete *T*-beam. Some designers consider it economical to replace the steel joists with concrete *T*-beam construction. However, the apparent economy is not entirely realized because steel struts must then be used between the columns perpendicular to the direction of the girders to brace the steel frame before the concrete floor is poured. If steel joists are used, they serve this purpose.

Preformed Steel Deck. Thin sheet metal formed into a series of continuous channel sections (known as *Q-decking*) provides an inexpensive flat roof deck spanning between steel joists. One surface is nearly a continuous flat plate while the other side presents a deep corrugated appearance. Either side may be turned up or down. The flat surface may be turned up and covered with a multi-ply roof covering, or the corrugated side may be turned upward and filled with light weight concrete. The depth of the channel-shaped corrugations may be from 3 or 4 in. up to 12 in. or more which provides for a great range of spans and loadings.

256. Concrete Joist Floors — Metal Pan Construction. Rows of metal pans may be placed on a temporary wood deck or otherwise supported as shown in Fig. 223(a). Then when filled with concrete a continuous slab and *T*-beam floor is produced. The pans are removed after the concrete has set. Consequently a suspended ceiling becomes necessary. Although Fig. 223(a) shows the pans continuous over several 20-in. spans, the use of individual pans is common. Then, the distance between pans controls the width of the beam, and the bottom of the beam is formed with a wood plank. Two-way con-

struction (pans with closed ends are placed 4 in. apart in both directions) will be found economical for warehouse floors that carry extremely *heavy* loads. Steel pans are readily obtainable in depths from 4 to 12 in. and in widths of 20, 22, and 24 in. Several types are patented.

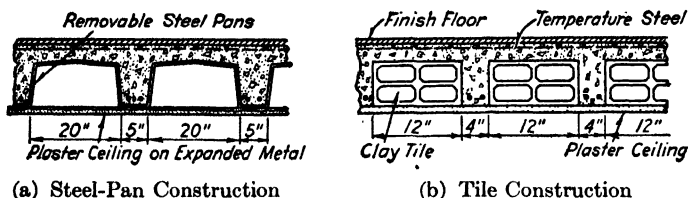


FIG. 223. CONCRETE JOIST CONSTRUCTION.

Slab Construction. The use of high strength concrete has permitted the use of relatively thin two-way reinforced slabs of concrete supported only on edge beams. If a slab 9 in. thick or less can be designed to carry the load, it will often prove less expensive than pan construction with a suspended ceiling.

Tile and Concrete Joist Construction. This once widely used type of floor is illustrated by Fig. 223(b). Rows of 12-in. square tiles are laid down on a temporary wood deck in a direction perpendicular to the main girders. The rows of tiles are placed from 4 to 6 in. apart to form the stems of the Tees. Reinforcing rods are supported between the tiles an inch or more above the deck. Finally, concrete is poured over the entire floor to a depth of from 2 to 4 in. above the tiles. When the concrete has set, the deck is removed from below and the under surface plastered directly on the tiles. The tiles used are soft burned and are formed with grooves, so that both concrete and plaster obtain a good bond. This saves the cost of a suspended ceiling. Tiles may be obtained in depths from 4 to 12 in., which controls the strength of the stem. The slab may be made any depth desired. Despite these advantages the increasing cost of tiles, and their dead weight which requires temporary support during construction and ultimate support by girders, columns and footings, has led to adoption of pan construction with suspended ceilings.

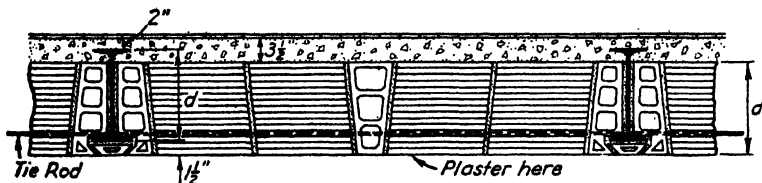


FIG. 224. OBSOLETE FLAT-ARCH FLOOR CONSTRUCTION.

257. Arch Floor Construction. Either segmental or flat-arch floor construction illustrated by Fig. 224 is entirely obsolete, but the engineer may have

to check such a floor to determine its safe load capacity. The thrust of unbalanced live loads is resisted by the tie rods which were usually $\frac{3}{4}$ -in. rods spaced uniformly across the floor. Dead-load thrusts are balanced, and dead load therefore stresses the tie rods only near the exterior walls. The rise h of the flat arch was usually taken to be $2\frac{1}{2}$ in. less than the depth of the tile. See. Fig. 224.

Stress in Tie Rod. The pull to be furnished by the tie rod for any floor arch can be determined quite simply if the loading is taken as uniform. The bending moment at the crown will be set equal to zero since this assumption will give the maximum possible value of the pull to be resisted by the tie rod. By referring to Fig. 225, we may write

$$M_o = \frac{wL}{2} \frac{L}{2} - \frac{wL}{2} \frac{L}{4} - Ph = 0,$$

from which

$$(1) \quad P = \frac{wL^2}{8h}.$$

Note that if w is in pounds per square foot, then L and h must both be expressed in feet. The value of P will be in pounds per foot width of floor.

The Floor Arch Compared with a Reinforced Slab. Since $wL^2/8$ is the bending moment in a simple span we see that the stress in the tie rod ($wL^2/8h$) is the same stress that would be computed in a reinforced concrete slab for which the effective depth jd corresponded to the rise h of the arch. The neglect of dead load for interior spans gives the arch an advantage which is not entirely justified. Actually when the dead load stresses the tie rods of exterior spans, this tension would carry through the entire set of tie rods from wall to wall except as the rods may be helped out by the wood floor or subfloor.

258. Welded Plate Decks. Small I-beams spaced about 2 ft. apart and covered by thin steel plates which are welded together and also welded to the I-beams form a strong rigid deck construction. It has been found* that 5-in., 10-lb. I-beams spaced at 2-ft. centers and covered by a $\frac{1}{4}$ -in. plate will carry a total load of 135 lb. per sq. ft. on a 20-ft. span with a 0.174-in. deflection. The weight of steel in this deck is but 15.2 lb. per sq. ft. The beams are designed as T-sections, the upper flange being formed by the steel plate. There is no objection to this procedure if the computed stress in the plate does not exceed about 4000 lb. per sq. in. Higher compressive stresses might produce *buckling*.

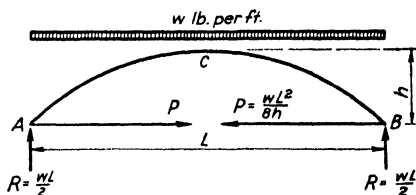


FIG. 225. TENSION IN THE TIE ROD OF A FLOOR ARCH.

* Engineering News-Record, August 29, 1923, p. 327.

Tests of Steel-Plate Decks. Load tests were made upon a **battledeck** floor by the U. S. Department of Commerce. This floor of 18-ft. span was constructed with 4-in., 7-lb. I-beams spaced 24 in. apart and covered by a $\frac{1}{4}$ -in. rolled steel plate. End restraint reduced the moments approximately to those of a 14-ft. simple span. The following conclusions were reached.*

- (1) The continuous manual arc welds united the beams and plates so that they behaved as a unit when loads were applied.
- (2) The measured stresses and measured deflections were in substantial agreement with values computed by the ordinary theory of beams.
- (3) This floor carried a load of 420 lb. per sq. ft. for $5\frac{1}{2}$ days without any indication of collapse. The total deflection was $3\frac{1}{8}$ in.; the permanent sag was $2\frac{3}{4}$ in.
- (4) The results gave no positive indication of the width of plate which may be considered effective. There is strong presumption, however, that under conditions of loading and length of span not totally unlike those of the test, an effective width up to 24 in. may be assumed in designing with $\frac{1}{4}$ -in. plate.†

The welded steel-plate deck seems ideal for ships, drydocks and for structures where great strength and light weight are desirable. Moreover, the dead load on columns and footings of structures will be reduced greatly by the use

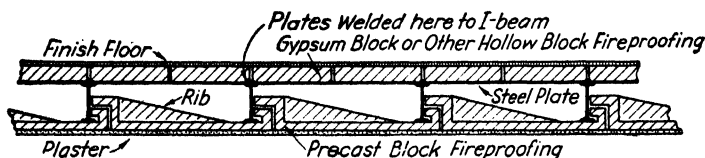


FIG. 226. BATTLEDECK FLOOR OF WELDED STEEL PLATES ON I-BEAM JOISTS.

of this type of construction. However, if precast-block fireproofing (or noise proofing) must be used both above and below the deck, as shown in Fig. 226, the advantages of the welded steel-plate floor are greatly reduced. One of the types previously discussed may then prove to be more economical.

259. Fireproof Construction. The need for fireproofing the structural steel members in tall buildings has been mentioned frequently. The following specifications cover the important requirements of most *building codes*.‡

(a) Fireproof material shall consist of brick, concrete, hollow tile or gypsum blocks, or other material passing the standard fire test.

(b) Wall columns shall be protected by 4 in. of brick or by 3 in. of concrete on their outer surfaces. The interior surface of a wall column shall be protected according to specification (c).

* U. S. Department of Commerce, Bureau of Standards, Paper RP 662.

† The author suggests $100t$ as the effective width of plate where the direct compressive stress in the plate does not exceed 4000 lb. per sq. in. It would be satisfactory to reduce the effective width by $5t$ for each 1000-lb. increase in plate stress above a working stress of 4000 lb. per sq. in. Thus the maximum compressive stress of 15,000 lb. per sq. in. would be permitted for a plate span of $45t$. This is consistent with the requirements for cover plates on compression members.

‡ For instance, *Building Code*, National Board of Fire Underwriters or National Fire Protection Association.

(c) Interior columns shall be protected by a 3-in. casing of hollow tile or other fireproof blocks or 2 in. of non-siliceous poured concrete or cast-in-place gypsum. Galvanized steel wire, No. 12 gage or larger, shall be wrapped around the column crossing each block at least once.

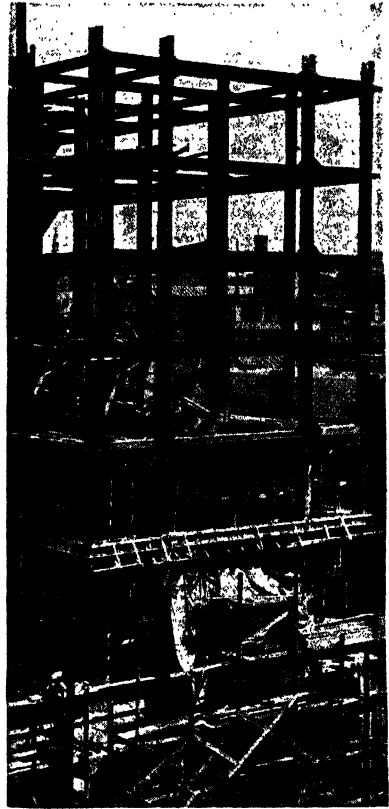
(d) Wall girders or spandrel beams shall be protected on the outer face by 4 in. of brick or by 3 in. of concrete except for flanges or outstanding plates, which may extend to within 2 in. of the outside face of the wall. The inside surface of the girder must be covered by 2 in. of fireproof material. This same specification applies to lintels.

(e) Interior girders and trusses shall be protected by a 2-in. casing of fireproof material. The minimum protection permitted for beams, joists, and secondary structural members, or for lugs or brackets attached to main structural members shall be 1½ in.

(f) Plaster is not to be considered as fireproofing unless it is specifically accepted as such.

Concrete Fireproofing. Concrete is now almost universally used for fireproofing steel members. See Fig. 215 and Fig. 227. Its cost would be considerably less than the cost of precast blocks except for the expense of forms. Some building codes allow the concrete around steel columns to be considered as carrying load when it is properly hooped. If this provision exists in the building code, the use of hooped concrete columns with structural steel cores becomes an economical solution of the fireproofing problem. The objection to this construction is that the concrete is affected by *time yield* so that it eventually transfers a considerable part of its load *onto the steel core*. A.I.S.C. specifications permit a steel beam encased in concrete to be treated as a reinforced concrete beam with increased carrying capacity.

Precast-Block Fireproofing. With the change in specifications permitting concrete fireproofing to be calculated as an aid in carrying load, all types of precast block fireproofing became practically obsolete. However, the com-



Courtesy American Bridge Co.

FIG. 227. END VIEW OF A BUILDING UNDER CONSTRUCTION.

Notice the formwork used for pouring the floors and for encasing the columns in fireproof concrete. Observe the locations of the splice plates on the columns at the top of the picture. Since the girders are connected to the columns by riveted gussets, it is clear that this building is to be many stories high.

mon methods of applying tile blocks to structural members are shown in Fig. 228. Precast blocks of any type would be applied in a similar manner. It is particularly required that all joints between the tile shall be filled with cement mortar and that all space between the tile and the steel shall be completely filled with mortar. Solid porous blocks are sometimes used. They are commonly required to be embedded in cement mortar in two layers.

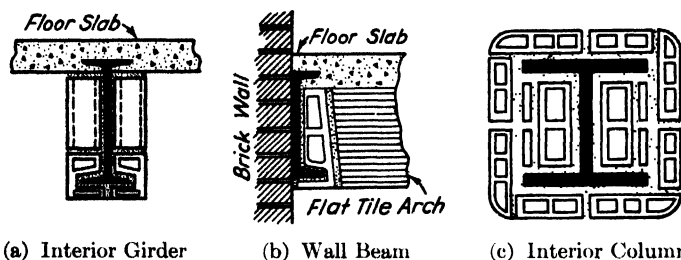


FIG. 228. OBSOLETE TILE FIREPROOFING OF STRUCTURAL STEEL MEMBERS.

Enclosure Walls. An important feature of fireproof construction is the enclosure of all elevator shafts, stairways, and chutes from one floor to another by fireproof walls. The purpose is to prevent such openings from becoming flues by which the fire might travel from floor to floor. When enclosed by such walls, the shaft also provides safe exit from the burning building provided that the exit is directly to the street.

260. Walls and Partitions. Exterior walls of steel-frame buildings are commonly of brick, although tile or concrete blocks, reinforced concrete, and metal walls also are in use. Walls of brick or of building blocks must be a *minimum* of 12 in. thick. Such walls are increased 4 in. in thickness for each additional height of 15 ft. above a normal height of 15 ft. Reinforced concrete enclosure walls may be made two-thirds of the thickness of brick walls but never less than 8 in. thick. Metal walls, such as aluminum or stainless steel, are usually required to be backed up by masonry not less than 8 in. thick for fire protection. Recent construction is gradually adopting materials of lighter weight to this purpose. Since three-quarters of the exterior wall area of a modern office building or factory may be of glass, it is clearly absurd to require more than a half hour of fire resistance for the small area of masonry that remains. Some codes specify four hours fire resistance for exterior walls but permit unlimited glass providing only ten-minute protection.

Partitions are of lighter construction than exterior walls. They are classed as permanent or temporary. *Permanent partitions* include those which enclose halls and the floor spaces occupied by different tenants. These partitions are of brick, tile, gypsum, or concrete blocks not less than 4 in. thick, or of reinforced concrete or solid plaster not less than 3 in. thick. *Temporary*

partitions are made 3 in. thick of any building block or 2 in. thick of solid plaster. It is assumed that these walls will not be extended above a single story of 15-ft. height without lateral support. The minimum thickness of a brick interior fire wall should be 12 in.

261. Weights of Materials for Floors and Walls. The following data may be used in making a preliminary estimate of the dead weight. After the design has been made, the first estimate should be checked by use of weights taken from manufacturers' catalogues. Much data on the available types of doors, windows, elevators, etc., can be obtained from *Sweet's Architectural Catalogue*, while other essential data are given in *Sweet's Mechanical File*. These are publications by Sweet's Catalogue Service, 119 West 40th St., New York.

Aluminum		165 lb. per cu. ft.
Cast Iron		450 "
Steel		490 "
Wood	Yellow pine, oak, or walnut	40 "
Brickwork	100 to 150 lb. per cu. ft. Medium grade building brick	125 lb. per cu. ft.
Concrete	Broken-stone aggregate	140 "
	Cinder aggregate	110 "
	Burned-clay aggregate	80 "
Tiles	Fire protection around beams	
(Hollow)	60 to 80 lb. per cu. ft. decreasing with increase in size of piece	70 lb. per cu. ft.
Terra Cotta	Walls and partitions	
(Hollow)	3 in. thick plastered both sides	22 lb. per sq. ft.
	4 in. thick "	25 "
	6 in. thick "	30 "
	8 in. thick "	35 "
Gypsum Block	Walls and partitions	
(Hollow)	3 in. thick plastered both sides	17 lb. per sq. ft.
	4 in. thick "	20 "
	6 in. thick "	25 "
	8 in. thick "	30 "
Plaster	On metal lath, 2 in. thick, plastered both sides	20 lb. per sq. ft.
	Suspended ceiling plastered one side	10 "
Partitions	Lightest construction, wood or pressed steel studs with expanded metal, plastered both sides	18 lb. per sq. ft.
	Studs 2" x 4" metal lath, wood lath or plaster board, plastered both sides	18 "
	Steel sheets stiffened, no plaster	4 "
Walls	Brick 8", 12"	84, 121 lb. per sq. ft.
	Windows, glass, frame, sash	3 "
	Tile 6", 8", 12"	30, 33, 45 "
Std. Wall	Brick 4", tile backing 8"	75 "

262. Weight of Steel and Cost of Office Buildings. The *total weight* of structural steel will commonly vary from 1.0 lb. per cu. ft. of interior volume for a low tier building up to 2.5 to 3.5 lb. per cu. ft. of interior volume. The unit weight increases with increased height of the building. The unit weight also increases with increase in the wind load and the live load. For data on the weight of steel in several well known buildings, see Ketchum's *Structural Engineer's Handbook*. These data may be useful in making a preliminary estimate of costs, but, if the designer works downward from the top of the building in making his design, accurate figures for estimating the dead weight above any story are always available.

The *cost of structural steel* varies greatly, depending upon shipping charges. It will be of interest to study the figures of a 1934 survey of building costs published by Engineering News-Record. The following represented average costs in 1934 for a hypothetical building of 7,400,000 cu. ft. volume (about 40 stories) costing \$5,600,000, or 76 cents per cu. ft. By 1949 these costs had approximately doubled. Hence it is clear that costs should be revised month by month.

(a) Foundations	8.0%	of total = 5.9 cts. per cu. ft.
(b) Steel frame	13.2%	= 9.9
(c) Concrete and cement work	8.9%	= 6.7
Subtotal for the structural features	30.1%	= 22.5
(d) Exterior finish	10.8%	= 8.2
(e) Interior finish	19.4%	= 14.7
(f) Mechanical and electrical fixtures	23.4%	= 17.8
(g) Miscellaneous, including fees, tests, etc.	16.3%	= 12.7
Total	100.0%	= 75.9 cts. per cu. ft.

263. Live Loads for Building Floors. Live loads as specified by the building codes of cities vary a great deal. Because of this lack of uniformity, the loadings given below simply represent an average range for each type of structure. The designer should select a load in agreement with the particular building code controlling the design. Specified live loads represent minimum values, not maximum values.

Apartments, hospitals, hotels, etc.	40 to 75 lb. per sq. ft.
Office buildings	50 to 100 "
Public buildings where crowds gather	75 to 125 "
Stores	100 to 150 "
Warehouses	150 up "
Flat roofs	30 to 50 "

The live loads specified for building floors include an allowance for impact. However, where a crowd may be densely packed together, as in a hall leading to a fire escape, a static load of 175 lb. per sq. ft. becomes possible. Again, where a crowd may move in unison, as in a stadium or possibly in an auditorium, the impact or vibration effect may possibly necessitate an increase in the live load specified by the building code.

Permissible Reduction of Live Load for the Design of Girders and Columns.

Each member must be designed to resist the entire dead load that comes on it. The live loading, on the other hand, is controlled by probability. Specifications usually require that a beam shall be designed to carry the entire tributary loading. A main girder receives its load as the reactions of several beams, not all of which probably will be loaded at one time. Some specifications accordingly allow the live load for the design of the girder to be reduced as much as 15 per cent. The engineer should study the *probability* of the occurrence of full loading before he takes advantage of such an allowable reduction.

When considering the *live loading on a column*, we should make an allowance for the extreme improbability that all floors above will be loaded at one time. Again specifications vary, but a common requirement is that all of the live load on the roof must be used along with 90% of the live load on the top floor. Then the live load on floors below is *reduced successively* by 10%, that is, we use 100% of the roof load and then 90, 80, 70, 60 and 50% of the live loads for the upper floors. All lower floors are considered to carry a live load of 50% of the full specified value. In certain types of storage warehouses, more nearly *full* live loading of all floors may be probable. Hence the reduction mentioned above would be excessive.

Surveys of Building Loads. Actual field surveys have been made of the live loads occurring on office buildings and warehouses. One study reported by C. W. Dunham* reaches the following conclusions.

(1) The basic live load for any panel of a building shall be the maximum average live load that it is intended shall be allowed on the panel.

(2) When the basic live load is 100 lb. per sq. ft. or less, for any occupancy except assembly occupancy, the basic live load may be reduced for the design of any member at the rate of 0.08% per sq. ft. of tributary area; but the maximum reduction shall not exceed $w_{DL+LL}/4.33w_{LL}$.

(3) For assembly occupancies the basic live load shall not be reduced for the design of any member.

(4) When the basic live load is more than 100 lb. per sq. ft. it shall not be reduced for the design of floors, beams, girders, or trusses. It may be reduced at the rate of 0.08% per sq. ft. of tributary area for the design of columns, but the maximum reduction for these heavier live loads shall be limited to 20%.

Although the sudden change in treatment above the load of 100 lb. per sq. ft. is objectionable, this paper is an important one and deserves careful attention. Another paper by C. W. Barber† proposes the following formula for live-load reduction R in per cent.

$$R = 74 - \frac{w}{5.4} - \frac{5000}{A}$$

where w is the unit live load and A is the tributary area excluding the roof.

* C. W. Dunham, *Design live loads in buildings*, Transactions, A.S.C.E., Vol. 112, 1947, pp. 725-744.

† C. W. Barber, *A proposed live load reduction formula for buildings*, Civil Engineering, April, 1942, p. 194.

264. Spandrels or Wall Beams. In skeleton construction, it is necessary for the wall beam to support the weight of the wall. Hence the spandrel beam must be placed inside the wall. However, fire regulations require that the

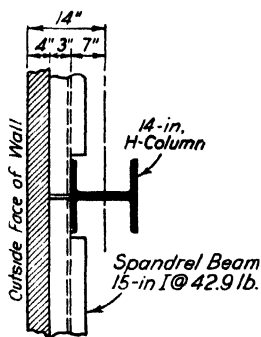


FIG. 229. PLACEMENT OF THE WALL COLUMN.

face of the beam shall be protected by not less than 4 in. of brick or by 3 in. of concrete. The same requirements apply to the column. It follows that the arrangement of spandrels affects the placing of the exterior columns. For instance, if a spandrel beam is a standard 15-in., 42.9-lb. I-section which is placed on the outside of a 14-in., 87-lb. column as shown in Fig. 229, the common minimum distance from the center line of the column to the outside face of the wall will be 14 in. The absolute minimum distance will be 12 in. if the building code permits the flange of the spandrel beam to be placed at 2 in. from the face of the wall. In case the outside face of the wall is of stone or other material which is not considered to be satisfactory for fire-proofing,

this distance of 12 or 14 in. will be increased by the thickness of the outside course. In all cases, the wall columns are placed as near the outside face of the building as possible. An I-beam would be preferred to a wide flange beam of equal strength for spandrel use because its narrow flange reduces the eccentricity of the wall load on the column. However, if the wall load is necessarily eccentric to the center line of the spandrel beam, a section of greater torsional resistance may be required.* Observe the spandrel beams in Fig. 219.

Use of Two or More Spandrel Beams. It will be noticed in Fig. 229 that the load transferred from the spandrel beam into the column is eccentric 7 in. from the center line of the column. In order to reduce the bending moment caused by this eccentricity, it may be desirable to use *two* spandrel beams with one placed as in Fig. 229 and with the other placed either on the center line of the column or even to the right of the center line. The outer beam will

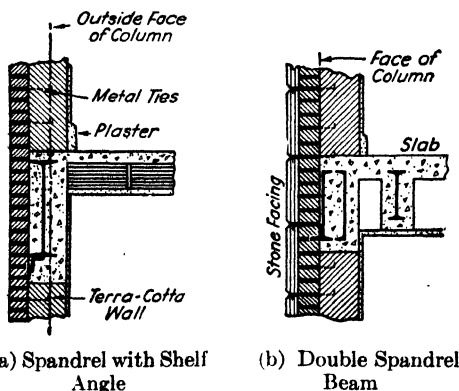


FIG. 230. SPANDREL SECTIONS.

of the column or even to the right of the center line. The outer beam will

* J. E. Lothers, *Torsion in steel spandrel girders*, Transactions, A.S.C.E., Vol. 112, 1947, pp. 345-376.

support the outside wall while the inner beam will act mainly as a floor joist, although it may also carry a part of the wall load.

Figure 230(a) shows a spandrel where a single beam serves to support both the wall and the floor. Two angles attached to the lower flange form a shelf for support of the outer brick facing of the wall. The lower angle may also be acting as a lintel over windows which do not show in the figure.

Figure 230(b) shows a spandrel section where three beams are used. A steel beam adjacent to the wall supports the edge of the floor. A channel on the center line of the column supports a part of the wall load. A channel framed to the outside face of the column is used to support the outer brick wall and the stone facing on the building. Note the use of *metal anchors* or *ties* to hold the three parts of the wall together.

265. Lintels. A lintel is a beam spanning an opening in a masonry wall and supporting the masonry above. Due to the *corbelling* or *arch* action of the masonry above the opening, it is only necessary for the lintel to support a

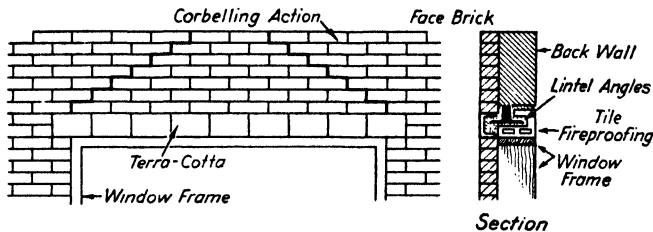


FIG. 231. FIREPROOFED LINTEL.

triangular or trapezoidal shaped piece of the wall. See Fig. 231. The lintel will be a steel angle or a channel for a wide opening, which spans the opening as a simple beam and carries the wall loading. This loading commonly is calculated for a 45° slope, although, as shown in Fig. 231, the slope is actually much flatter. It should be noted that either arch action or corbelling action requires that the wall extend for a considerable distance to *each* side of the opening. Where this requirement is not met, it would not be wise to allow for such action in the design of the lintel. The steel lintel usually is hidden in the brickwork both to secure a good appearance and to fireproof it.

266. Windows, Doors, etc. The selection of the inside equipment of the building is the province of the architect rather than that of the structural engineer. For current information on the merits of the many types of doors, windows, elevators, lighting fixtures, plumbing equipment, etc., refer to Sweet's Catalogues and to manufacturers' trade catalogues. Handbooks and textbooks are seldom adequately up to date. For example, the following table is typical of information only available from the manufacturer.

TABLE 7
FREIGHT ELEVATOR SIZES AND CAPACITIES

STANDARD PLATFORM SIZE		NET AREA SQ. FT.	STANDARD DOOR SIZE		STANDARD CAPACITY RATING — LB.
Width	Depth		Width	Height	
5 ft. 4 in.	7 ft.	32.5	5 ft.	8 ft.	2,500, 3,000
6 ft. 4 in.	8 ft.	45	6 ft.	8 ft.	2,500, 3,000, 4,000
8 ft. 4 in.	10 ft.	76	8 ft.	8 ft.	4,000, 5,000, 6,000, 8,000
8 ft. 4 in.	12 ft.	92	8 ft.	8 ft.	5,000, 6,000, 8,000, 10,000
10 ft. 4 in.	14 ft.	135	10 ft.	8 ft.	10,000

Note: Clear width inside car is 4 in. less than given above. Clear depth is 6 in. less and 8 in. less for openings at both front and rear. Data from Westinghouse Electric Co.

Stairs require special structural framing around the stair well. The loads from stair stringers can be estimated and the supporting members designed. Frequently their strength requirement is relatively small, in which case they are made of light 8-in. or 10-in. channels. The latter section provides better connections for the stair stringers. Framing around other openings, such as elevator shafts, produces similar problems.

Landings may be supported by ties from above, by columns from below, or in some cases by cantilever beams at their own levels. Such design involves complicated details but does not constitute a serious structural problem.

Roofs may be constructed in the same manner as floors. In fact, the structural roof is commonly a duplication of the floor except that the sections used are somewhat lighter. If a concrete roof slab is used, it must be covered by a waterproof membrane roofing material. A tile floor can be laid on top of the roofing material as a wearing surface. Sheet insulation should be attached to the bottom of the roof joists if the upper story is to be used as rentable space.

PROBLEMS

212. Select the beam sizes and compare weights of steel for the two floor panels shown in Fig. 216. Assume that the floor consists of a 5-in. reinforced concrete slab covered with 1-in. flexible floor tile which weights 9 lb. per sq. ft. Include a live load of 125 lb. per sq. ft. Encase the beams with 2 in. of concrete fireproofing as in Fig. 222 and allow 10 lb. per sq. ft. for a suspended ceiling. The columns are 12-in. sections. Neglect wind stresses and allow a working stress in flexure of 20,000 lb. per sq. in. Reduce the live load 15% for the design of the girders. In Fig. 216(b) one wide flange beam can take the place of the two channels shown.

213. Make out a table listing the requirements of three Building Codes, including that of the National Board of Fire Underwriters, covering the thickness of fireproofing for interior columns, wall columns, interior beams and spandrel beams.

214. Make out a table listing the requirements of three Building Codes, including the National Board of Fire Underwriters, covering the minimum live loads listed in § 263.

215. A battledoor floor is constructed of 5-in., 10-lb. I-beams spaced at 24-in. centers and covered with a $\frac{1}{4}$ -in. plate. The span is 20 ft. Compute the live-load carrying capacity of the floor if its total dead load including a suspended ceiling for second-rate fire-proofing and a 1-in. concrete terazzo floor reinforced with punched rings welded to the floor plate is 40 lb. per sq. ft. Limit the fiber stresses to 18,000 lb. in tension and 4000 lb. in compression in the plate produced by a moment of $\frac{1}{12}wL^2$. The weight of a tile floor with concrete joists would be about 75 lb. per sq. ft. to carry the same live load, but it would be fully fireproof.

Ans. 90 lb. per sq. ft.

216. Same as Problem 215 except use 3-in., 5.7-lb. I-beams and a $\frac{3}{16}$ -in. plate for a beam span of 15 ft. Assume that the dead load is 35 lb. per sq. ft.

Ans. 50 lb. per sq. ft.

217. Obtain costs from one of the construction magazines for a building recently erected and compare with the data as given in § 262.

218. Make up a table of sizes of double-angle lintels to span openings of 4, 6, 8, and 10 ft. to carry a 12-in. brick wall. Allow a fiber stress of 20,000 lb. per sq. in. Base your calculations upon a superimposed triangular loading of 35° slope.

STRESSES IN MEMBERS AND CONNECTIONS

267. Stresses in Moment Resisting Connections. The usual form of split-beam connection offers a simple case to investigate. Consider the connection shown in Fig. 232 where there are eight rivets through each flange of the girder. Similar connections are shown attached to the column in Fig. 233.

Rivet Stress. Apparently the horizontal shear in each rivet group is $170,000 \div 2.0 = 85,000$ lb. The shear per rivet is $85,000 \div 8 = 10,600$ lb. If the allowable shear per rivet is 15,000 lb. per sq. in. increased $33\frac{1}{3}\%$ for wind, $\frac{7}{8}$ -in. rivets should be used. The same stress of 85,000 lb. must be resisted in tension by the upper group of rivets through the column flange.

Since tests have shown the safe tension value of a rivet to be at least equal to its value in single shear,* eight $\frac{7}{8}$ -in. rivets are required in each group through the face of the column.

Naturally, there is always the question as to whether the tension pull is equally distributed to the several rivets of the joint. Tests made at the

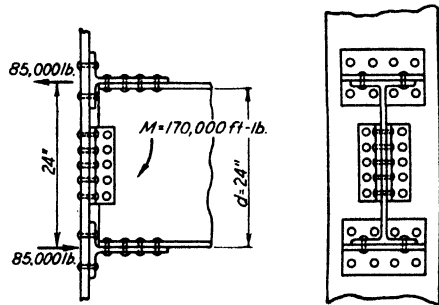


FIG. 232. RIVETED SPLIT-BEAM CONNECTION.

* W. M. Wilson and W. A. Oliver, *Tension Tests of Rivets*, Bulletin No. 210, University of Illinois Engineering Experiment Station, p. 35.

University of Toronto* show that a properly designed connection will develop a tension resistance of at least 90% of the sum of the tension values of the individual rivets.



Courtesy American Bridge Co.

FIG. 233. THREE-STORY COLUMN WITH SPLIT-BEAM CONNECTIONS.

The open holes at the left end are for the splice-plate rivets. The section consists of a 305-lb. rolled H-column with four cover plates 16 in. by $\frac{7}{8}$ in. Two-story columns are more usual.

EXAMPLE. Flexure of the Split-Beam Connection. The main stress in the split-beam connection is flexure caused by the pull of the tension rivets. There is considerable question as to whether the beam flange should be considered to be bent as a *single cantilever* or as a *double cantilever*.

The reversed deformation curve of Fig. 234(a) illustrates the actual situation *before* initial tension in the rivet is overcome by the pull P . *After* the initial tension has been overcome, the sketch (b) illustrates the action of the beam flange which may be described as simple cantilever flexure. In Fig. 234(a), the bending moment in the split-beam flange will be nearly 50% less than that which exists in (b). However, since the forces, C_1 and C_2 (bearing of the ends of the split-beam flanges on the face of the column) act *with* the pull P , it is clear that the rivet stresses T_1 and T_2 are each considerably more than $P/2$. In fact,

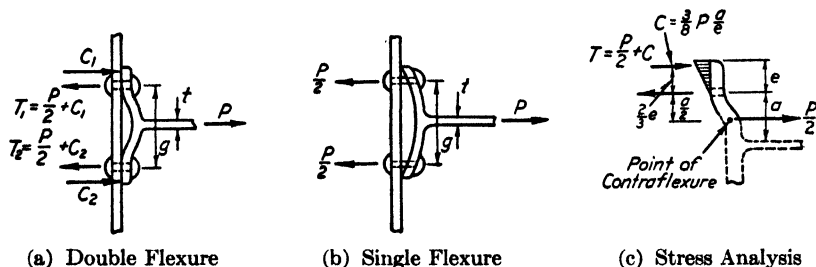


FIG. 234. ANALYSIS OF THE SPLIT-BEAM CONNECTION.

if the pressure under the edge of the flange is assumed to have a triangular variation, as shown in Fig. 234(c), an equation of moments about the point of contraflexure gives the expression

$$(2) \quad C = \frac{3}{8} P \frac{a}{e},$$

* C. R. Young and J. B. Jackson, *The relative rigidity of welded and riveted connections*, Canadian Journal of Research, 1934, pp. 62-134.

from which we obtain

$$(3) \quad T = \frac{P}{2} + C = \frac{P}{8} \left(4 + 3 \frac{a}{e} \right).$$

It has *not* been customary for engineers to design the rivets for this increased tension. Hence Fig. 234(b) should form the basis for the design of the split-beam flange.

The bending moment in the split-beam flange of Fig. 234(b) is

$$(4) \quad M_f = \frac{P}{2} \left(\frac{g}{2} - \frac{t}{2} \right).$$

Since g is always $5\frac{1}{2}$ in. for large rolled beams and t is at least $\frac{1}{2}$ in., the flange moment will not exceed $1.25P$. The required section modulus for a pull of 85,000 lb. would be $(85,000 \times 1.25) \div (20,000 \times 1.33) = 4.0$. If the length of the beam stub is 12 in., the thickness may be obtained thus:

$$S = \frac{12d^2}{6} = 4.0, \quad d^2 = \frac{6 \times 4.0}{12} = 2.0, \quad d = 1.4 \text{ in.}$$

The only rolled beams of this flange thickness are the 33-in., and 36-in. sections which weigh approximately 250 lb. per ft. If it is necessary to use a *lighter* split beam, the flange moment can be reduced approximately 50% (and the flange thickness in the example above may be reduced from 1.4 to 1.0 in.) by the use of extra rivets to resist the increased tension force of equation (3) and Fig. 234(c).*

268. Welded Beam and Column Connections.

The University of Toronto tests mentioned above show clearly that uniformity of stress distribution for a welded connection is far more difficult to obtain than for a riveted connection. For instance, a welded split I-beam connection to the flange of an H-column section failed at but 49% of its design value. This was undoubtedly due to the non-uniformity of the stress distribution through the weld. Horizontal stay plates welded between the column flanges at the level of the

split-beam connection improved the stress distribution through the length of the weld by preventing deflection of the column flanges. Nevertheless, these test results were not such as to encourage the use of welded split-beam connections.

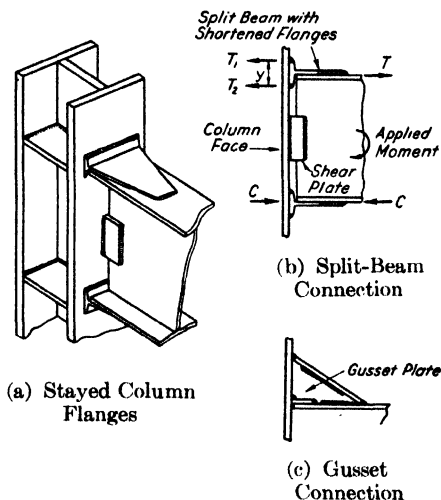


FIG. 235. WELDED CONNECTIONS PRODUCING RIGID JOINTS.

* For a further study of the action of the split-beam connection, see the author's discussion in Transactions, A.S.C.E., 1933, p. 734; also see discussion by U. T. Berg, p. 711.

Shortened T-Connection. An improvement in the welded split-beam connection is shown in Fig. 235, where the *flanges* of the split beams or T-sections are shortened to prevent the action illustrated in (b) of Fig. 234, which obviously would produce severe cross flexure in welds placed along the sides of the split-beam flange. It might be thought that the substitution of a plate for the split beam would result in a further improvement. However, the *eccentricity* of the force T with respect to the welds on the column face indicates that it would be desirable to separate these welds by a greater distance y than the thickness of a plate in order to equalize the division of T between T_1 and T_2 . See Fig. 235(b). We might attempt to adjust the relative lengths of the welds opposite T_1 and T_2 to improve further the distribution of stress. This is not to be taken to mean that the plate connection of Fig. 221(b) is unsatisfactory. The use of a split beam with shortened flanges is better design, however, where the moment of eccentricity to be resisted is large.

Gusset Plates and Knee Braces. As an alternate to the split-beam connection with shortened flanges, we might use the gusset connection, as shown in Fig. 235(c). Such gusset connections showed excellent resistance in the Toronto University tests. A second alternate is the welded knee brace shown in Fig. 236. The analysis of the knee-brace connection for moment is made clear on the figure. It might be preferable to change the bottom plate connection to a split beam with shortened flanges. In most cases, the vertical-shear connection to the column face has been of bolted clip angles for convenience in erection. The bolts no longer function after field welding. It is assumed that the entire end reaction of the beam will be resisted through the welded shear connection to the web.

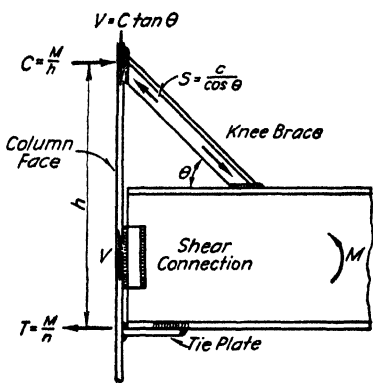


FIG. 236. WELDED KNEE-BRACE CONNECTION.

269. Riveted Gusset-Plate Connection. Another important connection is the gusset-plate type shown in Fig. 237. The analysis of stresses in the *horizontal lines of rivets* connecting the girder angles to the gusset plate is the same as has been discussed above. The moment is divided by the vertical distance h' , between horizontal rows of rivets to obtain the pull that must be resisted by one horizontal line of rivets acting either in double shear or in bearing on the gusset web.

Rivets through Column and Gusset. The vertical rows of rivets through the column flange act in *tension* above the neutral axis, while below the neutral axis the angles bear against the face of the column. The effective area of

cross-section consists of tension-rivet areas above and bearing area below the neutral axis, as shown in Fig. 237(b). The neutral axis is at a height of about $\frac{1}{4}$ of the depth h above the bottom of the connection angles. Initial rivet tension would raise the neutral axis to about the mid-height at ordinary working stresses, but not at the point of failure. The stress in the farthestmost rivet is computed by an application of the Mc/I formula using I as the moment of inertia of the effective section and c as the distance from the neutral axis to the farthest tensile rivet as shown in (b).

The vertical row of rivets through the gusset plate, Fig. 237(a), should contain approximately as many rivets as are used in a single row through the column flange, since these rivets through the gusset plate are in double shear and the tension value of a rivet is taken as equal to its single-shear value. Naturally, then, a gusset plate must be chosen thick enough so that the bearing value of a rivet on the gusset plate will at least equal the rivet value in double shear.

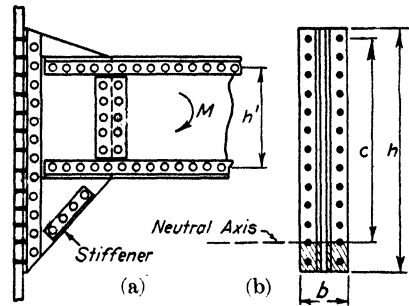


FIG. 237. RIVETED GUSSET-PLATE CONNECTION.

270. Good Design of Moment-Resistant Connections. It is unsatisfactory to use two lines of tension rivets in one leg of an angle. Since the angle leg is relatively flexible, practically all of the pull will come on the row of rivets nearest the back of the angle. The same reasoning leads us to question the value of four rows of tension rivets through the flange of a split-beam connection. In this case, however, the thick flange justifies us in allowing some value for the outside rows of rivets. It is suggested that a reasonable procedure would be to reduce the tension value of the outside rivets by 50 per cent.

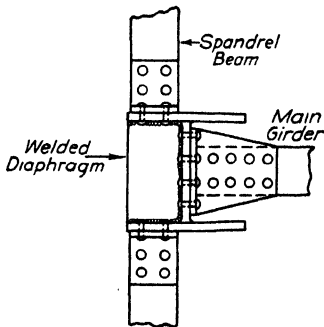


FIG. 238. DIAPHRAGM STIFFENER FOR A WALL COLUMN.

The column itself may need *local stiffening* to resist the stresses introduced by a heavy moment-resistant connection. The column web may be too thin to resist the pull of the connection, while a column flange may need stiffening, particularly if the pull is eccentric.

Figure 238 illustrates the use of a welded diaphragm on a wall column for the purpose of strengthening both the web and the flange. Such diaphragms must be located where they will not interfere with field riveting. It is good practice to weld such details as stiffeners even where all main connections are riveted.

271. Combined Stresses for Girders. On account of the fact that the girders must resist both the gravity loads and the wind moments, their *combined stresses* are important. Figure 239 shows moment curves caused by gravity loads, wind loads, and combined loads. A beam with end connections that resist moment is neither *simple*, as in (a), nor *fixed*, as in (b), but it is *restrained*, as shown in (c). Since the degree of restraint varies and in any

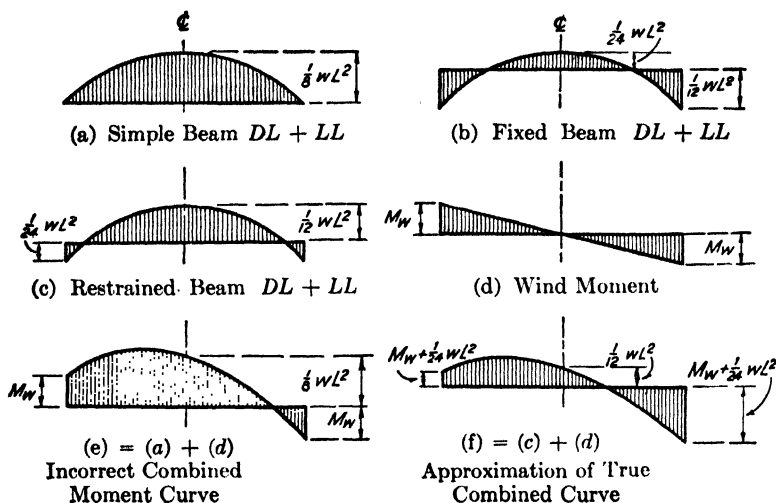


FIG. 239. COMBINED MOMENT DIAGRAM FOR GIRDER DESIGN.

case is not fully known, the moment curve of Fig. 239(c) was chosen to represent an average condition *half way between simple and fixed ends*. Figure 239(d) shows a theoretical moment diagram for wind with the point of contraflexure or of zero moment at the center of the girder.

Combined Moment Curves. The common practice has been to combine the curves (a) and (d), which gives rise to the curve (e), despite the fact that the curve (e) may not represent the maximum moment either in the end connection or in the girder itself. The curve (f), which is obtained by combining (c) and (d), would be more reasonable. In either case, notice that the maximum positive moment occurs to one side of the center of the span.

A study of the curves (e) and (f) of Fig. 239 shows that the common practice (use of the curve (e)) has been to *overdesign* the girder and to *underdesign* the end connection, except where the wind moments were relatively high, in which case both the girder and the connection would be underdesigned. Since the end connection would therefore be over-stressed, there would be a slip in the rivets and a tendency to allow the beam to act *more nearly* as a simple beam under gravity loads. At least this theory has formed the basis for the use of

the curve (e). Now, however, engineers have concluded that rigidity of the end connections is nearly as important as strength. It follows that any overstressing of the connections which produces slip should be outlawed, and the curve (f), or a similar curve, preferably determined to agree with the actual condition of end restraint, should be made the basis for the design of the girder and its connections.

Span Length. The length of a girder must be considered to be the clear distance between its end connections unless gusset plates are used. In that case, the girder section required to resist negative moment is controlled by the moment at the face of the gusset. The moment in the connection is the theoretical maximum moment when the connection is riveted to the column web, but this theoretical moment may be reduced for the shortening of the span when the connection is to the flange of the column. Of course, the wind moment is equal to the wind shear at the point of contraflexure times the distance to the point at which moment is computed.

272. Combined Stresses for the Columns. Under the action of gravity loads, an interior column receives no bending moment if both adjacent girder spans are of the same length, and if both spans are loaded equally. Some moment will be introduced into the column when the live load extends over but one of the adjacent spans, or where the adjacent spans are of unequal lengths. Evidently both the dead load and the live load will produce similar moments in the exterior columns. The importance of the moments due to gravity loads is dependent upon the end connections of the girders. Standard clip-angle connections (Fig. 217(a) and Fig. 220(a)) could not introduce gravity moments into the columns, but neither could they resist wind moments. If moment-resistant end connections are used, the neglect of gravity-load moments in the columns involves the idea that rivets will *slip* and connections will *rotate* when overstressed, which is rather a disquieting assumption and one that is difficult to justify. A proper design should, of course, be based upon moments calculated for the structure as a rigid frame which requires a knowledge of the methods of analysis of statically indeterminate structures. (See Vol. 2.)

Eccentric Column Loads. The bending moment in an exterior column produced by the eccentric gravity loads is always considered in design. Since the girder normal to the wall may frame into the flange rather than the web of the exterior column, and since the spandrel beams that carry the exterior walls may even be supported by brackets on the outside of the columns, the moments produced by such eccentric loads may become very important. Their action produces a point of contraflexure near the mid-height of the exterior column (Fig. 240). Hence these moments can be combined directly with the wind moments which have the same distribution. It is of importance that the moments due to eccentricity do not accumulate from story to story. They are

counteracted instead by moments introduced by the direct stresses (Re/h) in the girders. Figure 240 is self-explanatory. The moment of eccentricity is Re . The corresponding column bending moment is $Re/2$ for the point of contraflexure at the mid-height.

Stress Combinations. The columns must be designed to resist (1) the direct stresses caused by the gravity loads, (2) the flexure due to the eccentric gravity loads, (3) the direct stress produced by the overturning effect of the wind,

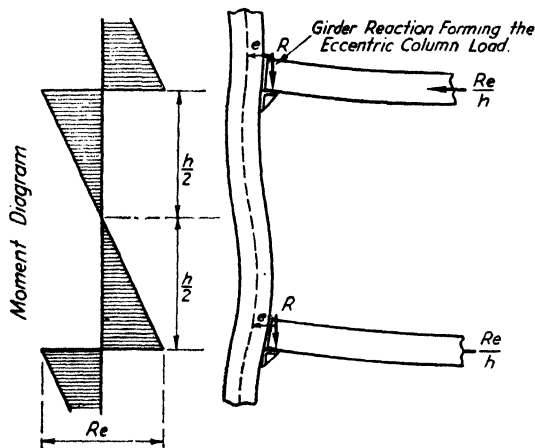


FIG. 240. COLUMN MOMENTS CAUSED BY ECCENTRIC LOADS.

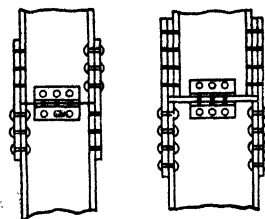
and (4) the flexure caused by the wind forces where deep diagonal bracing is not used. The maximum combined stress in the column may be calculated from the flexure formula: $f = P/A + Mc/I$. Sometimes it is convenient to express the eccentric load as an equivalent central load. If M represents the bending moment due to eccentricity (equivalent to $Re/2$ for the multi-story column of Fig. 240), then the equivalent central load is $P + Rec/2r^2$. Here r is the radius of gyration of the column section about the axis of flexure and $2c$ is its corresponding depth. This formula is convenient if the size of the column is known but its area is unknown. Then r may be estimated and the equivalent central load determined. A section selected to carry this load gives a first approximate design.

Since allowable working stresses are usually increased $33\frac{1}{3}\%$ when wind is considered, the columns in buildings of ordinary height will not need to be increased in size for wind. The same is true for the upper columns of tower buildings. In any building, it is rather unusual for wind flexure to overcome the dead-load compression and to produce tension in the extreme fiber of a column. Only in the most slender towers could the overturning effect of the wind overcome the dead-load compression and make it necessary to anchor the columns of their foundations. Nevertheless, specifications frequently require column anchorage simply as an added factor of safety.

273. Column Splices. Columns commonly are fabricated in two-story lengths that are spliced from 2 to 4 ft. above the floor at a place where the column splice and the girder connection will not conflict. The two-story col-

umn length wastes a small amount of material that could be saved by changing the section at every floor level, but the saving in erection costs and the cost of the splice more than offsets this small waste of material. Longer columns would be used despite the added material except for the fact that they require special erection equipment. Three-story columns (Fig. 233) have been used in several buildings.

Typical column splices are shown in Fig. 241. If there are no fill plates, the field rivets should be arranged as in (a) for ease of erection. The joint is riveted together by use of the side plates, which are not ordinarily assumed to transfer any stress, but which are used simply to



(a) (b)

FIG. 241. COLUMN SPLICES.

hold the sections in line. Where the section changes in width, a *horizontal plate* as well as *side plates* and *fills* are required, as shown in (b). Many engineers design the splice plates to transfer 25% of the column stress merely to guarantee a stiff joint. Actually, the ends of the pieces are milled to a perfectly flat surface, which assures us that practically all stress is transmitted by bearing. Of course, in the unusual case where flexure reverses the stress to tension in the extreme fiber, the splice plates must be designed to reproduce the moment resistance that is needed.

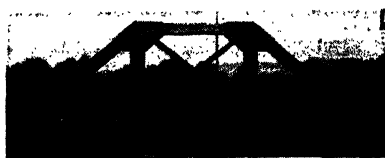
274. Irregularity of Column Arrangement. The removal of a column near the bottom of a building to obtain a clear ballroom floor is not unusual. This



Courtesy American Bridge Co.

FIG. 242. HEAVY PIN-CONNECTED TRUSS.

These trusses were used in the Stevens Hotel (Chicago) to span over the ballroom. The trusses are two stories high, and they carry the intermediate column loads.

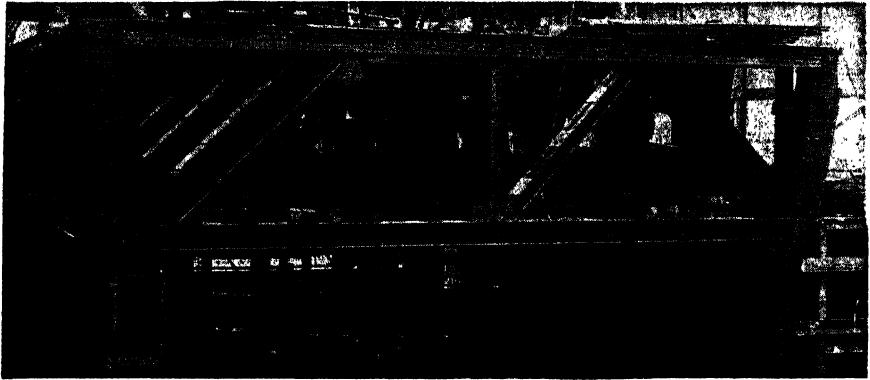


Courtesy American Bridge Co.

FIG. 243. WELDED TRUSS FOR A POWER HOUSE OF THE GENERAL ELECTRIC CO.

necessitates the introduction of a heavy girder (Fig. 190) or a truss to carry this column load to the adjacent columns. Such a pin-connected building truss is shown in Fig. 242, while a similar welded truss is shown in Fig. 243.

Occasionally the architect's plans require a change in the column spacing. Offset columns can be arranged as illustrated in Fig. 244, where the truss is also used to span over a wide lobby.



Courtesy American Bridge Co

FIG. 244. TRUSSES DESIGNED TO CARRY INTERMEDIATE COLUMNS.

Note that the columns are flared outward to permit a change in the spacing of the columns in the upper floors.

PROBLEMS

219. Design a riveted split-beam connection to develop 50% of the moment resistance of a 30-in., 108-lb. beam for a fiber stress of 18,000 lb. per sq. in. Rivets can be stressed to 13,500 lb. per sq. in. in single shear or in tension (increased $33\frac{1}{3}\%$ for wind). Base a design first upon Fig. 234(b) and then upon Fig. 234(a) or (c). Use a beam stub about 12 in. long with 1-in. rivets.

Ans. $\begin{cases} 36\text{-in., 260-lb. } WF \text{ with 8 rivets in tension, or,} \\ 36\text{-in., 160-lb. } WF \text{ with 12 rivets in tension.} \end{cases}$

220. Same as Problem 219 except that the connection is to develop the full moment resistance of a 24-in., 76-lb. beam. Use current A.I.S.C. working stresses.

221. Design an alternate welded connection for the beam of Problem 219. Welds may be stressed to 1200 lb. per lineal in. per $\frac{1}{8}$ in. of fillet leg. This working stress can also be increased $33\frac{1}{3}\%$ for wind.

222. Design an alternate welded knee-brace connection for the beam of Problem 220. The knee brace is at 45° and has a length along its center line of 36 in. Study Fig. 236. Follow current A.I.S.C. working stresses.

223. Assume that the vertical spacing of $\frac{3}{4}$ -in. rivets in the gusset connection of Fig. 237 is 4 in. and that the $4 \times 4 \times \frac{1}{2}$ -in. clip angles to the face of the column are 4 ft.-3 in. long. Compute the moment resistance of this connection based upon the cross-section shown in (b) if the tension in a rivet is not to exceed 10,000 lb. per sq. in.

Ans. 1,750,000 in.-lb.

224. Repeat Problem 223 based upon the assumption that the neutral axis is at *one-seventh* of the height h above the bottom of the clip angles. This is usually a fairly satisfactory assumption. What percentage of this moment resistance would be obtained by a

calculation based upon the neutral axis at the mid-height with rivet areas taken as furnishing the effective cross-section both above and below the neutral axis. *Ans.* 61%.

225. Same as Problem 223 except use current A.I.S.C. working stresses.

226. Assume that w in Fig. 239 is 4000 lb. per lineal ft., that the girder span is 20 ft., and that M_w is 75,000 ft.-lb. Plot the curves (e) and (f) and comment upon what effect the use of the incorrect curve (e) would have upon the design both of the girder and of the connection.

227. Repeat Problem 226 for these changed conditions: $w = 3000$ lb. per lineal ft., $M_w = 200,000$ ft.-lb. In what way would the use of the curve (e) affect this design differently than the design studied in Problem 226?

275. Evolution of Building Design. It is an undisputed fact that building design remains on a *less conservative basis* than bridge design. There are several contributing factors. Of greatest importance, perhaps, is the fact that bridge design has always been dominated by the conservative railway-bridge engineer through the influence of the American Railway Engineering Association, while building design has been influenced by the architect's liberal and sometimes even careless attitude toward the structural features of the design. As a case in point, consider the use of tension rivets. Conclusive tests became available about 1930 from two sources* that justified the use of rivets in tension at a working stress equal to the single-shear value of the rivet. However, without the justification of such tests, and in direct opposition to the bridge engineer's specifications outlawing the use of tension rivets, designers of tall buildings for the preceding three decades had been making the wind resistance of their structures dependent upon the resistance of tension rivets. On the other hand, it is most likely that a new generation of bridge designers will have to be developed before the prejudice against tension rivets, resulting in low working stresses even where reversal is not a consideration, will disappear from the field of bridge design.

The building designer has been proved correct in his acceptance of the use of tension rivets, but in other matters his overly liberal attitude has been unjustified. Buildings have been designed to resist a 20-lb. wind pressure in localities where the occurrence of a 30-lb. wind pressure could not reasonably be questioned. Moreover, working stresses in such cases have been increased 50% as a wind-stress allowance. Such inadequate structures were in the past greatly strengthened by the resistance of the outside walls; but with lighter walls, which may be practically of no assistance because of the large percentage of the wall surface that is glazed, a more conservative design is required. The building codes of most cities now make a more adequate structure necessary.

* C. R. Young and W. B. Dunbar, *Permissible Stresses on Rivets in Tension*, Bulletin No. 8, University of Toronto School of Engineering Research, 1928.

W. M. Wilson and W. A. Oliver, *Tension Tests of Rivets*, Bulletin No. 210, University of Illinois Engineering Experiment Station.

CHAPTER 11

WIND STRESSES IN TIER BUILDINGS

276. The Wind-Stress Problem. In buildings whose height is more than twice the least width, we may have to add material merely to resist wind forces. Earthquake forces may produce equally serious effects in low buildings. This extra material to resist lateral forces may be in the form of diagonal bracing or knee braces, or it may appear as an increase in the size of columns and girders when moment-resistant connections are introduced. Since we are allowed to increase working stresses $33\frac{1}{3}\%$ due to wind or other lateral forces before sections are increased, moment-resistant connections may be utilized to provide considerable wind resistance without increasing the weights of members. Beyond this limited range it is always cheaper to resist lateral forces by diagonal bracing, but the use of diagonal bracing may have to be restricted to a few locations around elevator shafts and stairways by architectural limitations. If sufficient diagonal bracing cannot be introduced, the use of moment-resistant connections may be the only solution.

The stresses in diagonal bracing may be calculated with reasonable accuracy by statics; but the structure with moment-resistant connections is highly redundant. The calculation of the wind moments by statics requires the acceptance of several simplifying assumptions. Hence the wind stresses when calculated are subject to serious inaccuracies. Nevertheless, such crude analyses have more frequently than not been the only basis of design for wind resistance. Also, such methods must be understood before a more exact analysis of wind stresses by the methods of indeterminate structures could be attempted. Hence, the study of such methods is considered doubly important.

Stability. As a first consideration we must provide that the building shall have sufficient stability that it will not be overturned by the wind. The question may be raised as to whether the resisting moment of the dead weight might be computed about the leeward line of columns. However, since these columns would be unable to carry the entire weight of the building, such dead load overturning resistance will not be developed. Instead the tensile wind stress in a windward column should be limited to two-thirds of its dead-load compressive stress unless it is adequately anchored to a footing which is able to resist $1\frac{1}{2}$ times the calculated net uplift of the column.

277. Wind Loads for Buildings. The tall office building is one of the few structures that extends to a sufficient height to reach above the disturbed wind

zone near the surface of the earth. Considerable information is available in regard to the probable wind to be encountered near the ground (see § 88, Chapter 3), but much less is known about the action of the wind at 1000 ft. above the ground. The highest permanent anemometer was at an elevation above the ground of less than 500 ft. until the construction of the Empire State Building* with its pressure recording apparatus. An excellent discussion of wind pressure is given in a book by Robins Fleming, *Wind Stresses in Buildings*.

A study of such data as are available leads to the following observations.

- (a) The wind velocity increases with the height above the ground.
- (b) The wind velocity becomes more uniform at greater elevations, that is, the maximum gust velocity and the average five-minute velocity approach each other.
- (c) Maximum wind velocity is very different in different parts of the country.
- (d) Winds of hurricane intensity are to be expected in only a few localities.

Code Requirements — Wind Load. The following represent the requirements of several building codes before 1940 in regard to wind pressure for the design of wind bracing for tall buildings. The lack of definite knowledge of wind pressure is evident in these specifications. Since 1940 the specified wind pressure in these and other city building codes has become more consistent due to the influence of the recommendations of the A.S.C.E. Sub-Committee on *Wind Bracing in Steel Buildings* which is given as the final specification below. See Transactions A.S.C.E. Vol. 105, 1940, pp. 1713-1739.

<i>New York.</i>	20 lb. per sq. ft. on the area above a height of 100 ft. 33⅓% allowable increase in working stress.
<i>Chicago.</i>	20 lb. per sq. ft. over the entire area. 50% allowable increase in working stress.
<i>St. Louis.</i>	20 lb. per sq. ft. up to 100 ft. and 30 lb. per sq. ft. above 100 ft. 25% allowable increase in working stress.
<i>Atlanta.</i>	20 lb. per sq. ft. on the exposed surface. Wind bracing not required unless the height of the building exceeds 2½ times the least width. Anchorage required for the overturning moment in excess of one-third of the moment of resistance.
<i>Boston.</i>	10 lb. per sq. ft. up to 40-ft. elevation; 15 lb. per sq. ft. between 40-ft. and 80-ft. elevation; 20 lb. per sq. ft. above 80-ft. elevation. 30 lb. per sq. ft. for structures having unusual exposure.
<i>Philadelphia.</i>	20 lb. per sq. ft. increased to 25 lb. per sq. ft. for unusual exposure. 30 lb. per sq. ft. on tanks, signs, etc. Anchorage required for the overturning moment in excess of two-thirds of the moment of resistance.
<i>Los Angeles.</i>	20 lb. per sq. ft. for structures above 60 ft. and 15 lb. per sq. ft. below 60 ft. 33⅓% allowable increase in working stresses. Overturning moment limited to 50% of the moment of resistance. Also rigid design for earthquake resistance.

* *Empire State and Manhattan Co. Buildings*, Engineering News-Record, February 19, 1931, pp. 331-334.

Nat'l Board of Fire Underwriters. 15 lb. per sq. ft. up to 40 ft. and 30 lb. per sq. ft. above 40 ft. 33⅓% allowable increase in working stress.

A.S.C.E. Sub-Committee (1940). 20 lb. per sq. ft. up to 300 ft. increased 2½ lb. per sq. ft. for each 100 ft. of added height. 33⅓% allowable increase in working stress.

Observed Data on Wind Pressure. An interesting comparison of observed wind pressures — extrapolated theoretically above the 500-ft. elevation — with the recommendations of the Sub-Committee of the A.S.C.E., is shown in Fig. 245. These curves are plotted from data presented by Mr. S. P. Wing.*

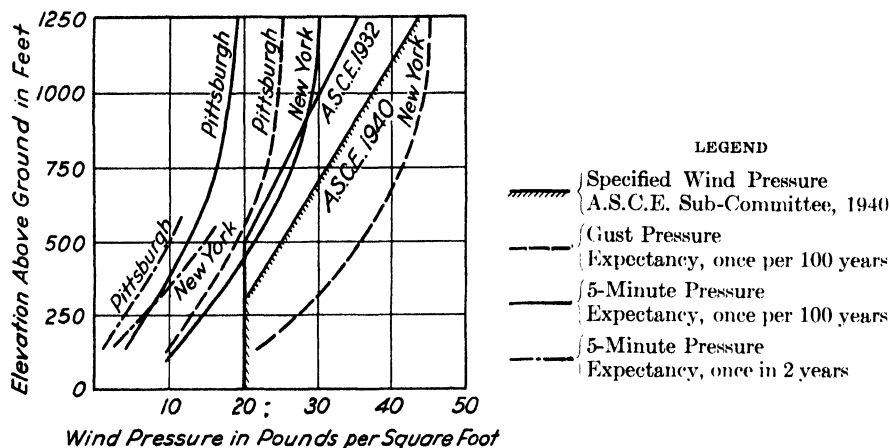


FIG. 245. PROBABLE VARIATION OF WIND PRESSURE WITH HEIGHT.

There is a comparison in Fig. 245 of the wind pressure to be expected once in two years with the abnormally high wind pressure to be expected once in 100 years or during the life of a building. There is a further comparison of maximum gust velocity with maximum 5-min. velocity. These curves tend to show that the recommendations of the Sub-Committee on Wind-Bracing in Steel Buildings (A.S.C.E. 1940) were reasonable.

In regard to the variation in wind pressure with the locality, data furnished by Mr. Wing are of interest. Table 8 shows the maximum expected wind pressure in the full life of the structure for several places in the United States as compared with the Sub-Committee's recommendation of 20 lb. per sq. ft. below the 300-ft. elevation. These factors should probably be increased to apply at the 1000-ft. elevation where the maximum 5-min. velocity and the maximum gust velocity are believed to approach each other. It is thought that the maximum 5-min. wind pressure is a safe value for use in design since the maximum gust pressure would not cover the entire face of a large building.

* S. P. Wing, Discussion of Second Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division of the A.S.C.E., Proceedings, August, 1932, p. 1108.

TABLE 8
RELATIVE WIND PRESSURE FOR VARIOUS LOCALITIES

LOCATION OF OBSERVATORY	GUST	5-MIN.
Recommendation of A.S.C.E. Sub-Committee		1.0 or 20 lb. per sq. ft. below the 300-ft. elevation
Los Angeles, Calif.	0.6	0.4
Pittsburgh, Pa.	1.0	0.6
San Francisco, Calif.	1.1	0.6
Seattle, Wash.	1.2	0.7
St. Paul, Minn.	1.5	0.9
Chicago, Ill.	1.75	1.0
New York, N. Y.	1.75	1.0
Dallas, Tex.	2.0	1.2
Mobile, Ala.	2.3	1.3
Point Reyes, Calif.	3.5	2.0

278. Strength and Rigidity of Wind Bracing. It has been considered satisfactory in the past to design the wind bracing of a tall building for strength, with little regard to rigidity or allowable deflection. The amount of lateral deflection seldom is of serious consequence for towers up to 500 ft. in height; hence a design which provided adequate strength was satisfactory. Modern towers may approach or exceed 1000 ft. in height. Since the deflection increases at least as rapidly as the height, a design of a 1000-ft. tower for strength alone may be inadequate.

Vibration. Clearly it is not only the static deflection but also the vibration that is important. Vibration has two important characteristics, *period* and *amplitude*. *Vibration may become objectionable either when the amplitude is large, or when the period is small. There is not sufficient information available to furnish limits of allowable period or amplitude, but it is clear that if sufficient rigidity is designed into the structure to reduce the static deflection to a low value, objectionable vibration will not occur. The Sub-Committee on Wind Bracing of the A.S.C.E. has suggested that a maximum static wind deflection of $0.002h$ (*two one-thousandths of the height*) will be a low enough limit to insure the absence of objectionable vibration. The static deflection mentioned is the *computed deflection* of the steel frame when the effects of walls, floors, and partitions are neglected. The actual sidesway under a steady wind will therefore be considerably less than the calculated deflection; with masonry walls perhaps not over 25% of the computed value. The amplitude of vibration, which is due to rapid fluctuations of the wind pressure, will amount only to a small fraction (probably *one-twentieth* or less) of the

* The *period* is the time required for a complete vibration cycle. The *amplitude* is the distance moved in a quarter period from a neutral position to the position of maximum displacement.

theoretical sidesway. Methods of computing deflections of building frames will be developed.

Damping Effects. Many factors contribute to the reduction of serious vibration from wind effects. Four are listed below.

(a) Rapid fluctuations of wind pressure are of small magnitude as compared with the wind pressure itself.

(b) Such fluctuations of wind pressure will not be uniform over the height or width of the building.

(c) Even wind pulsations are not of sufficient regularity to produce cumulative vibration. Vibration, therefore, is produced by the release of stored energy when the wind pressure is reduced suddenly.

(d) The dead weight of floors, which may offer little resistance to static deflection, produces a tremendous damping effect upon vibration. Their mass retards acceleration and increases the period of vibration. Walls and partitions reduce both static deflection and vibration.

In order to keep the calculated static deflection within two one-thousandths of the height ($0.002h$) of a building for which the height exceeds five times its width, we will find the use of diagonal bracing necessary. In fact, the advisability of using knuckle connections in a building whose height is five times its least width is questionable. In this range, the use of deep knee braces seems good practice, while for a more slender structure full diagonal bracing should be used instead. Naturally, both the *height-width ratio* and the *actual height* are critical factors in the selection of the wind bracing.

279. Earthquake Resistance. All types of wind bracing make a building resistant either to wind pressure or to earthquake forces. However, the wind pressure is not properly distributed to represent the action of an earthquake upon the building. An earthquake produces acceleration of the structure. Accordingly, its destructive effect is due to the development of *inertia forces*. Inertia forces are proportional to *mass* and to *acceleration*. It is estimated that the acceleration produced by earthquake vibration may locally become as high as $0.3g$ in the most highly destructive earthquakes although the San Francisco earthquake reached only $0.20g$.* Nevertheless, except for isolated instances, a horizontal acceleration of about $0.1g$ will probably represent a satisfactory estimate of earthquake severity away from the fault line. The vertical component of the acceleration usually is discarded as of little importance.

SPECIFICATIONS. Japanese specifications require the design of buildings on the basis of an acceleration of at least $0.1g$. The Los Angeles building code specifies an acceleration as a percentage of gravity expressed by the formula

$$\% \text{ of } g = \frac{60}{N + 4.5}$$

* Henry D. Dewell, *Damage to buildings in Tokyo in the 1923 earthquake*, Engineering News-Record, July 10, 1930, p. 51.

where N is the number of stories above the one under consideration and the percentage factor obtained is to be applied to the summation of all weights above that story. Masonry walls and smoke stacks must be tied into the building frame to resist an acceleration of $0.20g$ while a parapet or any ornament must be designed for a horizontal force equal to its full weight.*

The design of a building to resist a horizontal acceleration of $0.1g$ merely means to design the structure as a vertical cantilever frame resisting horizontal static loads equal to 10% of the weight of the structure, which should include about one-half of the specified live load. A heavy mass, such as a swimming pool, will produce a very large inertia force which will not be cared for properly if the building is designed for lateral wind pressure alone. A swimming pool located high in a building would produce severe earthquake stresses.

Earthquake Effects. An excellent discussion of earthquake effects is given in Part 2, Proceedings of the A.S.C.E. for May, 1932. This paper gives observations by Dr. Kyoji Suyehiro, Japanese seismologist.

(a) Each locality has a natural period† of vibration of the ground which represents the major period of vibration due to large earthquake tremors. Common periods vary from 0.3 sec. for a firm subsoil to 1.0 sec. for plastic or alluvial subsoil. Shorter periods of vibration corresponding to harmonics may be expected under the effect of slight disturbances. Maximum amplitudes will probably range from 1 to 3 in. Observations indicate that the acceleration will probably increase from $0.05g$ for firm subsoil to $0.1g$ or more for soft alluvial subsoil. It is of interest that the acceleration felt in riding in a motor car over very rough roads or in an old street car varies from $0.15g$ to $0.20g$.

(b) Cumulative vibration of a building having a natural period corresponding to the natural period of the ground might be highly destructive.‡

(c) A low rigid building vibrates with the natural period of the earth since its own period is extremely short. This statement would apply unquestionably to well constructed buildings of three and four stories.

(d) The skyscraper is a flexible structure having a period probably exceeding 2 sec. It is not susceptible to cumulative vibration because the period of the ground motion is much shorter. A ten-story building might be affected seriously by cumulative vibration. The height-width ratio and the type of construction would be of more importance in this connection than the actual height.

(e) It is not possible to associate acceleration, amplitude, and period by means of the ordinary formula for simple harmonic motion, that is, acceleration = amplitude $\times (2\pi/\text{period})^2$, because earthquake vibration is irregular and choppy. However, we can form an understanding of the relative importance of amplitude and period from a study of this formula. Note that acceleration is commonly given in ft. per sec.².

* See Engineering News-Record, November 7, 1935, p. 653 for photographs of the wreckage of brick veneer walls caused by the earthquake at Helena, Montana, October 31, 1935. The measured acceleration, $0.16g$ at a period of 0.2 sec., continued for only 4 seconds. Comments on the satisfactory action of structures designed to resist earthquake forces, that survived the Imperial Valley earthquake of 1940, are found in Engineering News-Record June 20, 1940, p. 45 and July 18, 1940, p. 43. Such structures were designed for forces of $0.08g$ and $0.10g$. Also see *Effect of foundation on earthquake motion*. R. R. Martel, Civil Engineering, January 1940, pp. 7-10.

† The *period* is the time required for an entire vibration cycle; the *amplitude* is the distance from a neutral position to a maximum displacement or the distance moved in a quarter period.

‡ See author's discussion of resonance, Engineering News-Record, September 10, 1936, p. 383.

280. Resistance of Walls and Partitions. Walls and partitions may furnish adequate wind resistance for a building less than ten stories high where the height does not exceed twice the least width. Nevertheless, the steel frame of a *tall* building of skeleton construction is designed to resist the *entire* prescribed wind force. Actually, considerable resistance is obtained at *low loads* from the action of walls and partitions, but it is customary to neglect such resistance in design. One reason is that partitions are subject to removal.

The careless construction of walls and partitions keeps them from fitting tightly against the steel frame. Hence the lateral resistance added by the vertical walls *in any one story* or *in any one bent* is so highly conjectural that it is not safe to allow for such resistance in design. Besides, walls are commonly constructed of *brittle materials* which rupture under severe distortion, with the result that practically all wind or earthquake forces are then transferred onto the structural frame. We must conclude that the walls can not even be depended upon to furnish an appreciable safety factor against ultimate failure. Reinforced concrete or steel-plate walls would serve as wind bracing, although in many modern buildings almost the entire area of the outside walls is removed by the windows. The most important action of the walls and the interior partitions is in reducing lateral deflection and vibration in the manner already discussed.

PROBLEMS

228. Present a report of imported data gleaned from the following articles:

(a) *Wind pressure in the St. Louis tornado*, Transactions, A.S.C.E., 1897, p. 221.

(b) *Effects of the Florida hurricane*, Transactions, A.S.C.E., 1931, p. 1118.

(c) *Florida storm damage confined largely to poorly built structures*, R. S. Tilden, Engineering News-Record, October 4, 1928, p. 515.

(d) *Damage to buildings in Tokyo in the 1923 earthquake*, Henry D. Dewall, Engineering News-Record, July 10, 1930, p. 51.

Multi-story rigid frame Army hospital in Hawaii designed to resist earthquakes, Engineering News-Record, July 12, 1945, p. 114.

229. Replot from Fig. 245 the curve which refers to the 5-min. wind pressure for New York City of an expectancy of once in 100 years by use of the following formulas. Maintain the pressure of 22 lb. per sq. ft. at the 500-ft. elevation constant. Transfer velocity into pressure by use of the formula $P = 0.0032V^2$.

(a) Stevenson formula (1880): $V = V_1 \left(\frac{h + 72}{h_1 + 72} \right)^{\frac{1}{2}}$, where V_1 is the observed velocity at a height h_1 and V is the velocity at a height h .

(b) Archibald formula (1886): $V = V_1 \left(\frac{h}{h_1} \right)^{0.26}$.

(c) Wing formula (1921): $P = 0.00126h + 1.16P_0$, where P is the pressure at an elevation h and P_0 is the ground pressure.

230. Determine the amplitude of vibration for a tall building having a period of 2 sec. in order to produce an acceleration of 0.1*g*. Use the formula for simple harmonic motion Repeat for an extremely tall building having a period of 8 sec.

Ans. 3.9 in. and 62.5 in.

231. In order to investigate the possible severity of cumulative vibration on a building of medium height, compute the acceleration by use of the formula for simple harmonic motion for a building placed on alluvial soil where the period of the structure corresponds to the fundamental period of 0.3 sec. of the ground. Assume an amplitude of $\frac{1}{4}$ in.

Ans. 0.29g.

281. Choice of Type of Wind Bracing. The structural steel frame may be made resistant to lateral forces in several ways: *diagonal bracing* may be enclosed in walls and partitions, or *rigid joints* may be formed by riveting or welding the columns and girders together. See Fig. 217 and Fig. 221. Several complex wind frames are shown in Fig. 246. The knee-braced bents may

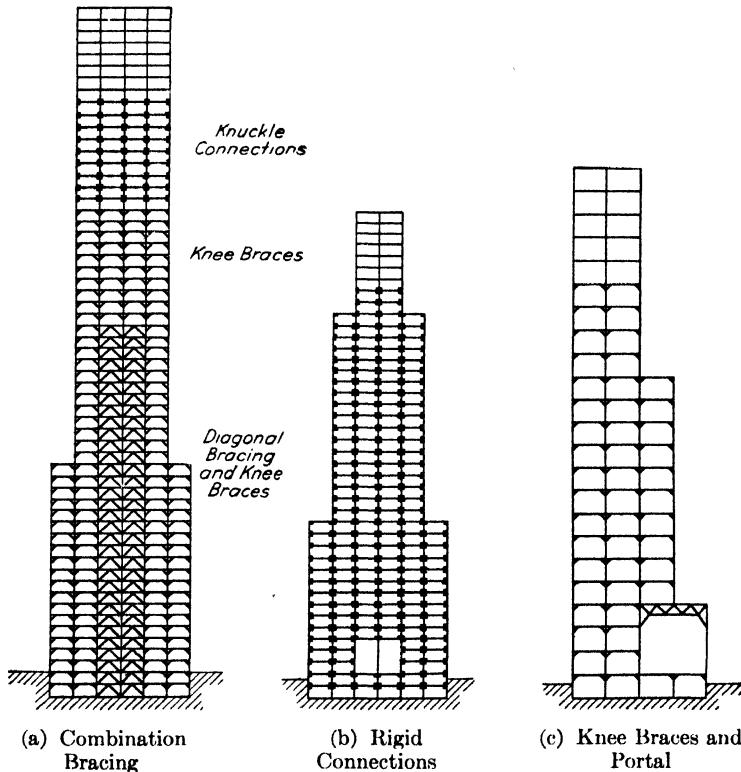


FIG. 246. BUILDING FRAMES WITH WIND BRACING.

be regarded as intermediate between bents with rigid joints and bents with full diagonal bracing. The *knee braces* act in direct stress, but introduce bending into both columns and girders.

Diagonal Bracing. The use of full-length diagonals produces the most effective type of wind bracing, but such bracing is of limited usefulness be-



FIG. 247. THE FEDERAL COURT HOUSE—NEW YORK CITY.

Double knee braces form the tower bracing; they continue to the basement level. The wings, being only a few stories high, do not need wind bracing. The left half of the building resembles the part shown. Notice the stiff-leg derrick in the center of the picture.

cause it can not be placed in a wall or partition where numerous doors or continuous lines of windows exist. In modern office buildings, the exterior walls are at least 50% window area. Full diagonal bracing usually can not be placed in such walls. Interior partitions may be placed temporarily to suit the desires of a particular tenant. Since they may be moved, the use of diagonal bracing in such partitions is impossible. Accordingly, the only places where diagonal bracing can nearly always be used are around elevator and stairway shafts and in a few permanent interior walls.

Diagonal bracing placed on all four sides of an elevator shaft from top to bottom of the building would form a *trussed tower*. If the bracing must be limited to three sides of the shaft to provide for large doors, its effectiveness is reduced. Nevertheless, where several such shafts exist over the building, it may be possible to introduce sufficient diagonal bracing to provide the necessary resistance.

Knee Braces. When the architectural requirements make the use of sufficient diagonal bracing impossible, the introduction of knee braces should be considered. Knee braces may be used in exterior walls and still allow a large percentage of the face of the building to be glazed. When placed in permanent interior walls, they do not greatly restrict the sizes of doors and passage ways. In fact, if they are placed only below the girders, there is no objectionable restriction of area ways. The details of knee-brace connections (Fig. 221) are greatly simplified and their cost is reduced by welding. It is to be expected that the wider adoption of structural welding will increase the use of knee braces for wind bracing. Observe the use of double knee braces in Fig. 247. Knee braces are not needed in the upper stories.

Portals. The use of portal bracing was an early development of wind bracing. Its cost was found to be excessive. A single portal is shown in the building frame of Fig. 246(c). The use of a portal is justified there because of the need for a truss to support the column at the middle of the span. In place of a simple-span truss, it is possible to introduce either sloping or horizontal knee braces, thus producing a portal to help resist the wind forces.

Rigid Joints. The wind bracing most commonly used for buildings other than tower structures consists of the introduction of *moment-resistant connections* between the columns and the girders. Both riveted and welded connections are in use. Several types are illustrated in Fig. 217 and Fig. 218. Specifications commonly allow the dead-load and live-load stresses in the columns and girders to be increased $33\frac{1}{3}\%$ by wind without requiring an increase in their sections. In order to take the greatest possible advantage of this specification, we should attempt to introduce moment-resistant connections on as many column lines as is needed to prevent the wind stresses in the girders and columns from exceeding *one-third* of the dead-load and live-load stresses. This arrangement would permit the use of the **dead-load and live-load frame**, an ideal design that can seldom be achieved. In any case, however, maximum economy is obtained by the arrangement of wind bracing to add the least possible weight to the sections that are required to resist the dead-load and live-load stresses.

282. Location of Wind Bents. The proper position of the wind bents on the plan of the structure depends upon the type of wind bracing used. In Fig. 248, it seems probable that sufficient diagonal bracing could be introduced around elevator and stairway shafts as indicated by the dotted lines. In place of such diagonal bracing, we might consider the use of knee braces in the exterior walls at the points indicated by the check marks. A third alternative would be the use of moment-resistant connections along all or possibly alternate column lines for bents parallel to *A-B* in the short direction of the building and perhaps only in the outside walls, *A-D* and *B-C* in the long direction. It is evident that wind pressure on the wide face, which must be resisted by the narrow bents parallel to *A-B*, produces a more serious wind-stress problem than the action of wind on the narrow face, which is resisted by the wide bents parallel to *B-C*.

Action of the Floors. When identical tiers of bracing are placed symmetrically across the building, it is assumed that the floors, which act practically as rigid horizontal plates, will distribute the wind force equally to all parallel tiers of bracing. If the floors are inadequate, diagonal floor bracing should be

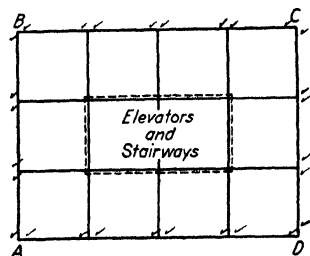


FIG. 248. POSITION OF THE WIND BRACING ON PLAN.

used. Hence all tiers of bracing should be constructed as nearly alike as possible, so that they will deflect equally under identical loads and will resist the wind forces equally. Unsymmetrical placement of wind bents will allow the wind to *twist the building about a vertical axis*, thus producing unequal shears even when the wind bents are alike. The distribution of wind shears to unlike bents or to unsymmetrically placed bents will be discussed toward the end of this chapter.

283. Forms of Diagonal Wind Bracing. Several forms of so-called deep bracing are shown in Fig. 249. Types (a) to (g) inclusive are the most satisfactory, since they form

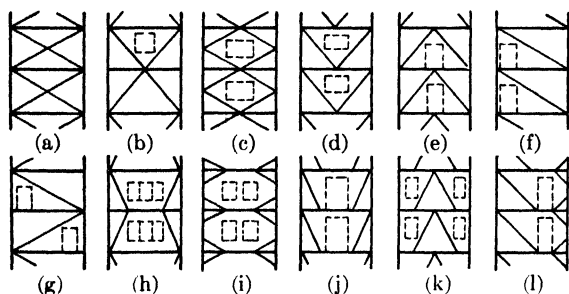


FIG. 249. TYPES OF DIAGONAL BRACING.

fully trussed systems which may be made both highly resistant to stress and to lateral deflection. Of these, the K-bar bracing shown in (d) and (e) offers the additional advantage that elastic shortening of the columns under dead load and live load does not introduce

heavy stresses into the diagonals because the girder can deflect slightly at its center and relieve this stress. Types (h) to (l) inclusive provide large door and window openings. Such systems of partial bracing produce wind moments either in the girders or the columns, or in both.

Other Considerations. There is a possible economy in the use of types (b) to (e) and (h) to (l) which can be arranged to form *interior supports for the girders*. The girders then act as continuous beams of shortened span. Their bending moments both from dead load and live load are reduced greatly. This reduction in size of the girders is an important economy which will nearly compensate for the cost of the wind bracing. All wind bracing ought to be made of stiff sections capable of resisting compression, or, for types (a) and (b), where tension diagonals are permissible, *initial tension* should be produced in the diagonals so that they will resist compression by a reduction of tension stress. Otherwise, the bracing will tend to loosen up and allow the building to drift sideways under the pressure of the wind. The initial tension also permits the diagonals to shorten as the columns deform elastically.

WIND-STRESS ANALYSIS

284. Stress Analysis of Diagonally Braced Frames. The analysis of the stresses in a wind frame with deep diagonal bracing is comparatively simple.

It will be assumed that all tiers of bracing are identical and that they are placed symmetrically across the building, in which case the wind shear resisted by each tier is the same. For unlike wind frames the wind forces resisted by each bent will have to be computed from the relative stiffnesses of the various wind bents, a procedure that will be discussed in a later section. Relative stiffness is measured by the deflection under a given load.

Stresses in the Diagonals. For types (a) to (e) of Fig. 249, there are two diagonals cut by a horizontal section through each bay. These diagonals are either stiff compression members or else they carry an initial tension which makes them capable of resisting a compression stress by a reduction of the initial tension. Accordingly, the horizontal component of the stress in a diagonal is one-half of the story shear resisted by the bay under consideration. The corresponding wind stress is this horizontal shear times $\sec \phi$, where ϕ , as shown in Fig. 250, is the angle that the diagonal makes with the horizontal. When initial tension exists, the total stress in the diagonal is the wind stress added to or subtracted from the value of the initial stress.

Direct Stresses in the Girders. The direct stresses in the horizontal girders produced by the wind forces are small in pounds per unit of cross-sectional area. They are seldom considered to be worth calculating. The direct stress

in a girder can always be found from the known stresses in the diagonals by an application of the equation $\Sigma H = 0$ to a joint where the diagonals and the girder join. For instance, in Fig. 250(a) or (b), it is evident that the maximum direct stress in a girder is equal to the horizontal component of the wind stress in the diagonal that connects to the leeward end of the girder.

Direct Stresses in the Columns. The column stresses, which are of sufficient importance to justify calculation in most cases, will be particularly serious for tall wind frames. If we pass a horizontal section below the girder CD for the wind bent shown in Fig. 250(a), it is apparent that the two equal stresses in the diagonals produce equal and opposite moments about the point A or the point B . Thus we conclude that the direct stress in either column of the first story is equal to the overturning moment of the wind about $A-B$ divided by b , the width of the bent. Evidently the stresses in the columns AC and BD are equal but of opposite signs. A similar analysis of Fig. 250(b) shows that the stress in the column AC is equal to the overturning moment about $A-B$ divided by the width of the bent while the stress in the column BD is obtained by dividing the overturning moment about $C-D$ by the width of the bent. These column stresses are opposite in sign and unequal in value.

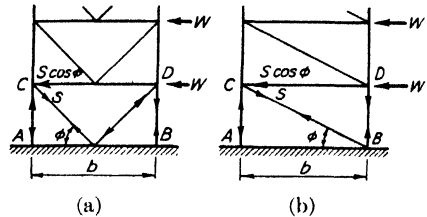


FIG. 250. WIND STRESS IN THE GIRDER.

Multiple Bays. If identical bays are placed side by side, as shown in Fig. 251(a), each bay is assumed to resist the same amount of shear. Hence the column CD will obtain a compression as a member of the bay CE and an

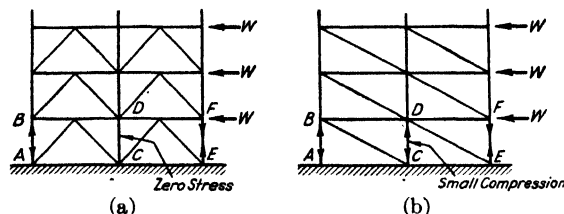


FIG. 251. DIRECT STRESSES IN THE COLUMNS FROM WIND.

slightly larger than its tension calculated as a member of the bay AC . For practical purposes its wind stress is negligible.

It is still possible to show that interior columns receive little or no *direct stress* for wind frames more than two bays in width. Evidently this statement assumes that all bays will be of the same width and of the same stiffness, so that they will resist equal shears. For other cases, each interior column stress should be calculated by taking the difference of its tension and compression stresses produced by its action as a member of two adjacent bays. When stresses in interior columns are zero, the direct stress in an exterior column can be computed by dividing the total overturning moment of the wind about any story level by the *entire width* of the bent.

It should be realized that a diagonally braced frame more than one bay in width is an indeterminate structure. Hence the estimation of the shear division between bays is merely a device to reduce the labor of making calculations. Probably the procedure suggested here is as accurate as can be justified except for the highest tower buildings where the wind stresses become of major importance in the design not only of the wind bracing but of the columns as well.

285. Stress Analysis of Partially Braced Frames. Knee braces and diagonal braces that do not join together form partially braced frames. The diagonal braces act in direct stress. In addition, there is flexure introduced into the girders or the columns, or into both. The three frames shown in Fig. 252 will be discussed. The partially braced frame in (a) has two diagonals that resist the entire horizontal shear. Hence the vertical component of each diagonal stress acts as a vertical load on the girder producing the moment diagram, as shown, with a maximum moment of $Whb/(2a)$. In Fig. 252, W is not a single wind force but instead represents the total shear acting on the story under consideration. The columns do not resist flexure. The direct stress in a column below any floor level is equal, in this case, to the overturning

moment about the floor, of all wind forces above the level, divided by the width of the bent and increased by the vertical component of the stress in the adjacent diagonal. This may be shown from an equation of moments about the far end of the girder. Partially braced frames placed side by side may be analyzed by the procedure discussed for multiple frames with full diagonal bracing.

Analysis of Bents with Knee Braces. Two types of knee-braced bents are shown in Fig. 252(b) and (c). When the two bents are lettered as shown, the

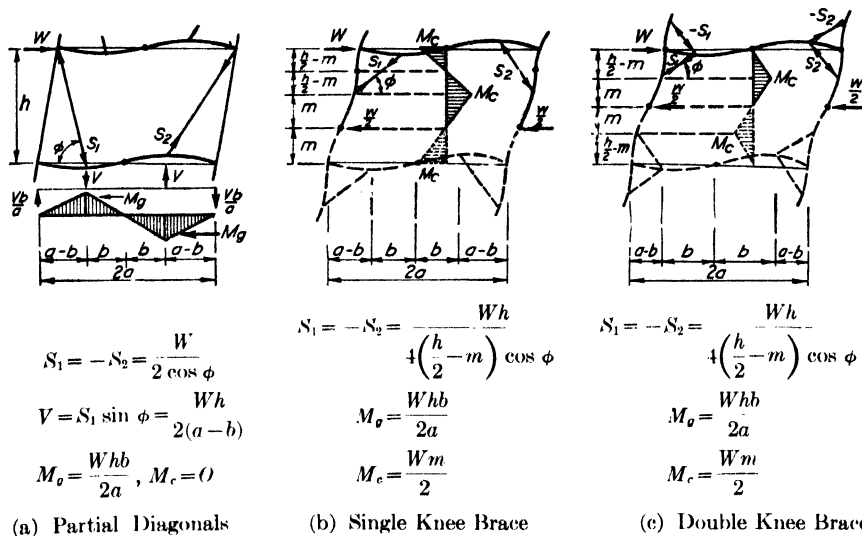


FIG. 252. ANALYSIS OF PARTIALLY BRACED FRAMES. W = WIND SHEAR.

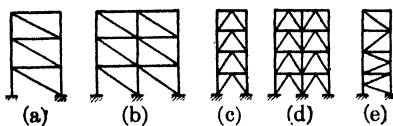
stresses in their knee braces and the maximum girder and column moments are given by the same expressions. It is evident that these structures are statically indeterminate. However, they will become determinate if the locations of the points of contraflexure are assumed and if the wind shear is taken as divided equally between the columns. In the frame with knee braces placed below the floor level, Fig. 252(b), it is certain that points of contraflexure must exist in the columns approximately half way from the center line of the girder *up* to the foot of the knee brace and again about half way from the center line of the girder *down* to the foot of the knee brace. When knee braces are placed both above and below the floor level, Fig. 252(c), one point of contraflexure for the column moves to the center line of the girder and the other exists approximately midway between the ends of the knee braces. Of course, *symmetry* requires that there be points of contraflexure at the centers of the girders for all three bents shown in Fig. 252.

Stress in Knee Brace. With the points of contraflexure located as shown in Fig. 252(b) or (c), each story of the wind bent is treated as a portal for the calculation of stresses in the knee braces and of moments in the columns and girders. To calculate the knee-brace stress, we should isolate that part of the column between the points of contraflexure above and below the knee-brace connection. We can then write a single equation of moments about the upper point of contraflexure which will involve the knee-brace stress as the *only* unknown and the wind shear in the column ($W/2$) as the only known force. The girder moment is obtained by loading the girder with the vertical component of the stress in one or two knee braces at each interior connection. There should be no difficulty in checking the expressions for moments and stresses, as given in Fig. 252, if these suggestions are followed. The direct stress in either column is obtained by dividing the overturning moment about the level of the lower points of contraflexure by the width of the bay (See Fig. 252(b)).

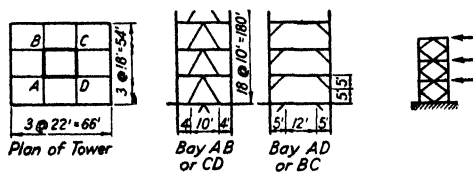
Multiple-Bay Bents with Knee Braces. The extension of this theory of portal action to a combined bent several bays in width follows the procedure discussed for diagonally braced bents in § 284. Clearly, each interior column will tend to resist shear as a member of each of two adjacent bays; hence it will be assumed to resist *twice* the shear of an exterior column. Interior columns receive no appreciable direct stresses when the bays are of equal widths. Points of contraflexure remain as shown by Fig. 252(b) and (c). The analysis of this structure will be clarified by the study of frames with moment-resistant joints that follows.

PROBLEMS

232. Determine the number of redundants for each of the diagonally braced frames illustrated. All joints are assumed to be pin-connected, so that the columns do not resist horizontal shears.



PROBLEM 232.



PROBLEM 233.

PROBLEM 235.

233. A building is constructed with a tower extending 18 stories or 180 ft. above the main shaft. The wind bracing is placed around an elevator shaft 22 ft. by 18 ft. as shown by the heavy lines in the plan view. Compute the wind moments and the direct stresses for the lower-story members of the bent placed in the bay *AB*. The wind pressure is 30 lb. per sq. ft. (Compute these moments and stresses for the 18th story below the top of the tower.)

Ans. Stress in diagonal = 195 kips.

234. Compute the wind stresses in the lower story of the bay *AD* of the wind bracing for the tower of Problem 233.

Ans. Stress in knee brace = 201 kips.

235. Study the bracing shown in the sketch. Would this bent be statically determinate? What reactions would be needed for stability if all joints are connected by pins?

236. Place wind frames of the type shown for Problem 235 in each outside wall panel of the tower of Problem 233. Assume that all column stresses are resisted by the four outside corner columns of the tower. Compute wind stresses in the 17th and 18th stories below the top of the tower. Which solution (Problem 233 or 236) is the more practical?

286. Analysis of Frames with Moment-Resistant Joints. The rigid-frame wind bent forms a highly indeterminate structure that can only be analyzed correctly by the methods of indeterminate structures. Nevertheless, it has been common practice in the past to analyze such frames by simple *approximate* methods. Some writers attempt to justify this procedure on the grounds that riveted joints do not form absolutely rigid connections, so that the assumption of perfect continuity would be about as seriously in error as the assumptions used in the approximate solutions. However this may be for riveted construction, such arguments evidently do not apply to welded frames. These simple approximate methods of analysis are useful (1) *for a preliminary design*, later to be revised to agree with the requirements of a more nearly exact analysis, or (2) as a time-saving device *for the analysis of a highly regular frame*, or (3) in the study of a frame *where the wind moments are comparatively unimportant*.

Special Assumptions. The early approximate methods of wind-stress analysis are now known as the **portal method** and the **cantilever method**. The wind

Assumptions fundamental to all common methods of wind-stress analysis are as follows:

- (a) Wind forces are applied to the steel frame by the spandrel beams as concentrated loads at the floor levels and *not* as a distributed load on the windward column.
- (b) The wind load is resisted *entirely* by the steel frame.
- (c) Floors are adequate to act as rigid horizontal plates and to divide the total wind load between the several wind bents in proportion to their relative resistances to lateral deflection. This means *equal loads* to identical bents.
- (d) Floors and walls do not affect the *relative* stiffnesses of columns and girders.
- (e) All joints are rigid or deform in such a manner that the wind stresses are *unchanged* by such deformations.
- (f) The lengths of members are the *theoretical* lengths center to center of columns or center to center of girders. These lengths can later be reduced when design moments are computed.
- (g) The gusset plates, knuckle connections, or welded joints do not affect the *relative* stiffnesses of columns or girders. Stiffness is measured by I/L , the moment of inertia divided by the length.
- (h) The changes in lengths of the columns due to *direct stress* are neglected in regard to their effects upon the bending moments.
- (i) Shortening of girders from direct stress and flexure is negligibly small; in other words, the columns of a single bent have *identical* deflection curves.
- (j) Columns are adequately fixed at their bases to justify the assumption of *fixed* ends. Column splices are capable of resisting moment if needed.

moments as computed by these two methods do not check. Evidently, then, the reader is not expected to be impressed by their accuracy. The *cantilever method* follows the procedure of analysis of the continuous portal in that column stresses are assumed to be proportional to distances out from the center or from the neutral axis of the bent. This assumption agrees with the action of a well-designed bent of a slender tower building. On the other hand, the *portal method* assumes that each bay acts separately from each other bay as an individual portal so that an *interior* column receives a compression from its action as a part of the windward bay and an equal tension from its action as a member of the adjacent leeward bay. Its resultant direct stress is zero. All direct stress therefore occurs in the exterior columns. This assumption agrees reasonably well with the observed action of a perfectly regular bent of medium height-width ratio.

287. Portal or Individual-Bay Method. This is the simplest method of wind-stress analysis. The fundamental assumptions are as follows:

- (1) Points of contraflexure exist at the mid-heights of all columns.
- (2) Points of contraflexure exist at the centers of all girders.
- (3) The direct stresses in the interior columns are zero.

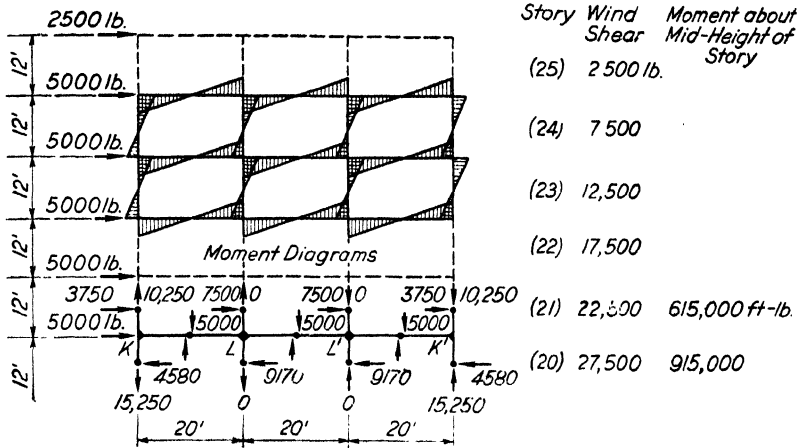
EXAMPLE. This method of analysis applied to a perfectly regular frame is illustrated by Fig. 253. The bent shown represents the upper six stories of a 25-story building frame. The overturning moment of the wind loads about the mid-height of the twentieth story is 915,000 ft-lb. Since the moments in the columns at this horizontal section are zero and since the stresses in the interior columns are also zero, the direct stresses in the exterior columns must form a couple of 915,000 ft-lb. value. The stress in an exterior column is $915\,000 \div 60 = 15,250$ lb. Similarly, a stress of 10,250 lb. is obtained for the exterior columns of the 21st story.

The Exterior Joint. By isolating the joint *K* at the exterior column between the 20th and 21st stories as in Fig. 253(b), we can show from the equation $\Sigma V = 0$ that the girder shear S_1 is $15,250 - 10,250 = 5000$ lb. Equilibrium requires that the algebraic sum of the moments in the two columns and in the connecting girder must be zero at their intersection. Thus we conclude that the sum of these column shears times half the story height must equal the girder moment. Hence $(S_2 + S_3)6 = 5000 \times 10$, or

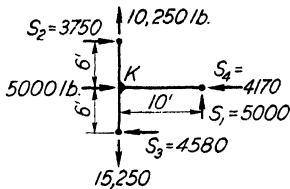
$$S_2 + S_3 = \frac{5000 \times 10}{6} = 8330 \text{ lb.}$$

For this regular symmetrical frame, it seems clear that the shears in a column in two adjacent stories should bear the same ratio to each other as the total wind shears in these two stories. It follows that the combined shear of 8330 lb. should be divided between the exterior columns of the 20th and 21st stories in the same ratio as the total wind shears of 27,500 lb. and 22,500 lb. The column shear S_2 is found to be 3750 lb. in the 21st story, and S_3 is found to be 4580 lb. in the 20th story. We now may apply the equation of equilibrium $\Sigma H = 0$ to the joint *K* to obtain the direct stress in the girder, $S_4 = 4170$ lb.

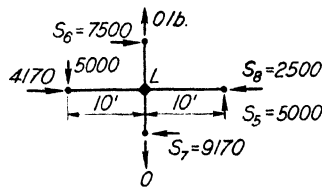
The Interior Joint. The succeeding step is to isolate the adjacent interior joint *L* as shown in Fig. 253(c). Since the direct column stresses are zero, the equation $\Sigma V = 0$ shows that the girder shear S_5 is 5000 lb. At this joint, the sum of the bending moments



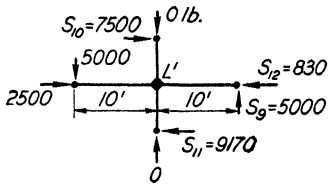
(a) Shears and Direct Stresses at the 21st Floor Level



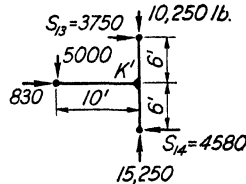
(b) Joint K



(c) Joint L



(d) Joint L'



(e) Joint K'

FIG. 253. EXAMPLE OF THE PORTAL METHOD OF WIND-STRESS ANALYSES.

in the girders must equal the sum of the bending moments in the columns. Hence $(S_6 + S_7)6 = 5000 \times 10 + 5000 \times 10$, or

$$S_6 + S_7 = \frac{100,000}{6} = 16,670 \text{ lb.}$$

Again the shears S_6 and S_7 are assumed to bear the same relation to each other as do the total shears in the corresponding stories. This shear-ratio is 22,500 lb. to 27,500 lb. The column shear S_6 is found to be 7500 lb. in the 21st story and S_7 is found to be 9170 lb. in the 20th story. The equation $\Sigma H = 0$ gives us the direct stress in the girder, $S_8 = 2500$ lb.

This procedure may be repeated by use of the other isolated joints L' and K' , as shown in (d) and (e) of Fig. 253.

288. Observations Concerning the Portal Method. It will be noticed that at each of the joints K , L , and L' in Fig. 253 there were four unknowns. Hence it was necessary to supplement the three equations of statics by assuming that the ratio of the individual column shears in adjacent stories would be the same as the ratio of the total story shears in order to provide *the fourth equation*. At the final joint K' , however, there were but two unknowns, so that it was possible to check the calculations by using the third equation of statics.

Column and Girder Shears. Referring again to Fig. 253(a), we observe that the shear in an interior column comes out to be *exactly twice** the shear in an exterior column of the same story. For bays of equal widths, the shears in all girders of a given floor are the same. Hence we can save time by distributing story shears between the columns in the *ratio of two to one* for interior and exterior columns. Then, when the girder shear has been computed in one girder, the other girder shears at that floor level are known to be the same.

Column and Girder Moments. The maximum bending moment in a column obtained by the portal method is equal to the column shear times one-half of the story height. Likewise, the maximum girder moment is equal to the girder shear times one-half of the panel length. In Fig. 253 the maximum girder moment is $5000 \times 10 = 50,000$ ft-lb. The maximum column moment in the 21st story is $7500 \times 6 = 45,000$ ft-lb. The corresponding moment in the 20th story is 55,000 ft-lb. These moments are reduced 50% for exterior columns. The actual moments used in design would be reduced to allow for the fact that the *clear span* of a girder is less than the theoretical width of a bay and that the length of a column between girders is less than the story height. The straight-line moment diagrams, shown in Fig. 253(a), are plotted on the tension sides of the members as caused by flexure.

289. Improved Assumption for Locating Points of Contraflexure. After the use of the portal method or any method of wind-stress analysis that assumes the points of contraflexure to be located at the mid-points of the members, it is wise to compute bending moments upon the basis of normal displacements of the points of contraflexure from the centers of the members. For lower story columns of multi-story frames the points of contraflexure are commonly about $0.55h$ above the base. They would be at $0.60h$ or even $0.65h$ above the base if the designer did not stiffen the first floor girders to force the

* The resistance of a column to wind shear depends mainly upon its end restraints. A column with pinned ends could not resist any wind shear. An interior column is restrained by *two girders*; consequently, it resists about twice the wind shear of an exterior column which frames into a *single girder*.

points of contraflexure downward. The point of contraflexure of a wall girder is normally located more than one half way from the wall column to the first interior column, say $0.55L$. For all other columns and girders compute the wind moment at each end from $0.52h$ or $0.52L$ to allow for an unknown but probable shift in the point of contraflexure in one direction or the other. The arm $0.5L$ should be reserved for use in computing moments where symmetry controls the location of the point of contraflexure.

290. Slightly Irregular Frames Analyzed by the Portal Method. Where widths of bays vary and columns have different cross-sections, the use of any approximate method must be looked upon as a very crude estimate of the wind stresses. Further, to be of any use in such cases, the original assumptions of the portal method must be revised.

Unequal Bays. If the only irregularity consists of bays of unequal widths, a revision is necessary either:

(a) to allow for the points of contraflexure to shift from the centers of the girders; which is unrealistic since the shift toward the wall column (Fig. 254) to maintain a 2:1 ratio of shear between interior and exterior columns is just opposite to observed action of models.

(b) to allow for the interior columns to resist less than twice the shear resisted by the exterior columns; or

(c) to allow for direct stresses in the interior columns, which is inconsistent with the original conception of the portal method.

Approximate Shear Distribution to Identical Columns. As mentioned before, the resistance of a column to shear is dependent largely upon the combined

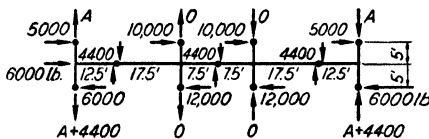


FIG. 254. POINTS OF CONTRAFLEXURE ADJUSTED—PORTAL METHOD.

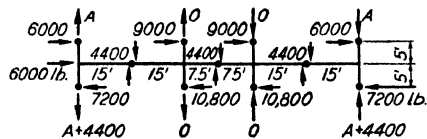


FIG. 255. COLUMN SHEARS ADJUSTED—PORTAL METHOD.

stiffness of the girders into which it frames. Since the dead-load and live-load moments that control the design of the girders vary as L^2 , it is clear that the moments of inertia I of the girders also vary approximately as L^2 (for girders of the same depth and the same fiber stress). Hence the stiffness factor or I/L value varies directly with the *length of the girder*, so that the column shear has often been made dependent upon the combined widths of the adjacent bays. Thus the interior column in Fig. 255, which is tied into a 30-ft. girder and a 15-ft. girder, would be expected to resist about 50% more shear than the exterior column which is tied into one 30-ft. girder. This is the same shear ratio that will exist between the columns if each bay is assumed to resist shear in proportion to its width.

291. Improved Assumptions as to Shear Division between Columns. A subdivision of the total shear between the columns (known as the Bowman Method *C**) is an improvement over the portal method. From Fig. 253 it may be seen that the story shear divides itself in a 2:1 ratio between interior and exterior columns when the portal assumption of zero direct stress in all interior columns is made. For equal widths of bays the same result is achieved if the story shear is divided equally between bays and then the bay-shear is divided equally between its adjacent columns. However, another assumption, long ago abandoned, was that the wind shear could be divided equally between the columns of any story.

The Bowman method recognizes the fact that equal shear division between columns is as much in error as equal shear division between bays, but that the truth lies between these two extremes. Hence a percentage p of the wind shear at any level is divided between the columns of that story according to their K -values (equal shears to identical columns), and the remaining percentage $(100-p)$ of the wind shear is divided between the bays of that story according to the K -values of the girders above the bays. Of course, $K = I/L$, the moment of inertia divided by the length of the member. Then the shear assigned to a bay is divided equally between its two adjacent columns. This type of subdivision of shear has a long history but the contribution of Professor Bowman is in establishing the percentage p of the shear to be assigned directly to the columns.

$$(1) \quad \text{For lower story columns, } p = \frac{100 (b - \frac{1}{2})}{c}$$

$$(2) \quad \text{For columns above the lower story, } p = \frac{100 (b - 2)}{c}$$

where b is the number of bays and c is the number of columns. In several cases where it was tried out this subdivision of wind shear proved satisfactory for design. In fact, D. F. Baker working under the author's direction (1949) applied this method successfully to unsymmetrical two-bay wind frames, a critical case. Except for lower stories it assumes that the girder stiffnesses (i.e., K -values) at the top and bottom of a given story are about equal or at least provide the same relative resistances to end rotation at top and bottom of all the columns in that story.

Unlike Columns. If the irregularity is such that the columns are highly dissimilar, it seems unlikely that any approximate method will give satisfactory results. The design of such a building requires the services of an expert who is familiar with the analysis of continuous-frame structures. This is

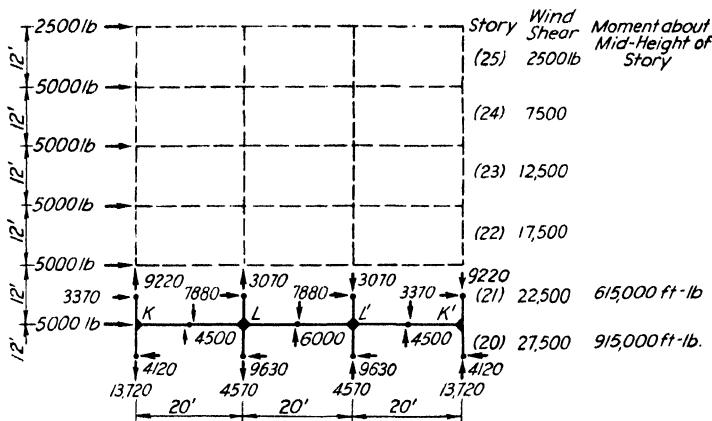
* See "Structural Theory," Hale Sutherland and H. L. Bowman, Third Edition, Method *C*, p. 273, John Wiley & Sons.

always true for irregular frames with columns or girders omitted or with setbacks.

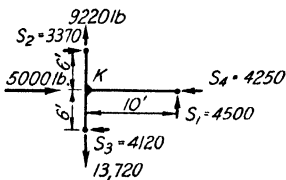
292. The Cantilever or Continuous-Portal Method. The fundamental assumptions of this method of wind-stress analysis are:

- (1) Points of contraflexure exist at the mid-heights of all columns.
- (2) Points of contraflexure exist at the centers of all girders.
- (3) The direct unit stresses in the columns vary as the distances of the columns from the centroidal axis of the bent.

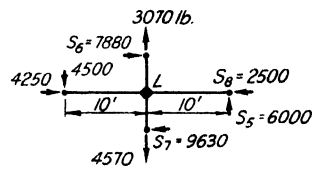
EXAMPLE. This method of analysis is applied to a perfectly regular bent in Fig. 256. This same bent was analyzed by the portal method in Fig. 253. The centroidal axis of this



(a) Shears and Direct Stresses at the 21st Floor Level



(b) Joint K



(c) Joint L

FIG. 256. EXAMPLE OF THE CANTILEVER METHOD OF WIND-STRESS ANALYSIS.

symmetrical bent is at the middle of the center panel. The *unit* column stresses are calculated by an application of the Mc/I formula as was explained for the continuous portal in Chapter 4. The *total* column stress is the unit stress times the column area. The analysis then follows directly from a study of the static equilibrium of the joints shown in Fig. 256(b) and (c).

The Exterior Joint From Fig. 256(b), with the column stresses known, the equation $\Sigma V = 0$ shows S_1 to be 4500 lb. Then, since the bending moment in the girder at the joint

must be equal to the sum of the column moments, we may write the equation $4500 \times 10 = (S_2 + S_3)6$, or

$$S_2 + S_3 = \frac{45,000}{6} = 7500 \text{ lb.}$$

The column shears, $S_2 = 3370$ lb. and $S_3 = 4120$ lb., are found by dividing their sum, 7500 lb., in the same ratio as that of the total story shears. Following the determination of column shears, we can apply the equation $\Sigma H = 0$ to the joint K to show that $S_4 = 4250$ lb.

The Interior Joint. The equation $\Sigma V = 0$ applied to the joint L shows that S_6 is 6000 lb. The sum of the moments in the two girders may be set equal to the sum of the column moments. Thus $4500 \times 10 + 6000 \times 10 = (S_6 + S_7)6$, or

$$S_6 + S_7 = \frac{105,000}{6} = 17,500 \text{ lb.};$$

whence $S_6 = 7880$ lb. and $S_7 = 9630$ lb. (divided as before). Then from $\Sigma H = 0$, we obtain the girder stress, $S_8 = 2500$ lb. The bending moment in any column or girder can now be determined by multiplying the corresponding shear by one-half of the length of the member.

Unequal Bays and Unlike Columns. The cantilever method automatically cares for the case of columns of different areas and of bays of unequal widths. However, the use of this method should be limited to frames of relatively slight irregularity. There is no approximate method that has been found satisfactory for highly irregular frames.

293. Improved Cantilever Procedure. Theory and experiment both demonstrate the fact that the column reactions or the vertical shear between any two columns such as columns 1 and 2 of the wind bent in Fig. 257 will change with change in the total stiffness (ΣK_g) of the girders in any vertical tier. Hence we should be able to vary the girder stiffnesses and thus control the wind reactions, at least to some extent. This is indeed possible and is the basis of the Spurr and Witmer* methods of design for wind resistance.

A relationship established by Ross and Morris† states that the total stiffness of girders in a vertical tier (ΣK_g) controls the total vertical wind shear (V_g) acting on this tier of girders according to the relation

$$(3) \quad V_g \propto \Sigma K_g / L_g$$

or

$$(4) \quad I_g \propto V_g L_g^2.$$

These expressions are dependent upon a linear relation between column deformations as is the cantilever method. Hence we can compute the total column stresses by the cantilever method and compare the total vertical shear in the outside bay due to the reaction under the outside column (R_1 in Fig. 257) with the total vertical shear in the first inside bay due to the reactions under two

* F. P. Witmer, Wind-Stress Analysis by the K-Percentage Method, Transactions, A.S.C.E., Vol. 107, 1942, pp. 925-954.

† Bulletin No. 48, Engineering Experiment Station, Ohio State University, Columbus, Ohio.

columns, R_1 and R_2 in Fig. 257. Since the cross-sectional area of an inside column may be larger than the area of a wall column, it is reasonable to suppose in Fig. 257 that R_1 and R_2 might be of about the same magnitude. For that case $R_1 + R_2 = V_2 = 2V_1$; and if L_1 and L_2 are equal, $I_{g2} = 2I_{g1}$. This results in the necessity of placing much heavier girders in the inside bays than the outside bays although the dead- and live-load requirements may be identical for both girders.

We conclude then that a wind bent with girders in inside and outside bays designed to meet the requirement of equation (4) will act according to the cantilever method and may be analyzed quite accurately by this method. Before using cantilever analysis on a given bent it should be checked at several levels to see whether the summed stiffnesses of the girders in inside and outside tiers above each level give relative values of V_{g1} and V_{g2} from equation (3) in agreement with cantilever or straight-line variation of unit column stresses. If the variation from linear distribution of *unit* column stress is quite marked, the cantilever method of analysis is not useful. For minor variations from linearity, one may make use of the known values of ΣK_{g1} and ΣK_{g2} at any level to determine V_1 and V_2 at that level and compute therefrom the total column stresses. After column stresses are known above and below any girder level, all wind shears and moments of that level may be found from the procedure of Fig. 256.

Limitations on Cantilever Design. Since the vertical shear will increase for girders nearer the center of the bent and this requires increased stiffness of these girders according to equation (4), we can readily see that a bent of many bays could hardly be designed with reasonable economy to function according to the assumption of linearity of column stress. Hence a different assumption should be made in the analysis of a continuous portal of many bays for a low industrial building as contrasted to one of three or four bays. See Fig. 91 and also Fig. 92. In tall buildings the cantilever method fits the design of the slender tower which is likely to be only three or at most four bays wide.

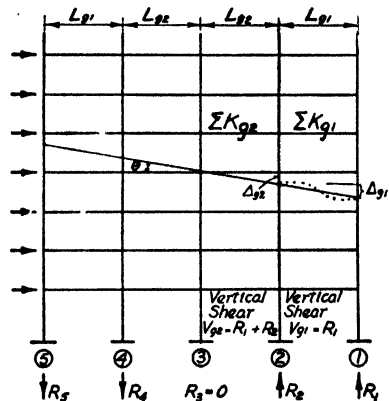
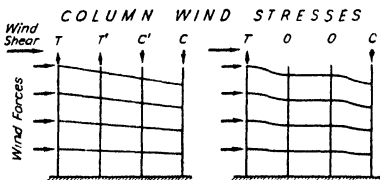


FIG. 257. REQUIREMENT FOR CANTILEVER ACTION, $\Sigma K_g \propto V_g L_g$

A proof of this relationship follows from the straight deformation line of slope θ shown on the illustration. The end moment of girder L_{g1} is $V_{g1}(L_{g1}/2)$ or $V_{g1} \propto M_{g1} \div L_{g1}$. From the double-cantilever deflection of L_{g1} we may write $\Delta_{g1} = M_{g1}L_{g1} \div 6EK_{g1}$, or $\Delta_{g1} \propto V_{g1}L_{g1}^2/K_{g1}$. But from the straight-line relationship $\Delta_{g1} = \theta L_{g1}$, or $\Delta_{g1} \propto L_{g1}$. Hence, by substitution, $V_{g1}L_{g1}^2/K_{g1} \propto L_{g1}$ or $V_{g1} \propto K_{g1}/L_{g1}$. Hence, $I_g \propto V_g L_g^2$.

294. Choice of a Method of Analysis.* The portal method, being the simpler to apply, has found the widest acceptance. Another reason for its common selection is that the calculated girder moments along a single floor line are equal when all bays are of the same width. This makes it possible to use *identical end connections* for all girders on one floor, which is a convenience to the designer and an economy to the owner. The portal method will be most nearly correct when all girders at one level are identical. The Bowman method (equations 1 and 2) should be used when proportions vary from this ideal arrangement. For buildings of height up to 20 stories even when very narrow (unsymmetrical 2-bay design) the Bowman method of § 291 fits the case better than the improved cantilever method of § 293.

Towers. The cantilever method forms a better basis for the design of an extremely tall structure, since it is necessary that the columns shall resist wind stresses in proportion to their distances from the center of the bent *in order to*



(a) Cantilever Action (b) Portal Action

FIG. 258. FLOOR DISTORTION FROM COLUMN DEFORMATION.

maintain floors plane.† If the bracing for an extremely tall building is designed so that interior column stresses are nearly zero, the deformation of the exterior columns will severely distort the exterior girders with resultant excessive flexural stresses due to wind. Probably this effect, as pictured in Fig. 258, is of little importance for buildings of a height-width ratio of less than 5.0.

It is also evident that one way to relieve exterior columns of heavy direct wind stresses is to design the girders so that the interior columns will be forced to resist direct wind stresses. As was shown by equation (4), interior girders then become deeper and stiffer than would be required by $DL + LL$.

PROBLEMS

237. Make use of the portal and cantilever methods to compute the column and girder moments for the lower two stories of the bent shown. Check your results with the values given in Tables 23 and 24 of Bulletin No. 80, Engineering Experiment Station, University of Illinois. Compare the column direct stresses with those given in Table 19 of this bulletin. Data for this bent are also given in Vol. 2 *Theory of Modern Steel Structures*, 2nd Ed., p. 136.

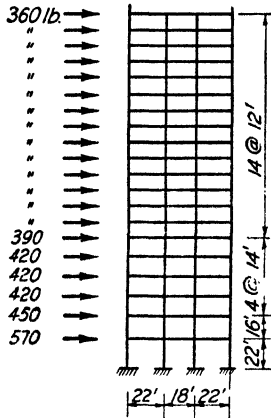
238. Check the moments given, Table 3, p. 937, Transactions A.S.C.E., 1942, against the moments obtained by use of equations (1) and (2) for the lower two stories of the Witmer frame illustrated on p. 927.

* W. M. Wilson and G. A. Maney, *Wind Stresses in the Steel Frames of Office Buildings*, Bulletin No. 80, Engineering Experiment Station, University of Illinois. Albert W. Ross and Clyde T. Morris, *Design of tall building frames to resist wind*, Proceedings, A.S.C.E., May, 1928, pp. 1395-1433.

† Henry V. Spurr, *Wind Bracing*, McGraw-Hill, 1930. This book treats exhaustively the subject of wind bracing for the 1000-ft. tower building.

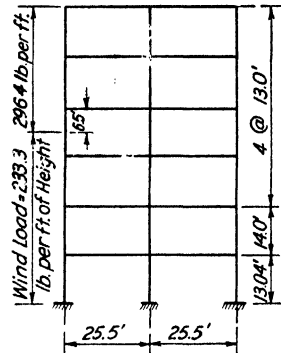
239. Apply the cantilever method to the determination of the direct stresses and moments in the lower five stories of the American Insurance Union Building. See Proceedings, A.S.C.E., May, 1928, p. 395. Check your results against the values from Tables 10 to 13 inclusive. Dimensions and wind loads are given in Fig. 8 and Table 8. Data for this bent are also given in Vol. 2, *Theory of Modern Steel Structures*, 2nd Ed., p. 142.

240. Make use of equations (1) and (2) for analyzing the building frame of Problem 239.



Wilson-Maney Bent

PROBLEM 237.



Coultas and Lawton Frame

PROBLEM 243

241. Make use of equation (4) to determine whether the bent of Problem 237 would obey the assumptions of cantilever analysis.

242. Make use of equation (4) to determine whether the building frame of Problem 239 would obey the assumptions of cantilever analysis.

243. Analyze for the wind moments in the Coultas and Lawton bent shown and compare with the values from Table 11, p. 661, Transactions, A.S.C.E., 1934. You may use either the portal or the cantilever method but check with equations (1) and (2). Data for this bent are also given in Vol. 2, *Theory of Modern Steel Structures*, 2nd Ed., Problem 117, p. 148.

244. Use the cantilever method to compute the wind moments for the frame shown in Fig. 19, p. 664, Transactions, A.S.C.E., 1934, and compare with the true moments shown in parenthesis on Fig. 20. First use equation (4) to determine probable accuracy of cantilever analysis for this frame. Interior columns have 1.5 times the cross-sectional area of wall columns.

RIGIDITY OF MULTI-STORY FRAMES

295. Distribution of Wind Shear to Bents. When the wind bents are all of identical construction and are placed symmetrically across the building, the wind shear is distributed *equally* to the various bents. In the discussion which follows, it will be assumed that the wind bents are placed symmetrically across the building but that all wind frames are *not* of identical construction,

The effect of an unsymmetrical arrangement of wind frames will be discussed in a later section.

It will be assumed in the calculations of lateral deflections that all members *may* be stressed to 15,000 lb. per sq. in. by wind action. In practical design this factor will need to be reduced for those members that are stressed by dead load or live load, such as beams, columns, and diagonal struts that form reactions for beams. From 5000 to 10,000 lb. per sq. in. would be a reasonable allowance for the fiber stress produced by the wind forces in a girder of a tall building. Columns will carry even a smaller unit wind stress except in extremely high towers.

Wind Stress Limited by Permissible Deflections. Based upon the conception of the floors as rigid horizontal plates that do not rotate because of the symmetry of the frame, we conclude that all the wind frames are deflected the same amount. Hence, if the deflections of the various wind frames can be estimated on the assumption that their members will be stressed to the maximum value of 15,000 lb. per sq. in., it is evident that their permissible relative stresses can then be found from the knowledge that all the wind frames must deflect the same amount. The reader may find this conception of *stress being limited by allowable relative deflection* to be new to him. It is always an important factor where members of different stiffnesses are *forced* to act together. As a simple illustration, consider the relative fiber stresses in two crossing beams of the same span but of unequal depths that are forced to deflect or rotate together.*

296. Deflections of Wind Frames — Spurr Method.† In the simple analysis of deflections that follows, it will be assumed that there are no actual changes in the lengths of the columns or girders. Such deformations do occur and they permit sidesway of the wind frames; but these deflections usually are small as compared to the deflections produced by the deformations of the diagonals and knee braces or by the flexure of the columns and girders. The side lurch produced by column deformation becomes important for an extremely tall building such as the Empire State Tower. Such monumental structures would naturally receive special study.

EXAMPLES. Deep Bracing. Three frames with deep bracing are shown in Fig. 259. These frames are similar in that the wind resistance of each is dependent upon one or more diagonals placed at an angle ϕ with the horizontal, as indicated in (a). When such a diagonal is lengthened or shortened by an amount δL , the accompanying side lurch of the frame is $\Delta = \delta L / \cos \phi$. But, since δL is equal to fL/E where f is the wind stress of 15,000 lb. per sq. in. in the diagonal, there is a permissible simplification in the relation $f/E = 1/2000$ which gives rise to the expression of Fig. 259(a)

$$(5) \quad \Delta = \frac{h}{2000 \cos \phi \sin \phi}.$$

* See the author's discussion of a paper on the *Relation of analysis to structural design*, H. Cross, Transactions, A.S.C.E., Vol. 191, pp. 1400-1405.

† Henry V. Spurr, *Wind Bracing*, McGraw-Hill, 1930.

It is apparent that the value of Δ for a diagonally braced frame of a given height is dependent only upon the slope of the diagonals when the working stress is maintained at the assumed value of 15,000 lb. per sq. in. It will be observed from Fig. 259 that the

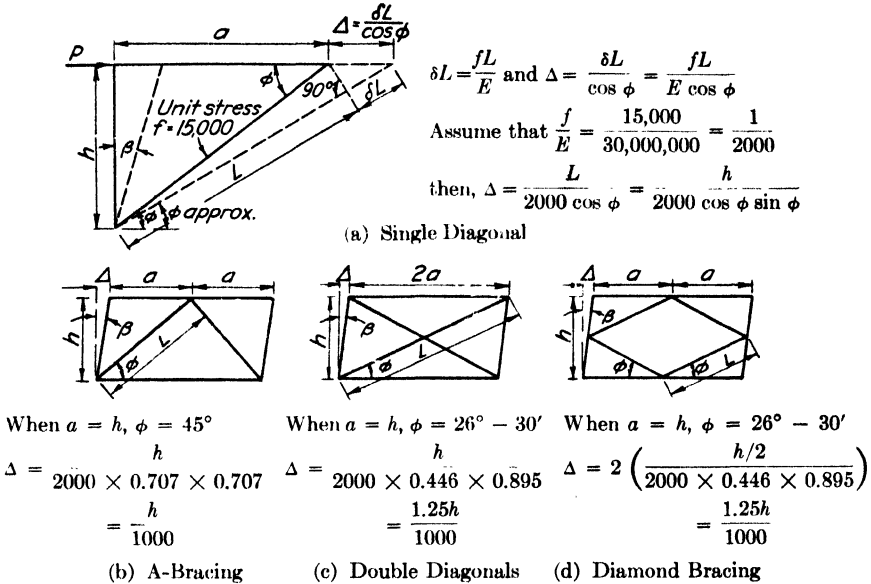


FIG. 259. SIDE LURCH OF DIAGONALLY BRACED WIND FRAMES.

value of Δ varies only from $1.00h/1000$ to $1.25h/1000$ for a reasonable range of ϕ below 45° . From equation (5) we conclude that the value of Δ will be the same for values of ϕ of 30° and 60° or of 20° and 70° , etc.

Shallow Bracing. Figure 260 shows the method of determining the value of Δ for a bent with rigid joints and Fig. 261 illustrates the development of the formulae for the

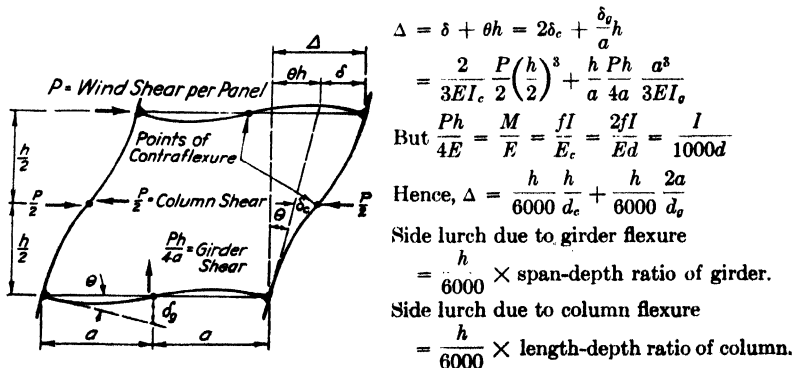
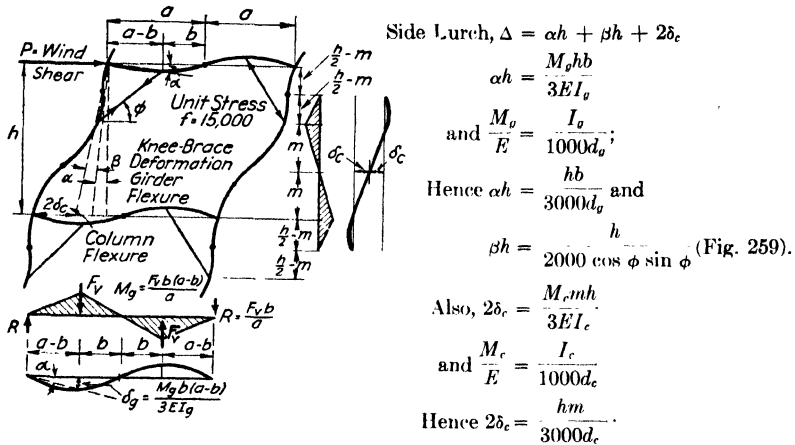


FIG. 260. SIDE LURCH OF A WIND FRAME WITH RIGID JOINTS.

sidesway of *partially braced frames*. Note that the maximum column and girder moments for the frame with rigid joints (Fig. 260) are assumed to be equal, which is approximately true for a regular building frame. If the reader is able to calculate the deflections of ordinary cantilever beams and overhanging beams, he should have no difficulty in following these derivations. Otherwise he should refer to a standard textbook on strength of materials.

297. Lateral Drift from Joint Slip. Another consideration in determining the side lurch of a frame with rigid joints is the probable deflection permitted



(a) Single Knee Braces

$$\text{Therefore, } \Delta = \frac{h}{2000 \cos \phi \sin \phi} + \frac{hb}{3000d_g} + \frac{hm}{3000d_c}$$

Similarly, we derive the expressions given below.

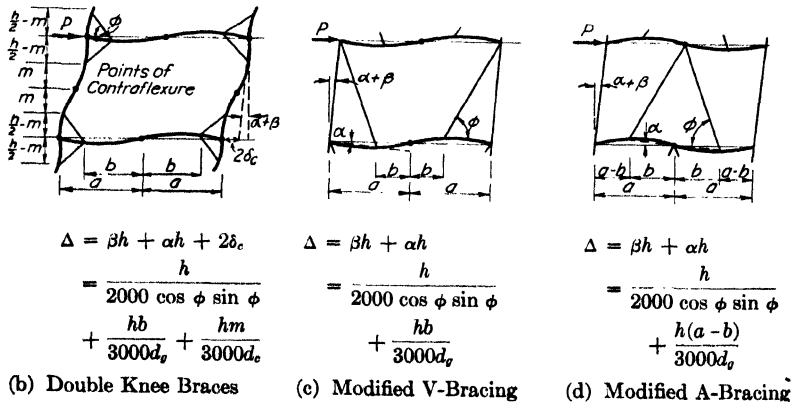


FIG. 261. SIDE LURCH OF PARTIALLY BRACED FRAMES.

by the *elastic deformation* of a welded joint and both the *elastic deformation* and the *slip* of a riveted joint. It has been shown by tests* that well made riveted joints will permit an angular change due to rivet slip of about 0.002 radian at 50% *above* standard working stresses. Such slip would account for a sidesway of $2h/1000$. However, the side lurch produced by elastic deformation at normal working stresses apparently could not exceed the side lurch produced by the deformation of a knee-brace at 45° , or $h/1000$. See Fig. 261(a) or (b). If it is necessary to limit deflection, this sidesway of $h/1000$ can be reduced by overdesign of the connection. Moreover, it has been shown that a proper welded connection to a column web, or to a stayed flange where flange deflection is prevented, will allow no appreciable rotation either from elastic deformation or from joint slip.†

298. Comparison of Sidesway for Different Bents. A comparison of the calculated values of sidesway for ten different wind frames is contained in Table 9. The values of the deflections corresponding to a working stress of 15,000 lb. per sq. in. range from $1.00h/1000$ to $3.67h/1000$. A more practical comparison is obtained from the last two columns of this table where the values of side lurch are calculated for a direct stress of 15,000 lb. per sq. in. and a flexural stress of either 7500 or 5000 lb. per sq. in. When the latter value is used, the range of deflection is from $1.00h/1000$ to $1.89h/1000$, which indicates that we have approached nearer to a balanced design where such frames might reasonably be expected to *work in parallel*. All frames considered in Table 9 are special cases for which $b = a/2 = h/2$ and $h/d_c = 2h/d_g = 8$. The terminology is that of Fig. 259, Fig. 260, and Fig. 261.

A.I.S.C. WORKING STRESSES. Current A.I.S.C. specifications permit a unit stress of 26,670 lb. per sq. in. in any member designed exclusively for wind force and 6670 lb. per sq. in. as the additional wind stress permitted without increase of section in a member designed for $DL + LL$. For Frame No. 7 of Table 9 we may compute the corresponding side lurch as follows:

$$\Delta = \frac{26,670}{15,000} \times \frac{1.00h}{1000} + \frac{6,670}{15,000} \left(\frac{0.67h}{1000} + \frac{0.67h}{1000} \right)$$

$$\Delta = \frac{1.78h}{1000} + \frac{0.59h}{1000} = \frac{2.37h}{1000}.$$

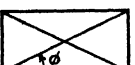
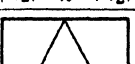
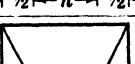
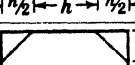
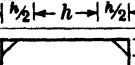
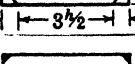
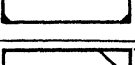
In like manner all data of Table 9 may be adjusted for any desired working stresses.

299. Wind-Shear Distribution to Unlike Bents. The data of Table 9 will be applied to the solution of a practical problem. In the plan view of Fig. 262, the placement of wind bents is as shown by the heavy lines.

* W. M. Wilson and H. F. Moore, *Tests to Determine the Rigidity of Riveted Joints of Steel Structures*, Bulletin No. 104, University of Illinois Engineering Experiment Station.

† C. R. Young and J. B. Jackson, *The relative rigidity of welded and riveted connections*, Canadian Journal of Research, July, 1934.

TABLE 9
COMPARISON OF THE SIDE LURCH OF TYPICAL FRAMES

No.	TYPE $b = \frac{a}{2} = \frac{h}{2}$ $\frac{h}{d_c} = \frac{2h}{d_o} = 8$	DIRECT STRESS	GIRDER FLEXURE	COLUMN FLEXURE	ELASTIC DEF. OF CONNECTIONS	MAXIMUM SIDE LURCH	SIDE LURCH WITH FLEXURE STRESSES REDUCED BY	
		Stress =	15,000 lb.	persq.in.*			50%	66.6%
1		$\frac{h}{1000}$	—	—	negligible	$\frac{h}{1000}$	$\frac{h}{1000}$	$\frac{h}{1000}$
2		$\frac{1.25h}{1000}$	—	—	"	$\frac{1.25h}{1000}$	$\frac{1.25h}{1000}$	$\frac{1.25h}{1000}$
3		$\frac{1.25h}{1000}$	—	—	"	$\frac{1.25h}{1000}$	$\frac{1.25h}{1000}$	$\frac{1.25h}{1000}$
4		$\frac{1.00h}{1000}$	$\frac{0.67h}{1000}$	—	"	$\frac{1.67h}{1000}$	$\frac{1.33h}{1000}$	$\frac{1.22h}{1000}$
5†		$\frac{1.25h}{1000}$	$\frac{0.67h}{1000}$	—	"	$\frac{1.92h}{1000}$	$\frac{1.58h}{1000}$	$\frac{1.47h}{1000}$
6		$\frac{1.25h}{1000}$	$\frac{0.67h}{1000}$	—	"	$\frac{1.92h}{1000}$	$\frac{1.58h}{1000}$	$\frac{1.47h}{1000}$
7		$\frac{1.00h}{1000}$	$\frac{0.67h}{1000}$	$\frac{0.67h}{1000}$	"	$\frac{2.33h}{1000}$	$\frac{1.67h}{1000}$	$\frac{1.44h}{1000}$
8		$\frac{1.00h}{1000}$	$\frac{1.00h}{1000}$	$\frac{0.67h}{1000}$	"	$\frac{2.67h}{1000}$	$\frac{1.83h}{1000}$	$\frac{1.55h}{1000}$
9†		—	$\frac{1.33h}{1000}$	$\frac{1.33h}{1000}$	$\frac{1.00h}{1000}$	$\frac{3.67h}{1000}$	$\frac{1.88h}{1000}$	$\frac{1.22h}{1000}$
10		$\frac{1.00h}{1000}$	$\frac{2.00h}{1000}$	$\frac{0.67h}{1000}$	negligible	$\frac{3.67h}{1000}$	$\frac{2.33h}{1000}$	$\frac{1.89h}{1000}$

* For a working stress more or less than 15,000 lb. per sq. in. each contribution to the side lurch is to be multiplied by the appropriate working stress and divided by 15,000.

† Note that all frames except No. 9 are assumed to have pin-connected joints.

EXAMPLE. The building extends 24 stories or 240 ft. *above* the story to be studied. Hence the total wind shear for a wind pressure of 25 lb. per sq. ft. is $25 \times 120 \times 245 = 735,000$ lb. The column sections shown are assumed to be stressed to their full allowable value of 16,700 lb. per sq. in. by dead load and live load. They can not be stressed above 5570 lb. per sq. in. by the wind load unless they are increased in size. The combined dead-load and live-load girder stresses are 19,800 lb. per sq. in. in *OP*, 18,300 in *ST*, and but 6900 in *NO*. Each diagonal strut or knee brace in the bents *AB* and *GH* carries a total stress from dead load and live load of about 21,000 lb. These are approximate figures based

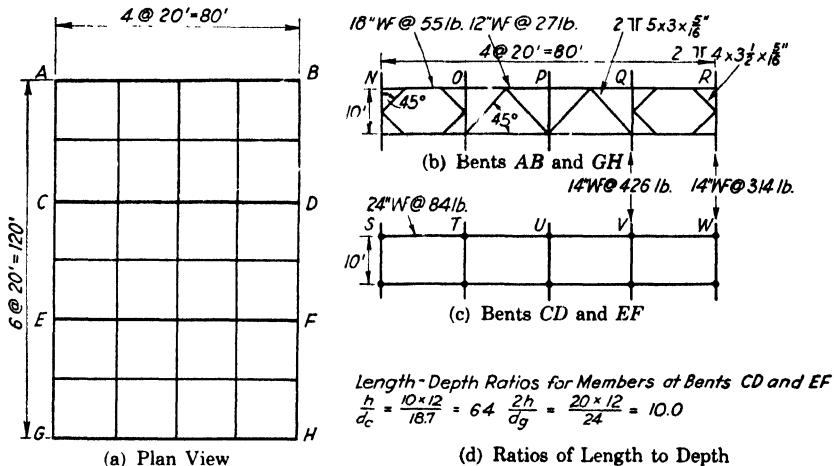


FIG. 262. WIND-SHEAR DISTRIBUTION—SYMMETRICALLY PLACED WIND FRAMES.

upon girder loads of 4500 and 6000 lb. per lineal ft. for wall frames and for interior frames, respectively. By turning the long legs of the diagonal struts outward and permitting the masonry wall to prevent buckling in its plane, the L/r reduction of the strut by the column formula will be quite small.

Bays *NO* and *QR* with Knee Braces. If it is desired to restrict the side lurch of the bays *OP* and *PQ* to $1.00h/1000^*$, the diagonals can be stressed to only 15,000 lb. per sq. in. by wind. See Frame No. 1 of Table 9. Since these are the stiffest bays, all other bays must be designed to deflect the *same amount*. At a stress of 15,000 lb. per sq. in. in the diagonals and the girder, bay *NO* would deflect $1.00h/1000$ because of the *knee-brace deformation* and $(h/3000)3.33$ from *girder deflection*, a total of $2.11h/1000$. This follows from Fig. 261(b) by letting the last term equal zero. In order to reduce this value to $1.00h/1000$, the wind stresses would need to be reduced to $15,000 \div 2.11 = 7100$ lb. per sq. in. The 18-in., 55-lb. girder can carry a bending moment of but 696,000 in.-lb. at this stress, from which it is possible to compute the allowable vertical component of the stress in the knee brace, F_v in Fig. 261(a), which is 23,200 lb.

$$F_v = \frac{M_g a}{b(a - b)} = \frac{696,000 \times 120}{60 \times 60} = 23,200 \text{ lb.}$$

* Of course this is rather a severe restriction on sidesway or drift which is usually limited to twice this value or $0.002h$.

With the knee brace at 45° , this value of 23,200 lb. also becomes the horizontal component of the knee-brace stress or one-half the wind shear resisted by the bay *NO*. In the entire building there are four such bays which will resist a total wind shear of 186,000 lb. The two $4 \times 3\frac{1}{2} \times \frac{5}{16}$ -in. angles of the knee brace are stressed to $(23,200 \times 1.41) \div 4.50 = 7300$ lb. per sq. in. This is a satisfactory value when compared with the permissible stress of 7100.

Girder Stress NO. The dead- and live-load stress in this girder was 6900 lb. per sq. in. based upon a 10-ft. span. The wind stress is $15,000 \div 2.11 = 7100$ lb. per sq. in. It is therefore understressed if full reaction support is obtained from the knee braces. A reduction in its size should be investigated.

Bents CD and EF with Knuckle Connections. In the study of the bents *CD* and *EF*, the calculation of the wind stresses will be made to depend upon the portal method of analysis. Hence the moment for an interior column will be nearly the same as the moment for an interior girder, and their relative fiber stresses will be in proportion to their section moduli, or as 196/707 or 1/3.6. If we assume the allowable wind stress to be 15,000 lb. per sq. in. for the girder, we find the corresponding flexure stress in the column to be 4200 lb. per sq. in. The side lurch produced by these flexural stresses would be found by summing the contributions of *columns*, *girders*, and *joints* from Fig. 260 and Table 9 as follows:

$$\Delta = \frac{h}{6000} \times \frac{6.4}{3.6} + \frac{h}{6000} \times 10.0 + \frac{h}{1000} = \frac{2.96h}{1000},$$

where the factor 6.4 is the *height-depth* ratio of the column and the factor 10.0 is the *span-depth* ratio of the girder.

From this calculation, it becomes clear that all stresses in the bents with stiff joints must be divided by 2.96 to reduce the sway to $1.00h/1000$. It is assumed that the wind connection is designed to develop nearly the full moment resistance of the girder, so that its contribution to the side lurch will be reduced with reduction of wind moment.

A more exacting study would take account of the action of the outside columns. However, we will accept the usual assumption that their smaller sections and reduced wind shears permit them to act the same as the interior columns. At a working stress of $4200/2.96$ or 1420 lb. per sq. in., the moment resistance of an interior column is 1420×707 or 1,000,000 in-lb. Its shear would be $1,000,000 \div (5 \times 12)$ or 16,700 lb. if we suppose that the point of zero moment is at its mid-height. Note that here as elsewhere theoretical lengths rather than clear depth or clear span are used. The designer is free to make consistent reductions in lever arms for moment as justified by the joint design. The shear resisted by three interior columns and two exterior columns in one bent would be $4 \times 16,700 = 67,000$ lb. A total shear of 134,000 lb. could be resisted by the two interior wind frames.

Girder Stress ST. The dead- and live-load stress in this girder was 18,300 lb. per sq. in., based upon a 20-ft. span. The wind stress is $15,000 \div 2.96 = 5100$ lb. per sq. in. The girder is therefore understressed by a small amount, and a reduction to the next lighter weight could be studied.

Bays OP and PQ with Diagonal Bracing. The total shear resisted by the two interior wind frames plus the exterior bays *NO* and *QR* of the two end or wall frames *AB* and *GH* is $134,000 + 186,000 = 320,000$ lb. The remainder of the wind shear is $735,000 - 320,000 = 415,000$ lb. This wind force will be divided equally between four bays identical with *OP*, or about 104,000 lb. per bay. The stress in one diagonal will be $(1.41 \times 104,000) \div 2 = 74,000$ lb. The unit wind stress in the two-angle strut ($5 \times 3 \times \frac{5}{16}$ -in. angles) is $74,000 \div 4.80 = 15,400$ lb. per sq. in., a value sufficiently close to 15,000 to assure a sideways of approximately $h/1000$. The dead-load and live-load stress of 21,000 lb. in the strut will

produce a unit stress of 4400 lb. per sq. in., thus making its total stress 19,800 lb. per sq. in. The two-angle strut should be fabricated with the 5-in. legs outstanding so that its effective radius of gyration will be 2.47 when the angles are separated by a $\frac{3}{8}$ -in. plate. It is adequately stiffened in the plane of the wall by the masonry. An investigation of this strut by A.I.S.C. specifications will show that it is capable of resisting a *combined* compressive stress of more than 19,800 lb. per sq. in.

Girder Stress OP. This girder carried a dead- and live-load stress of 19,800 lb. per sq. in. Its only wind stress is due to the horizontal components of the stresses in two intersecting diagonals which becomes a horizontal axial load on the girder. The girder should be investigated, and in this case it should be increased to the 12-in., 36-lb. section, to resist the axial force.

Column Stresses. Direct column stresses can be computed separately for each bay. A comparison of column stresses may be based upon an overturning moment equal to the wind shear times a height of 125 ft. from the center of the wind pressure

$$\text{For column } N, \text{ the direct stress} = \frac{23,200 \times 2 \times 125}{20 \times 92.3} = 3130 \text{ lb. per sq. in.}$$

$$\text{For column } S, \text{ the direct stress} = \frac{16,700 \times 4 \times 125}{20 \times 4 \times 92.3} = 1130 \text{ lb. per sq. in.}$$

$$\text{Wind flexure} \dots \dots \dots = 1420$$

$$\text{Total for Column } S \dots \dots \dots = 2550 \text{ lb. per sq. in.}$$

For column *O*, the direct stress is a compression from bay *OP* minus a tension

$$\begin{aligned} \text{from bay } NO &= \frac{104,000 \times 125}{20 \times 125.2} - \frac{23,200 \times 2 \times 125}{20 \times 125.2} \\ &= 5200 - 2300 = 3900 \text{ lb. per sq. in.} \end{aligned}$$

In all cases these wind stresses are less than the allowable stress of $0.33(DL + LL) = 0.33 \times 16,700 = 5570$ lb. per sq. in. Hence column sections will not be increased.

300. A Study of Proper Proportions for Wind Frames. The problem studied in the previous section was the division of the wind shear between bents made up of *known sections*. The selection of *proper sections* is more difficult. The columns are to be looked upon as fixed sections, since there is little probability that an unnecessary increase in column section to help resist wind flexure will prove economical. Therefore the column sections, except for high towers, are controlled by dead load and live load. Girders in bents with deep diagonal bracing will be designed for the dead-load and live-load moments, but, with knee braces or knuckle connections, the girders will need to be increased in section to resist wind stresses. Diagonals and knee braces will be selected to work at a stress of about 15,000 lb. per sq. in. in tension or compression unless it is necessary to reduce this stress in order to decrease the sidesway, to allow for the dead-load and live-load stresses, or because of the slenderness ratio of the diagonal strut. If sidesway must be reduced by increase of column and girder sections, it is necessary to increase both since the doubling of the stiffness of either columns or girders alone was found to reduce the lateral drift only 10 per cent.*

* Transactions, A.S.C.E., Vol. 107, 1942, p. 953.

EXAMPLE. *Unlike Bents Used in Parallel.* It should be clear from the example studied that the use of diagonally braced bents in parallel with frames having only knuckle connections will seldom result in an economical design. The stresses in the bents with knuckle connections must be reduced so low in order to maintain the small deflection which is permissible for the diagonally braced frames that the wind resistance of the former is comparatively small. Since knee-brace construction is *intermediate in stiffness* between shallow connections and deep diagonal bracing, it may be used with either. Diagonally braced bents could be made less rigid by the use of flexible end connections for the diagonals. On the other hand, wind frames with knuckle connections will be increased in stiffness by the concrete fireproofing. Either effect might produce a 25% change in stiffness, which certainly would require consideration.

Arrangement of Bracing. Evidently there are innumerable arrangements of wind bracing that might prove satisfactory to resist the wind loads acting on the building of Fig. 262. For instance, the girders of the interior wind frames might be increased in size, or the interior wind frames might be used on more column lines. In either case, the result would be to relieve the end frames. It would be possible also to increase the weights and depths of the girders *NO* and *QR* of the wall frames to provide for a larger shear resistance in these bays with a corresponding reduction in shear for the bays *OP* and *PQ*.

In any case, the problem consists of two studies: first, to arrange for equal sidesway in all bents, and second, to add as little material as possible to the column and girder sections that are required to resist *the dead load and the live load*. Note the wide variation in the weights of girders *NO*, *OP* and *ST*. This makes clear the fact that direct or inverted V-type wind bracing will in many cases add little or no cost to the frame and may even result in a *saving of weight* due to the great reduction in the weight of the girders. Of course, V-bracing can not be used where large clear openings are necessary.

301. Unsymmetrically Placed Wind Bracing. It is not always possible to place wind frames symmetrically across the building or to make the center of resistance of the wind bracing line up with the point of application of the wind load. The result is a *twisting action* that may prove extremely serious. This problem is most likely to be encountered where a tower is placed unsymmetrically on its base, although there is always the probability that the wind will itself produce a torque moment in the structure because of its *natural swirling action*. Common specifications do not seem to consider this possibility.

Effect of a Torque Moment. Again it will be assumed that a floor forms a rigid horizontal plate which will not be distorted by the wind forces, but which will *rotate and translate as a unit*. It follows that a torque moment applied above the floor will produce a simple rotation which will deflect the panels of wind bracing in proportion to their respective distances from the center of rotation measured at right angles to their respective planes. Such perpendicular distances are used in place of radii because a wind frame can not resist shear perpendicular to its plane.

Hence we may compute the wind shears that an applied torque will produce in the individual bents by use of the torsion formula, $S_s = Tr/J$. This formula will be rewritten in the form

$$(6) \quad F_1 = R_1 \frac{Tz_1}{\sum R_1 z_1^2}$$

where F_1 is the total shear resisted by bent No. 1, T is the torque moment of the wind force, z_1 is the perpendicular distance in the proper x or y direction of bent No. 1 from the center of rotation, and R_1 is a number representing the relative value of the shear resistance of bent No. 1.

Definition of Shear Resistance. Any bent can be designated as having a unit resistance R_0 . Then the other bents will be represented by other numbers depending upon the ratio of their respective resistances to deflection to the resistance of the standard unit bent. Resistance to deflection can be defined as the reciprocal of the deflection caused by a unit shear in the panel under consideration. The data from Fig. 259, Fig. 260, Fig. 261, and from Table 9 can be used to determine R . However, we should revise the Δ -contribution of each member by the introduction of its true stress caused by a unit shear in place of the assumed stress of 15,000 lb. per sq. in. To maintain reasonable figures, it would be preferable to use 10,000 lb. rather than 1 lb. as a unit of shear. This is permissible for calculating *relative R-values*.

302. Shears Produced by Translation plus Rotation. For simplicity, it is advisable to separate the translation in the direction of the wind pressure, which introduces wind shears proportional to the R -values, from the rotational effect controlled by the torsion formula. Then the center of rotation for pure torsion will be at a point defined by the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

APPLICATION. The center of rotation is located at Z in Fig. 263(a), where bents are assumed to have equal R -values. Evidently the wind diagonals in the 20-ft. bays will necessarily have different areas of cross-section than the diagonals in the 15-ft. bays if *equal deflections* are to be maintained under *unit shears*. The effect of the slope of the diagonal upon sidesway can be determined from the following formula for the deflection of a bent braced by diagonals and acted upon by a 1-lb. horizontal shear. We make use of the data of Fig. 259(a) to write for a stress f in the diagonal of 1-lb./ $\cos \phi$

$$(7) \quad \Delta_{1\text{-lb.}} = \frac{fL}{E \cos \phi} = \frac{h}{AE \sin \phi \cos^2 \phi} = \frac{hf}{E \sin \phi \cos \phi}.$$

The center of rotation in Fig. 263(b) lies at the intersection of perpendicular lines drawn through the centers of resistance of parallel bents. With this observation in mind, we have no difficulty in making the calculations for the wind-shear distribution, as shown in (a) and (b) of Fig. 263. Each individual shear produced by a translation plus a rotation may be calculated from the combined-stress formula

$$(8) \quad F_1 = \frac{R_1 W}{\Sigma R} \pm R_1 \frac{T z_1}{\Sigma R z^2}.$$

In applying this formula, we determine the signs by inspection. Note $z = x$ or y .

303. Effect of Unequal Resistances of Bents.* It will be noted that a considerable variation in wind-shear distribution exists between the two cases (a) and (b) of Fig. 263. In (a), all the bents must be equally stiff, which means

* This problem is one of the most important in the design of tall buildings; yet it has received little attention. It deserves special study.

that all must be designed to be as *rigid* as the most heavily stressed bent which carries the shear F_2 of $0.68W$. There is an apparent improvement in (b), where the bent F_2 must be designed to resist $0.84W$ and the bent F_3 must be equally rigid, but where the bents F_1 and F_4 can be designed for a stiffness of but 33% of the stiffness of F_2 or F_3 . The increased weight of bracing in the bents F_2 and F_3 should be more than compensated for by a decreased weight in

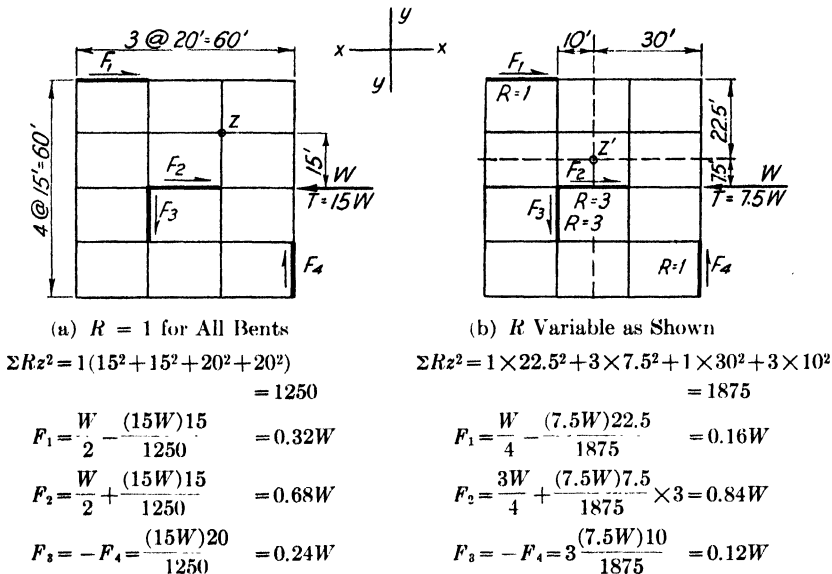


FIG. 263. DISTRIBUTION OF WIND SHEAR PRODUCED BY A TORQUE MOMENT. ($z = x$ or y .)

the bents F_1 and F_4 , which allows a probable saving over the requirements of case (a). Whether the design of bents of relative rigidities of 3 and 1 would be practical is a controlling factor, but in any case the calculations of Fig. 263 illustrate a cut-and-try method that can be applied to the economical design of wind bracing to resist torsion.

PROBLEMS

245. Revise the calculations for shear distribution of the building of Fig. 262 on the assumption that the interior frames such as CD and EF are used on all interior lines of columns. Knee braces and diagonals remain unchanged. On the basis of your calculations suggest how the design of this wind bracing might be improved. Permit the drift to equal $0.002h$.

246. Assume that a third wind frame similar to AB and GH is added on the center line of the building of Fig. 262. (Omit all rigid connections between columns and girders.) Determine the revision in size of the girders NO and QR and the revision in sizes of knee

braces and diagonals in order to make all the frames work together in resisting the wind shear. Note that the slenderness ratio of the long diagonal is an important factor. Permit the drift to equal $0.0015h$.

247. Calculate the maximum shear resisted by a single bay for the building of Fig. 263(b) if the R -values of the bays No. 2 and No. 3 are reduced from 3 to 2.

Ans. $0.79W$.

248. Repeat the calculations of Fig. 263 for the effect of wind pressure at 90° to the direction shown.

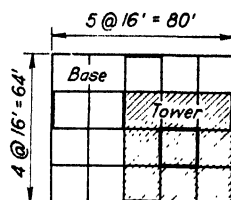
Ans. Case (a) $F_2 = -F_1 = 0.12W$; $F_3 = 0.66W$; $F_4 = 0.34W$.

249. The base of a building is 80×64 ft. in plan and 160 ft. high above the ground. There is a tower 48×48 ft. in plan rising 180 ft. above the base without set-backs. Diagonal bracing is placed between columns as indicated by the heavy lines on the plan of the base. If the wind pressure is 20 lb. per sq. ft. and all tiers of bracing are of the same stiffness, compute the shear resisted by each tier at the ground level. Take the wind pressure as acting on the 80-ft. face.

Ans. Maximum shear per bent = 90,000 lb.

250. Same as Problem 249, but give each bay of wind bracing parallel to the wind an R -value of 1.5 and each of the other bays an R -value of 1.0.

Ans. Maximum shear per bent = 90,500 lb.



PROBLEM 249.

304. The Study of Indeterminate Structures by Statics. A number of indeterminate structures have been analyzed in this book by statics with the aid of assumptions. The most complex is the wind-stress analysis of tall building frames, but we have met similar problems in simple bent analysis. Since the use of statics is an exact procedure it neither improves upon nor reduces the accuracy inherent in the original assumptions. Mathematics performs its function in the same manner. Therefore, we must always critically study the basic assumptions made in any analysis to form an idea of the usefulness of the results obtained. This observation applies in some measure to all analyses because we can hardly perform any stress analysis without the aid of assumptions. Hook's Law is an assumption that fortunately is almost exactly true for normal working stresses in steel, but Hook's Law is quite a crude assumption when applied to many materials. The treatment of supports as knife edges and of truss joints as pins are always imperfect and sometimes dangerous. The assumption that foundations neither move nor rotate is often untrue. The assumption of uniform distribution of floor loads bears only a slight resemblance to reality, while wind pressure fluctuates so rapidly from pressure to vacuum and back that it is fortunate for our sense of equilibrium that the phenomena is invisible.

Engineering is not an exact science but is the *application* of science based upon trained judgment. It is not entirely meritorious in engineering to have performed an exact analysis based upon a long list of crude assumptions. If you have to admit that your assumptions may be in error by 30 per cent, the use of an exact method of analysis from indeterminate structures cannot re-

duce this 30 per cent approximation. On the other hand, if you substitute a crude method of analysis that is itself known to involve errors of 20 per cent, may not the two inaccuracies combine and make some of your results wholly unacceptable?

As a procedure of common sense, engineers should set down their assumptions and estimate the probable inaccuracy involved. Then the method of analysis used should be chosen with the thought that exactness is not necessary but that additional errors of more than a few per cent should not be superimposed upon the inherent inaccuracy of the initial assumptions. For wind-stress analysis the original assumptions about the loads themselves are necessarily so conventionalized and so far from probable reality that no one could feel much interest in a method of computing wind stresses closer than ± 10 per cent to the mathematically *exact* values. However, the statically determinate methods given in this chapter will produce much greater inaccuracies than 10 per cent wherever an irregularity exists in the building. Hence, improved methods from the theory of indeterminate structures will be needed in such cases.

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